

## انتشار البولاريتونات في $GaAs/AlxGa_{1-x}$ كسطح فاصل بين شبه موصل و مواد أيسرية كتركيب الدليل الموجي في مجال مغناطيسي كمي.

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**الملخص:** يتم تقديم التحقيق النظري لسطح البولاريتونات غير المشعة في المواد الفوقية  $GaAs/AlxGa_{1-x}$  (أيسر المواد LHM) كدليل موجي. نقطة الاتصال المختلفة الحقيقية مع LHM في ظل الظروف وتفضيل العدد الصحيح الكمي لتأثير هول (IQHE) ووجود تبديد في طبقة الإلكترون ثنائية الأبعاد. انه وجد أنه في ظل ظروف IQHE ، فجميع جوانب سطح البولاريتونات هي كمية. وقد أظهرت أيضا أن منحنيات التبديد هي مقلوبة من قبل LHM الاصطناعي حديثا.

# Propagation of polaritons in GaAs/AlGaAs heterojunction with left handed materials waveguide structure in quantizing magnetic field

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**Abstract:** A theoretical investigation of nonradioactive surface polaritons in GaAs/Al<sub>x</sub>Ga<sub>1-x</sub> metamaterials (left-handed material-LHM) waveguide is presented. The real heterojunction with LHM under both conditions is favoring integer-quantum Hall effect (IQHE) and a presence of dissipation in a two-dimensional electron layer. It is found that under IQHE conditions all aspects of surface polaritons are quantized. It has also shown that the dissipation curves have been inverted by the newly artificial LHM.

*Key terms:* Polaritons, Left handed Materials (LHM), Metamaterials, Dissipations effect, and Quantizing magnetic field.

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## 1. Introduction

The interests in the two dimensional electron systems (2DES) in collective electromagnetic excitations (elementary excitation) have been attracting considerable attention (*Agranovich V. M. et al., 1982, Boardman A. D., 1982, Ando T. et al., 1982, Brazis R. S. et al., 1981 and Kosevich Yu. A. et al., 1988*). Surface polariton (SP) is considered as a  $p$ -polarized surface electromagnetic wave (a classical surface polariton) at the interface between the vacuum and a metal or left-handed medium. Surface polaritons have special interest as a non radiative electromagnetic waves localized at 2DES (*Boardman A. D., 1982 and Beletskii, N. N. et al. 2000*). The strong magnetic field effect is of particular of interest, as it leads to integer quantum Hall effect (IQHE) in 2DES. Under these conditions, the high-frequency 2DES-permeability tensor components are quantized, i.e. they are changed discontinuously with varying magnetic field. In this case, all characteristics of surface polaritons are always quantized. SP's properties under IQHE conditions were considered for a single and a double 2DES. The group velocity of surface polaritons in the IQHE regime and in the vicinity of the cyclotron resonance (CR) exhibits a stepwise behavior and undergoes jumps proportional to the fine-structure constant  $\alpha=e^2/c\hbar$  where  $e$  is the electronic charge,  $c$  is the velocity of light, and  $\hbar= h/2\pi$  ( $h$  is the Planck constant) (*Aronov I. E. et al., 1996 and Aronov I. E. et al., 1997*).

Surface polaritons properties under the assumption that the plane occupied by the 2DES is embedded in a homogeneous medium with permittivity  $\epsilon$ . In reality, the real structure of the GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  heterojunction simulated by a 2D electron layer is more complex. In fact, a doped  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer has a finite thickness. As a result, the electrons making up a 2DES are

sandwiched between GaAs (this layer forms the substrate and it may be considered infinitely thick) and the  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer, which is in contact with vacuum or air. The two layers  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  and GaAs have different dielectric permittivities, and their ratio was set to 0.95. The above factors can create qualitatively new features in the propagation of surface polaritons along a real GaAs/  $\text{Al}_x\text{Ga}_{1-x}$  heterojunction. Besides, under real conditions one should take into account dissipation in the 2DES (*Constantinou N. C. et al., 1986 and Wallis R. F. et al., 1988*).

This dissipation can give rise to a quantitative change in the surface-polariton spectrum, and in appearance of new types of nonradiative polaritons (both surface and bulk), whose electromagnetic field dies out exponentially away from both interfaces of the  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer.

