



XA0404401

IC/IR/2004/4
INTERNAL REPORT
(Limited Distribution)

40th anniversary
1964
2004

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

**DESIGN AND CHARACTERIZATION OF HOMOGENEOUS
TM NONLINEAR WAVEGUIDE SENSORS**

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Abstract

This work is devoted to a three-layer sensor bounded on one side by a nonlinear clad of intensity dependent refractive index. Sensitivity of this configuration is theoretically discussed and the exact condition to maximize this sensitivity is also determined. Behavior of sensing sensitivity is accounted for through power flow and cut-off considerations. Finally, we establish a method of determining the proper dimensioning of the sensor to execute its maximum sensitivity. We believe these concepts may be demonstrated and carried out for future novel sensors.

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July 2004

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1. Introduction

In recent years, increased attention has been paid to the nonlinear optical waveguide sensing after many of their promising properties [1-5] have been obtained. Optical waveguide sensors offer more attractive characteristics than other signal processing devices. They are field resistant, small sized, safe when used in aggressive environments and mechanically stable. Although light travels confined with the optical waveguide, some of the light travels in the surrounding media in so-called evanescent mode [5]. A measuring physical or chemical quantities appearing in the cover region is closely dependent on the strength and distribution of the evanescent field in that cover. If chemical or biological reaction result in a change in refractive indices of the surroundings, the evanescent field is modified giving rise to an alteration of the optical properties of the sensor. This process is referred to as sensing [1, 5]. All physical, chemical or biological properties that change in accordance with changes in refractive index can be detected this way. The type of waveguides currently used in chemical and medical sensing is the planar optical waveguide structure.

In designing sensors, it is important that the device works with its maximal sensitivity. In doing so, a designer must introduce the most fitting choice of waveguide thickness and the materials from which waveguide and surroundings are made so that the sensor is set to work with the desired efficiency [3, 6]. In most cases, the cover is chosen to be a liquid or a gas where the contact zone between the cover and waveguide film is zero (i.e. no bubbles or air films arise). Strong sensor response results from the use of thin films of higher refractive indices than those of substrates and covers. A normalized analysis for the design of linear evanescent wave sensors was carried out [6] and the condition for maximum achievable sensitivity was also studied. In most previous papers, substrate refractive index was reported to be higher than that of clad in order to have proper working sensors. This is called the normal symmetry (most currently used). Recently, attention has been paid to the reversed symmetry (i.e. refractive index of cladding higher than that of substrate). It was reported that reversed symmetry sensors are not only capable of light guiding, but they also have larger penetration depths in the cover region [1, 2, 5].

2. Mathematical Evaluations

2.1. Dispersion Equation

The Sensor under consideration is given schematically in fig.1. A linear waveguide film is surrounded by a linear substrate and a nonlinear cover with an intensity dependent refractive index whose dielectric function ϵ_c^{NL} is given through the approximate form:

$$\epsilon_c^{NL} = \epsilon_r^L + \alpha |E|^2 \quad (1)$$

where $\alpha = c\epsilon_0 n_c^2 n_{2c}$, c being the speed of light in vacuum, n_{cis} is the refractive index and n_{2c} is the nonlinearity constant.

The guiding film (g) occupies the region $0 \leq z \leq h$ while the two dielectrics (s,c) fill the regions $z < 0$ and $z > h$ respectively. The electromagnetic waves are considered to propagate along the x axis with guiding surfaces parallel to the xy plane.

Maxwell's equations for such a structure were solved in a variety of papers [7-9]. Matching the fields at the boundaries gives rise to the dispersion equation:

$$\tanh(k_0 h q_g) = - \frac{q_g (Q_c \tanh(k_0 z_c q_c) + Q_s)}{q_g^2 + Q_c Q_s \tanh(k_0 z_c q_c)}, \quad (2)$$

$$\text{where } q_i = \sqrt{N^2 - \epsilon_i}, \quad Q_i = q_i \left(\frac{\epsilon_g}{\epsilon_i} \right), \quad (3)$$

