Effect of Negative Permittivity and Permeability in The Transmission of Electromagnetic Waves Through a Left-Handed Material Waveguide

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Abstract—**We investigate the characteristics of electromagnetic wave transmission by multilayered structures consisting of a pair of left-handed material (LHM) and dielectric slabs inserted between two semi-infinite dielectric media. The theoretical aspect is based on Maxwell's equations and matching the boundary conditions for the electric and magnetic fields of the incident waves at each layer interface. We calculate the reflected and transmitted powers of the multilayered structure taking into account the widths of the slabs and the frequency dependence of permittivity and permeability of the LHM. The obtained results satisfy the law of conservation of energy. We show that if the semi-infinite dielectric media have the same refractive index and the slabs have the same width, then the reflected power can be minimized and the transmittance-frequency curve shows no ripple. On the other hand if the semi-infinite dielectric media have different values of refractive indices and the slabs have different widths, then under certain conditions the reflected power can be maximized.**

Keywords-electromagnetic waves; left-handed material; frequency; transmitted power

I. INTRODUCTION

Metamaterials (sometimes termed left-handed materials (LHMs)) are materials whose permittivity ε and permeability μ are both negative and consequently have negative index of refraction. These materials are artificial and theoretically discussed first by Veselago [1] over 40 years ago. The first realization of such materials, consisting of split-ring resenators (SRSs) and continuous wires, was first introduced by Pendry [2, 3]. Regular materials are materials whose ε and μ are both positive and termed right handed materials (RHMs). R. A. Shelby et al [4] have studied negative refraction in LHMs. I. V. Shadrivov [5] has investigated nonlinear guided waves in LHMs. N. Garcia et al [6] have shown that LHMs don't make a perfect lens. Kong [7] has provided a general formulation for the electromagnetic wave interaction with stratified metamaterial structures. M. M. Shabat et al [8] has discussed Nonlinear TE surface waves in a left-handed material and magnetic super lattice waveguide structure. I. Kourakis et al [9] has investigated a nonlinear propagation of electromagnetic waves in negative–refraction index LHM. H. Cory et al [10] and C. Sabah et al [11] have estimated high reflection coatings of multilayered structure. Oraizi et al [12] have obtained a zero reflection from multilayered metamaterial structures.

In this paper we consider a structure consisting of LHM and dielectric slabs inserted between two semi-infinite dielectric media. A plane polarized wave is obliquely incident on it. We use Maxwell's equations and match the boundary conditions for the electric and magnetic fields of the incident waves at each layer interface. Then we solve the obtained equations for the unknown parameters to calculate the reflection and transmission coefficients. We take into account the frequency dependence of permittivity and permeability of the LHM (in contradict with [10, 11]), widths of the slabs, refractive indices of the media, angle of incidence of the incident waves. Maximum and minimum transmitted power of the considered structure is proposed. It is found that the obtained results satisfy the law of conservation of energy given by [10, 13].

II. THEORY

We consider four regions each with permittivity ε_ℓ and permeability μ_ℓ , where ℓ represents the region order (see Fig. 1).

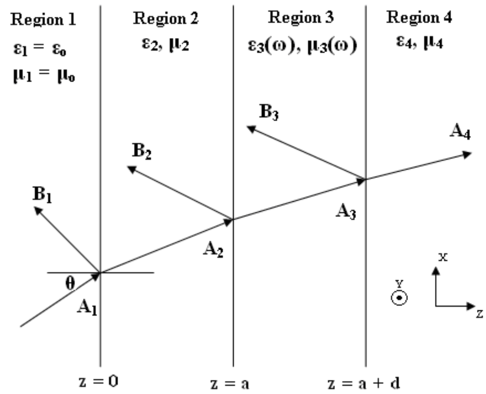

Figure 1. Wave propagation through a structure consisting of a pair of dielectric and metamaterial embedded between two dielectric semi-infinite media.

The electric field in each region is [7, 10]:

$$\vec{E}_\ell = \left(A_\ell e^{ik_{\ell z}z} + B_\ell e^{-ik_{\ell z}z}\right)e^{i(k_{\ell x}x-\omega t)}\ \hat{y} \qquad (1)$$

We use Maxwell's equation as done by [14] to find the corresponding magnetic field $\vec{H}_\ell$:

$$\vec{H}_\ell = \frac{1}{\mu_\ell \omega}\begin{bmatrix}(A_\ell k_{\ell x} e^{ik_{\ell z}z} + B_\ell k_{\ell x} e^{-ik_{\ell z}z})\hat{z} + \\ (-A_\ell k_{\ell z} e^{ik_{\ell z}z} + B_\ell k_{\ell z} e^{-ik_{\ell z}z})\hat{x}\end{bmatrix} e^{i(k_{\ell x}x-\omega t)} \qquad (2)$$

Where A_ℓ and B_ℓ are the amplitudes of forward and backward traveling waves. k_ℓ is the wave vector inside the material. Matching the boundary conditions for $\vec{E}$ and $\vec{H}$ fields at each layer interface, that is at z=0, $E_1 = E_2$ and $H_1 = H_2$ and so on. This yields six equations with six unknown parameters [10, 12, 14]. Letting A_1=1 and solving the obtained equations for the unknown parameters enables us to calculate the reflection and transmission coefficients B_1 and A_4 [10, 14]. The reflected power R and the transmitted power T are given by [10,14]:

$$R = B_1 B_1^*\ ,\ T = A_4 A_4^* \qquad (3)$$

Where B_1^* and A_4^* are the complex conjugate of B_1 and A_4 respectively.