The present research, we will study the properties of surface polaritons in GaAs/AlGaAs interface on the damping of nonradiative polaritons with left handed materials LHM in the quantized magnetic field. As it is known that LHM has negative refractive index over some frequency range (*Ki Young Kim et al., 2006 Ferrari and A. J. et al., 2011 and Chau J. K. et al. 2012*). A negative index metamaterial causes light to refract, or bend, differently than in common positive refractive index materials. The LHM layer over  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  layer was added for studying the properties of SP.

## 2. Theory

Electrodynamics of the GaAs/AlGaAs heterojunction in a high magnet magnetic field will be considered as a structure model consisting of two semi-bounded media 1 (LHM) ( $z > d$ ) and 3 ( $z < 0$ ) with permittivities  $\epsilon_1$  and  $\epsilon_3$ , separated a thin layer with thickness  $d$  (medium 2) and a dielectric

permittivity  $\epsilon_2$  as shown in fig.1. Let medium 2 be  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  semiconductor, and medium 3, the GaAs semiconductor. These form heterojunction at the interface  $z = 0$  at this interface a 2DES is shaped as shown in figure 1. The external quantizing magnetic field  $\mathbf{B}$  is directed along the  $z$  axis perpendicular to the 2DES. It is assumed that the polaritons in this structure to be nonradiative, i.e. that their electromagnetic fields in media 1 (LHM) and 3 decay evanescent exponentially with increasing distance from the interfaces of layer 2 into either medium.

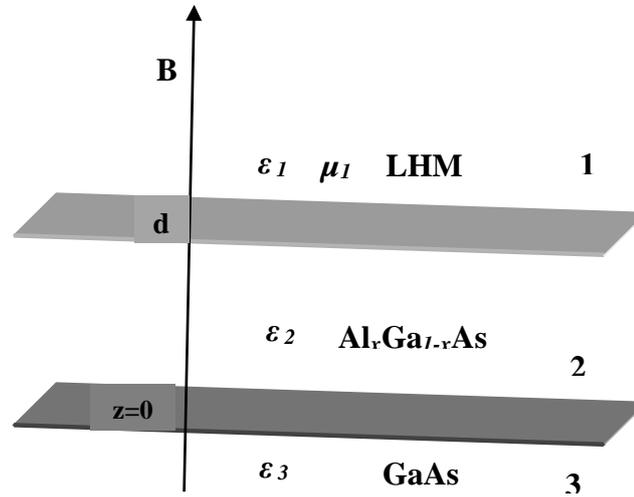


Figure 1. The GaAs/AlGaAs heterojunction. Waveguide structure of LHM semibounded medium 1 with permittivity  $\epsilon_1$ , medium 2 is an AlGaAs semiconductor slab with a thickness  $d$  and dielectric permittivity  $\epsilon_2 = 12.0$ , and semibounded medium 3 is GaAs semiconductor with dielectric permittivity  $\epsilon_3 = 12.9$ .

By assuming the nonradiative polaritons to propagate along the  $x$  axis, the dependence of all electromagnetic fields on coordinate  $x$  and time  $t$  will be described by the relation  $\exp [i(kx - \omega t)]$ , where  $k$  is the wave vector, and  $\omega$  is the frequency. To derive the dispersion relation describing propagation of nonradiative polaritons in a GaAs/  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  heterojunction, one has to

take into account two types of waves, TE and TM. This is due to the existence of a surface current at the  $z=0$  2DES interface, which acts on both TE and TM waves (Nakayama M., 1974).

Also assume the  $z=d$  interface the tangential components of the electric,  $\mathbf{E}$ , and magnetic,  $\mathbf{H}$ , fields of nonradiative waves are continuous, while at the interface  $z=0$  the tangential components of the magnetic field undergo a discontinuity (Aronov I. E. et al., 1996 and Aronov I. E. et al., 1997);

$$H_{x,2}^{(s)} - H_{x,3}^{(s)} = \frac{4\pi}{c} (\sigma_{xx} E_y^{(s)} - \sigma_{x,y} E_x^{(s)}) \quad (1a)$$

$$H_{y,2}^{(s)} - H_{y,3}^{(s)} = -\frac{4\pi}{c} (\sigma_{xx} E_x^{(s)} - \sigma_{x,y} E_y^{(s)}) \quad (1b)$$

Here  $\sigma_{ij}(\omega)$  are tensor components of the 2DES permeability, and the index  $s$  refers to the value of the electric and magnetic fields at the interface when  $z = 0$ .