N is the modal effective index. To simplify calculations, we define two asymmetry parameters (a_s and a_c) in addition to normalized variables (X_s and X_c) such that:

$$X_c = \sqrt{\frac{N^2 - \epsilon_c}{\epsilon_g - N^2}}, \quad X_s = \sqrt{\frac{N^2 - \epsilon_s}{\epsilon_g - N^2}} \quad (4)$$

$$a_c = \frac{\epsilon_c}{\epsilon_g}, \quad a_s = \frac{\epsilon_s}{\epsilon_g} \quad (5)$$

where ϵ_i is the dielectric constant for the i th medium. These normalized quantities are related to each other through the equation:

$$\frac{1 + X_c^2}{1 + X_s^2} = \frac{1 - a_c}{1 - a_s} \quad (6)$$

Substituting for q 's form (6) and making use of equations (4,5), one can end with :

$$k_0 h \sqrt{\epsilon_g} \sqrt{\frac{1 - a_s}{1 + X_s^2}} - \arctan\left(\frac{X_c \tanh(C)}{a_c}\right) - \arctan\left(\frac{X_s}{a_s}\right) - m\pi = 0 \quad (7)$$

$$\text{with } C = k_0 z_c \sqrt{N^2 - \epsilon_c}; \quad m = 0, 1, 2, \dots$$

In case of linear waveguides, the term $\tanh(C)$ in eqn.7 goes to 1 giving rise to the usual linear dispersion equation [6,10].

2.2. The Cut-off

As the waveguide width (h) decreases, the evanescent tail in the surrounding medium of higher refractive index is enlarged [1, 6] till it approaches infinity when the guiding film ceases to support the wave. This is called the cut-off. In this case the effective index N approaches n_s so that $X_s = 0$ in the normal asymmetry. Consequently; $X_c(\text{cut-off})$ is given for the fundamental mode ($m=0$) by:

$$X_{c(\text{cut-off})} = \sqrt{\frac{a_s - a_c}{1 - a_s}} \quad (8)$$

At the same mode, the waveguide width at cut-off is given by:

$$h_{cut-off}(a_c, a_s) = \frac{\arctan\left(\sqrt{\frac{a_s - a_c}{a_c(1 - a_s)}} \tanh(c)\right)}{k_o \sqrt{\epsilon_g(a_s - a_c)}} \quad (9)$$

There is no need to discuss the cut-off belonging to $X_c=0$ since it is achieved only in reverse asymmetry [1] when $a_c > a_s$ which is not our study here.

2.3. Evaluation of Sensitivity

Homogeneous sensing sensitivity (S_h) is defined as the rate of change of the effective refracted index (N) with respect to the change of the cover index, i.e.

$$S_h = \frac{\partial N}{\partial n_c} \quad (10)$$

We can express the dispersion equation, Eq.(2) in the form $f(X_c, n_c, N)=0$ so that the differential in Eq.(10) can be written[5] as:

$$S_h = \frac{\partial N}{\partial n_c} = - \frac{\partial f / \partial n_c}{\partial f / \partial N} \quad (11)$$

After some arrangement, sensitivity can be expressed as:

$$S_h(TM) = \frac{((a_c + X_c^2(2 - a_c)) \tanh(c) + H \sqrt{1 + X_c^2}) \sqrt{a_c}}{X_c (a_c^2 + X_c^2 \tanh^2(c)) \sqrt{a_c + X_c^2} \sqrt{1 + X_c^2} (F_{TM} + G_c + G_s) + H \sqrt{a_c + X_c^2}} \quad (12)$$

where

$$F_{TM} = \arctan\left(\frac{X_c \tanh(C)}{a_c}\right) + \arctan\left(\frac{X_s}{a_s}\right) + m\pi. \quad (12.a)$$

$$G_c = \frac{a_c (1 + X_c^2) t(c)}{X_c (a_c^2 + X_c^2 \tanh^2(c))} \quad (12.b)$$

$$G_s = \frac{a_s (1 + X_s^2)}{X_s (a_s^2 + X_s^2)} \quad (12.c)$$

$$H = k_o z_c X_c a_c \sqrt{\epsilon_g} \sqrt{1 - a_c} (1 - \tanh^2(C)) \quad (12.d)$$

Again; if the cover is linear, then $\tanh(C)$ is 1 and Eq.(12) coincides with the results given in literature[2,5,10].