The law of conservation of energy is given by [10, 13]:

$$R + \left(\frac{k_{4z}}{k_{1z}}\right)T = 1 \qquad (4)$$

III. NUMERICAL RESULTS AND APPLICATIONS

For the LHM in region 3 we employ a non-dispersive one with ε_3 and μ_3 appeared in [2, 3, 14]:

$$\varepsilon_3(\omega) = 1 - \frac{F_e \omega_{ep}^2}{\omega^2 - \omega_{eo}^2 + i\gamma_e \omega} \qquad (5)$$

$$\mu_3(\omega) = 1 - \frac{F_m \omega_{mp}^2}{\omega^2 - \omega_{mo}^2 + i\gamma_m \omega} \qquad (6)$$

where ω_{ep} and ω_{mp} are the electric and magnetic plasma frequencies, ω_{eo} and ω_{mo} are the electric and magnetic resonance frequencies. F_e and F_m are the scaling filling parameters. We have used the following parameters appearing in [14]:

$\omega_{mp} = 2\pi 10.95 GHz$, $\omega_{mo} = 2\pi 10.1 GHz$, $F_m = .26$, $\omega_{ep} = 2\pi 13.3 GHz$, $\omega_{eo} = 2\pi 10.3 GHz$, $F_e = .37$, with no loss case i.e. $\gamma_e = \gamma_m = 0$. In this case, the frequency range in which $\varepsilon_3(\omega)$ and $\mu_3(\omega)$ are negative extends from 10.3 up to 11.5 GHz.. T and R are calculated numerically as stated above. The transmitted power is plotted as a function of ω under different conditions as follows:

- The dependence of μ_3 on ω for the metamaterial in Region 3 is taken into account [2-5, 14-16], it is considered by [10, 11] to be $\mu_3 = -\mu_o$. The difference between the two cases can be noticed from Fig. 2.
- The thickness of the slabs is taken to be the same a = d, $n_2 = |n_3|$, $n_1 = n_4$, θ is kept constant. In this case the structure's transmitted power variation with frequency is smooth and shows no ripples as shown in Fig. 3.

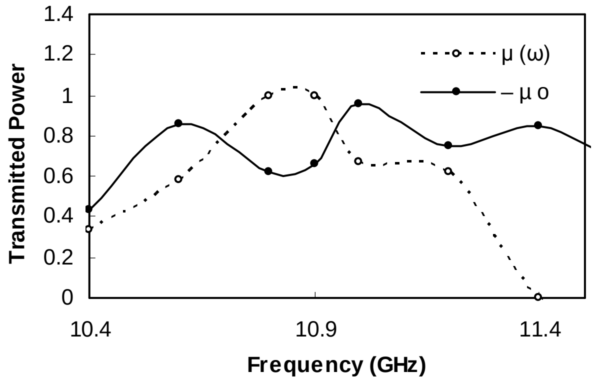

Figure2. Transmitted power variation with frequency. Two cases are taken into account: (a) μ_3 is a function of ω; (b) μ_3 is a constant ($-\mu_0$).

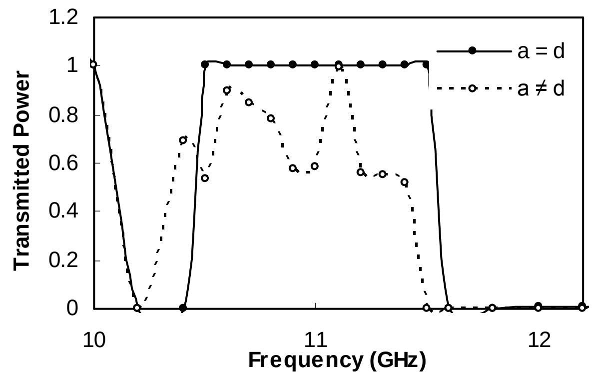

Figure 3. Transmitted power variation with frequency when $n_1 = n_4$, $n_2 = |n_3|$ and θ kept constant for two cases with respect to the widths a and d of the slabs: (a) a=b ; (b) a ≠ d.

- High transmitted power can be achieved if $n_1 = n_4$, $n_2 = |n_3|$ and a = d [10]. In this case R = 0 (minimum) and T = 1 (maximum) for any frequency and for any angle of incidence (Fig. 4). If $n_1 \neq n_4$

(and a = d, $n_2 = |n_3|$), then both R and T depend on the values of the refractive indices of the initial and final media n_1 and n_4, and on the angle of incidence θ see Figs. 5 and 6.