It is assumed that the spatial dispersion of the 2DES permeability tensor may be neglected, i.e. that  $kl \ll 1$ , where  $l = (c\hbar/eB)^{1/2}$  is the magnetic length. In that case the nonvanishing 2DES permeability tensor components can be given by (Aronov I. E. et al., 1996 and Aronov I. E. et al., 1997);

$$\sigma_{xx} = \frac{2e^2}{h} \frac{N\gamma}{1+\gamma^2} \quad (2a)$$

$$\sigma_{xy} = \frac{2e^2}{h} \frac{N}{1+\gamma^2} \quad (2b)$$

In Eq. 2a and 2b,  $\gamma = (\nu - i\omega)/\Omega$ , where  $\Omega = eB/mc$  is the electron cyclotron frequency,  $\nu$  is the electron-momentum relaxation frequency, and  $N = \pi l^2 n$  is the Landau level filling factor, which assumes integer values ( $N = 1, 2, \dots$ ) equal to the number of completely filled Landau levels lying below the Fermi level ( $n$  is the electron density in the 2DES).

The effect of electron damping is ignored, which corresponds to the condition  $\omega \ll \nu$ .

Using the above boundary conditions at interface  $z = 0$  and  $z = d$ , the following dispersion relation describing propagation of nonradiative surface polaritons in the heterojunction is obtained.

$$A_1 A_2 + (4\pi\sigma_{xy}/c)^2 p_1 p_2 B_1 B_2 = 0 \quad (3)$$

where

$$A_1 = [p_3 \varepsilon_2 - p_2 \varepsilon_3 - (i 4\pi p_2 p_3 \sigma_{xx}/\omega)][p_2 \varepsilon_1 - p_1 \varepsilon_2] + \exp(2p_2 d)[p_3 \varepsilon_2 + p_2 \varepsilon_3 + (i 4\pi p_2 p_3 \sigma_{xx}/\omega)][p_2 \varepsilon_1 + p_1 \varepsilon_2] \quad (4)$$

$$A_2 = [p_2 - p_3 + (i 4\pi\omega\sigma_{xx}/c^2)][\mu_1 p_1 - p_2] - \exp(2P_2 d)[p_2 - p_3 + (i 4\pi\omega\sigma_{xx}/c^2)][\mu_1 p_2 + p_1] \quad (5)$$

$$B_1 = [\mu_1 p_2 - p_1] + \exp(2P_2 d)[\mu_1 p_2 + p_1] \quad (6)$$

$$B_2 = [p_2 \varepsilon_1 - p_1 \varepsilon_2] - \exp(2P_2 d)[p_1 \varepsilon_2 + p_2 \varepsilon_1] \quad (7)$$

Here the variable  $p_i$  is given by

$$p_i = \sqrt{k^2 - \frac{\omega^2}{c^2} \varepsilon_i}, \quad i = 1, 2, 3. \quad (8)$$

where the variable  $p_i$  is a complex and refers to the transverse wave vector describing distribution of the electromagnetic field of nonradiative polaritons along the  $z$  axis. Because the polaritons are nonradiative, the following conditions should hold as:

$$\text{Re } p_1 > 0, \quad \text{Re } P_3 > 0. \quad (9)$$

If  $\varepsilon_2 = \varepsilon_3$ , then, in the limit as  $d \rightarrow \infty$ , the dispersion relation (3) coincides with that for surface polaritons in a 2DES embedded in a homogeneous medium with permittivity  $\varepsilon_2$  (Aronov I. E. et al., 1996 and Aronov I. E. et al., 1997).

### 3. Results and Discussion

Considering now the results of a numerical solution of dispersion relation in eq. (3) with inclusion of dissipation in the 2DES. It is assumed that the wave vector  $k$  is a real, and the frequency  $\omega = \omega' + i\omega''$ , a complex quantity. For convenience of the numerical solution of (3), the following dimensionless quantities were introduced:

$$\xi' = \omega' / \Omega, \quad \xi'' = \omega'' / \Omega, \quad \zeta = ck / \Omega, \quad \chi = z\Omega / c, \quad \delta = d\Omega / c, \quad \text{and } \Gamma = \nu / \Omega.$$

All calculations were carried out for a GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterojunction with  $\varepsilon_2 = 12.0$  and  $\varepsilon_3 = 12.9$ . The Al<sub>x</sub>Ga<sub>1-x</sub>As layer (medium 2) was assumed to be in contact with LHM permittivity  $\varepsilon_1$  (medium 1).

The damping  $\xi''(\zeta)$  for the case  $\delta = 0.1$  and  $\Gamma = 0.1$  is calculated for three Landau level filling factors  $N$ . In fig. 2, the damping curves (1'-3') for LHM in 2DES broadens exist for all values  $\zeta$  as for nonradiative polaritons in dashed 2DES curves (1-3) but in the presence LHM the curves (1'-3') show more sharpness. In the electromagnetic field region for nonradiative polaritons, the curves of damping decay exponentially away from the DES interface  $z=0$ , that means they are surface polaritons. In the LHM case the damping is associated with the generation of surface polaritons of the type of Brewster mode at  $z=d$  interface. The surface polaritons of Brewster mode have mixed polarization when  $z=d$  interface, where LHM exists due to damping in the 2D electron system located at  $z=0$  interface. Brewster mode is generated when  $\zeta \approx 2.5$ . One can see from Figure 3 the variation of the damping  $\xi''(\xi)$  of nonradiative polaritons with various  $N$ :  $N_1=1$ ,  $N_2= 5$ ,  $N_3= 10$  and dielectric permittivity  $\varepsilon$ :  $\varepsilon_1 = -2$ ,  $\varepsilon_2 = 12$ , and  $\varepsilon_3 = 12.9$ . The smooth damping curves of LHM are inverted and becomes very sharp when

the  $\zeta \approx 4$  because the permittivity  $\varepsilon_1$  has a negative value with the small value of  $\delta = 0.1$ .

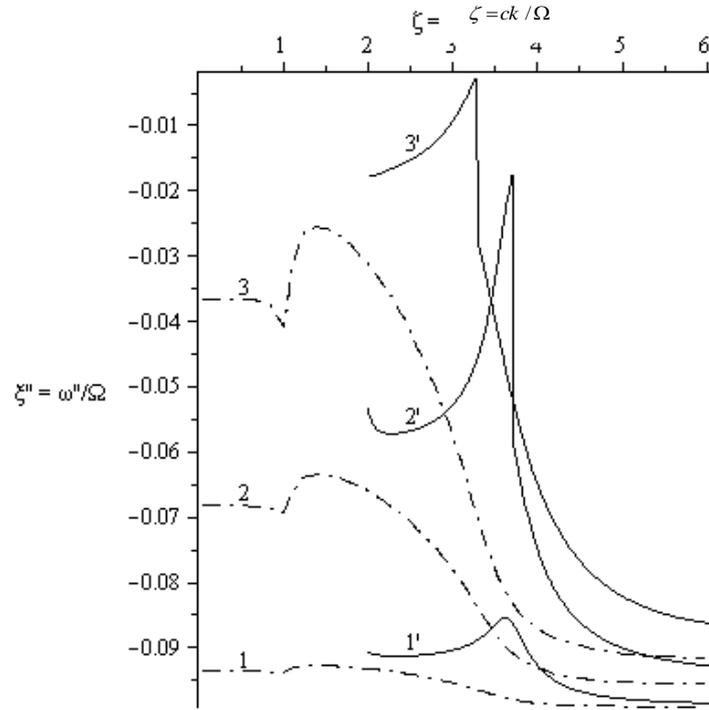


Fig. 2. Damping of GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterojunction in the case LHM  $\xi''$  for case  $\delta = 0.1$  calculated for three level filling factors  $N$ :  $N_1=1$ ,  $N_2=5$ ,  $N_3=10$  with dissipation  $\Gamma = 0.1$ .

Figure 3 shows the damping  $\xi''$  of LHM which is calculated when the dimensionless thickness  $\delta = 0.1$  and permittivity and permeability are the same with negative value .i.e.  $\varepsilon_1 = \mu_1 = -2$  with  $N = 5$  different value of dissipation  $\Gamma = 0, 0.05$  and  $0.1$ . The curves 1-3 emphasize, the LHM curves in figure, that they are inverted and sharp. When  $\Gamma = 0$  the curve 1 goes to the zero but when  $\Gamma = 0.1$  the curve is very sharp.