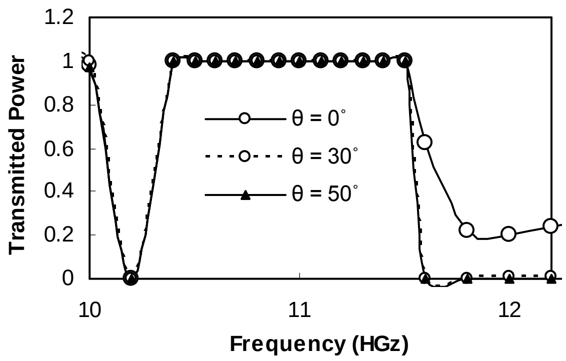

Figure 4. The transmitted power as a function of frequency when $n_2 = |n_3|$, $n_1 = n_4$ and a = d for various angles of incidence.

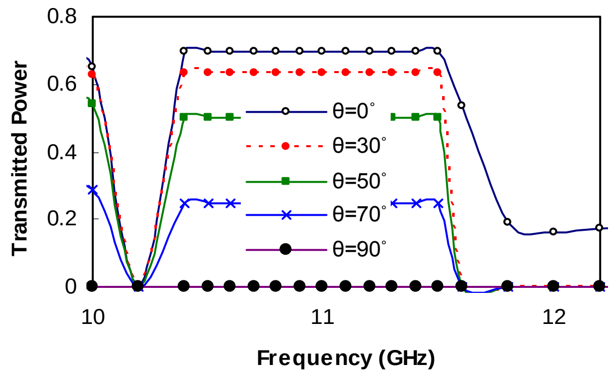

Figure 5. The transmitted power as a function of frequency when $n_1 \neq n_4$, $n_2 = |n_3|$ and a = d for various angles of incidence.

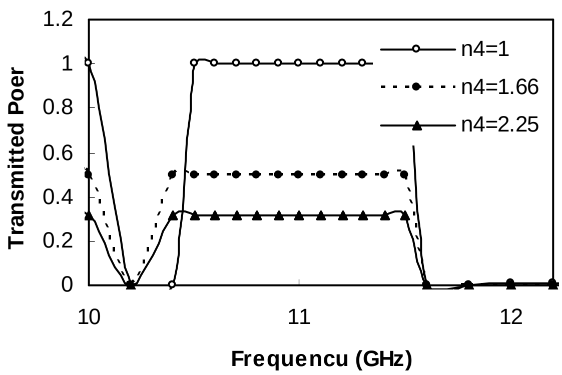

Figure 6. The transmitted power as a function of frequency when $n_1 \neq n_4$, $n_2 = |n_3|$, a = d and the angle of incidence θ = 30˚ (kept constant) for different values of n_4.

- Low transmitted power: in order to minimize T and maximize R, one has to choose a pair of adjacent dielectric and metamaterial slabs with highly contrasted refractive indices ($n_1 < n_2$, $n_2 > n_3$, $n_3 < n_4$) and $n_2 a = n_3 d = c\pi / \omega_o$ [10], where ω_o is the central frequency (ω_o = 10.9 GHz). In this case by a judicious combination of metamaterial and dielectric slabs, a low-transmitted power is achieved, for which the dependence of T on frequency and on the angle of incidence is consequently diminished (Fig. 7).

Note that the maximum value of T is .09 at 0^o. This value is smaller by a factor of 11 than that obtained in Fig. 4. Fig. 7 is different from that obtained by [10, 11] in the fact that, they had used a LHM with properties invariant with frequency. In our paper, the properties of the LHM in Region 3 depends on frequency (refer to (5) and (6)).

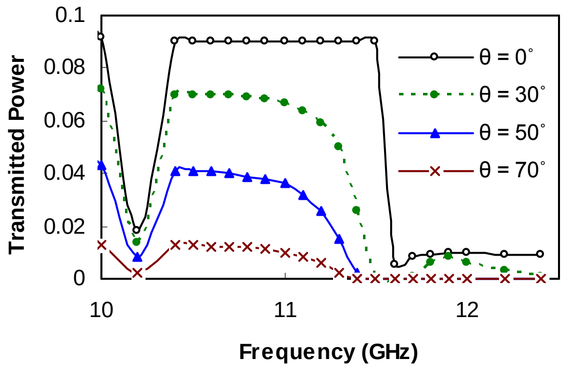

Figure 7. The transmitted power as a function of frequency where the condition ($n_1 < n_2$, $n_2 > n_3$, $n_3 < n_4$) is satisfied for various angles of incidence.

IV. CONCLUSIONS

The propagation of electromagnetic waves through multilayered structures consisting of a pair of LHM and dielectric slabs inserted between semi-infinite dielectric media has been studied. The followed method has been based on Maxwell's equations and matching the boundary conditions for the electric and magnetic fields at each interface layer. The frequency dependence of ε and μ of the LHM has been taken into account. The reflected and transmitted powers have been calculated numerically. The dependence of them on various parameters has been studied. Low and high transmitted powers has been achieved for any frequency and for any angle of incidence. The law of conservation of energy has been satisfied by the obtained results. The discussed problem is useful for applications which require controlling of reflected and transmitted powers like antenna radome, microwave, millimeter wave and optical devices.

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