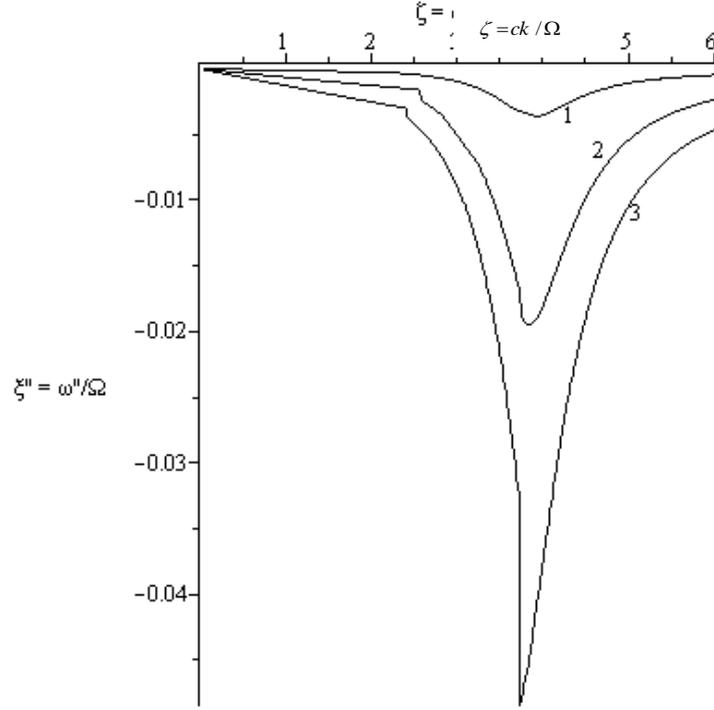


Fig. 3. The damping of GaAs/Al<sub>x</sub>Ga<sub>1-x</sub> ( $\xi''$ ) as heterojunction in the presence of LHM where  $\varepsilon_1 = \mu_1 = -2$  with three different values of  $\Gamma$

In figure 4 the damping  $\xi''$  of GaAs/Al<sub>x</sub>Ga<sub>1-x</sub>As heterojunction in the presence of LHM is calculated when the different values of dimensionless thickness are  $\delta_1 = -2$ ,  $\delta_2 = -1$ , and  $\delta_3 = -1/2$ , and three different values of permeability are  $\mu_1 = -2$ ,  $\mu_2 = -4$ , and  $\mu_3 = -8$  respectively. The permittivity of the second and third layers is  $\varepsilon_2 = 12$  and  $\varepsilon_3 = 12.9$  with  $N = 1$  and the value of dissipation  $\Gamma = 0.1$ . As shown in the figure 4 the curves 1 and 2 in are the same direction but the third one in opposite direction in the presence the LHM i.e. this is opposite in the presence of the vacuum or air.

It is interest to analyze the dependence of the frequency and damping of surface electromagnetic oscillations ( $\zeta=0$ ) for finite  $\delta$ . In figure 5.a (real part) and b (imaginary part) show the curves of  $\xi''$  and  $\xi'$  relation for the case  $\zeta=0$ ,  $N=5$  and different value of  $\Gamma$ . In this figure, the damping  $\xi''$  must be negative values in the presence of LHM. Therefore, the surface electromagnetic oscillations can be existing. Here the curves of the relations  $\xi''$  and  $\xi'$  are oscillating functions whose amplitude decreases with the dimensionless thickness  $\delta$  increases. When  $\delta \rightarrow \infty$ , the DES goes to be adjoining media with the permittivities  $\varepsilon_2$  and  $\varepsilon_3$ .

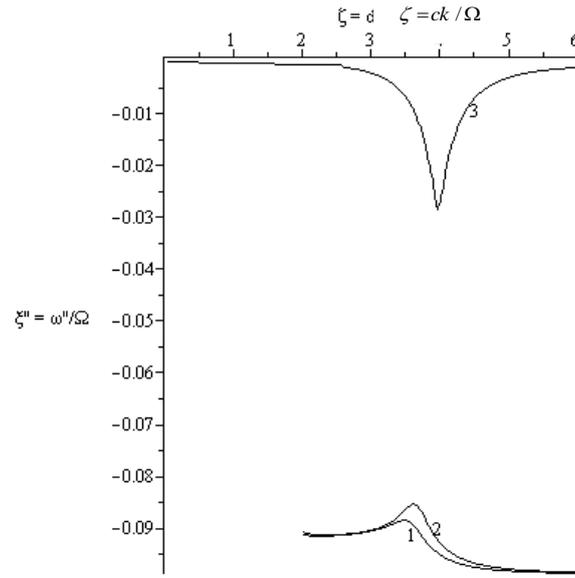
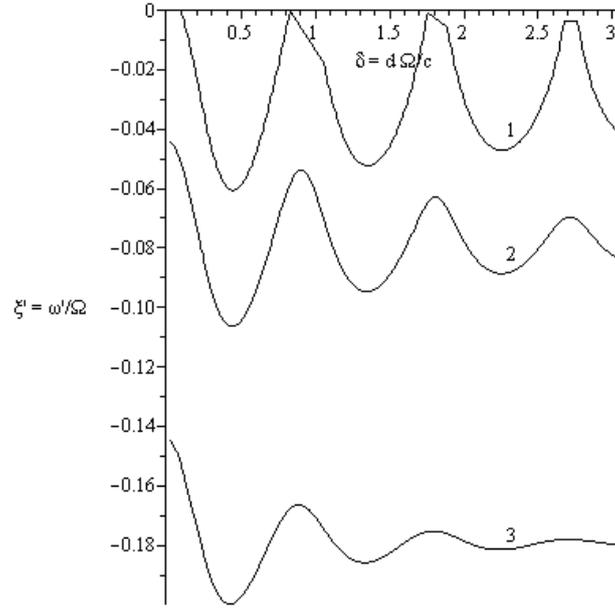
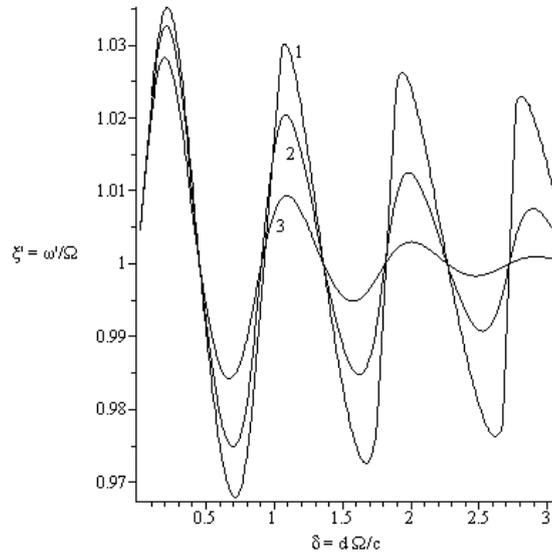


Fig. 4. The damping of GaAs/AlxGa1-x ( $\xi''$ ) as heterojunction in the presence of LHM where  $\varepsilon_2 = 12$ , and  $\varepsilon_3 = 12.9$  with three different values of the dimensionless thickness  $\delta$  and the three different values of permeability  $\mu$ .



(a)



(b)

Fig. 5. a- Damping of GaAs/Al<sub>x</sub>Ga<sub>1-x</sub> ( $\xi''$ ) as heterojunction and b- the relation  $\xi' = \omega'/\Omega$  in the presence of LHM where  $\varepsilon_1 = \mu_1 = -2$ ,  $\varepsilon_2 = 12$ , and  $\varepsilon_3 = 12.9$  with three different values of  $\Gamma$  in the case  $N=5$  and  $\zeta = 0$ .

## 4. Conclusion

The nonradioactive surface polaritons was investigated theoretically in GaAs/  $\text{Al}_x\text{Ga}_{1-x}$  metamaterials LHM waveguide structure. The heterojunction with LHM is favoring conditions IQHE with presence of dissipation. It was found that all aspects of SP are quantized and the dissipation curves are inverted. In the LHM case, the damping was associated with generation of surface polaritons of the type of Brewster mode at  $z=d$  interface. Moreover, The damping of GaAs/ $\text{Al}_x\text{Ga}_{1-x}$  as heterojunction in the presence LHM was calculated for different values of dimensionless thickness.

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