2.4. The condition for maximum sensitivity

An important aspect in designing sensors is the efficiency of transforming the chemical or biological reaction in a measurable signal. That is, we seek to bring the sensor to its maximum sensitivity provided a certain configuration. Given a configuration of constant ϵ_c, ϵ_g and ϵ_s , the condition for maximum sensitivity is achieved when the derivative of $S_h(TM)$ with respect to h (the waveguide width) vanishes [2], or :

$$\frac{\partial S_h(TE)}{\partial h} = \frac{\partial S_h(TE)}{\partial X_s} \frac{\partial X_s}{\partial h} = 0 \quad (13)$$

Applying this to eq.(12) and after some mathematical arrangement, the condition to achieve the maximum sensing sensitivity is given by the relation:

$$\begin{aligned} & 2 \left(\frac{X_c^2(2-a_c) \tanh(c)}{c(1-\tanh^2(c))} + \frac{a_c + X_c^2}{(1+X_c^2)} + a_c X_c^2 \right) - \frac{2X_c(E+F) \tanh(c)}{a_c(1+X_c^2)} \\ & \frac{X_c(a_c + X_c^2(2-a_c)) \tanh(c)}{c(1-\tanh(c)^2)} + a_c X_c(1+X_c^2) \\ & \frac{F}{X_c^2} + \frac{E(2X_c^2-1)}{X_c^2(1+X_c^2)} - \frac{2(E+F)^2 \tanh(c)}{a_c(1+X_c^2)} - \frac{X_c a_s^2(a_s^2 + X_s^2(3-2a_s^2))}{a_c X_s^3(a_s^2 + X_s^2)^2} \\ & \frac{a_c + X_c^2(2+2a_c+3X_c^2)}{X_c(a_c + X_c^2)(1+X_c^2)} = 0 \end{aligned} \quad (14)$$

where

$$A_{TM} = F_{TM} + \frac{E+F}{X_c} + \frac{(1+X_c^2)a_s}{X_s(a_c^2 + X_c^2)} \quad (15.a)$$

$$E = \frac{a_c(1+X_c^2) \tanh(C)}{a_c^2 + X_c^2 \tanh^2(C)} \quad (15.b)$$

$$F = \frac{k_o z_c X_c a_c \sqrt{\epsilon_g} \sqrt{1-a_c(1-\tanh^2(C))}}{\sqrt{1+X_c^2(a_c^2 + X_c^2 \tanh^2(C))}} \quad (15.c)$$

If $\tanh(C) = 1$, the term T goes to 1 and the optimizing condition becomes exactly that of linear waveguides proposed by Parriaux and Veldhuis[2,6].

3. Discussion and Representation

The configuration of our waveguide is represented schematically in Fig.1. The waveguide refractive index is taken to be silicon nitride ($\epsilon_g = 4$) such as silicon nitride and the wavelength of the light beam is chosen to have the value 1550 nm in most cases. Other values of λ were used for comparison. In all cases, a_s is taken to be greater than that of the cover medium which is the usual choice in sensors [1-3]. The dispersion equation was solved numerically for a fixed value

of $\tanh(C)$ and N is evaluated. Using Eq.(4), X_c and X_s are also evaluated. Substituting in the sensing sensitivity expression (Eq.(11)), we could have the required values of sensitivity. Given the asymmetry parameters a_c and a_s , one can substitute for X_s from Eq.(6) in Eq.(14) to end with a function of X_c only thus having the value of X_c . Substituting these values in the characteristic equation (7), we can compute the waveguide widths ensuring maximum. The values of these maxima are calculated through substituting the last values in the expressions of sensitivity (Eq.(12)) above. Plotting these results against a_c and a_s , a designer ends with a chart from which all parameters required for maximization of such sensors may be derived. Exact expression of the sensitivity is tedious with very large terms. The situation turns more and more difficult in evaluating the condition required for maximum obtainable sensitivity so that we developed a computer program capable of measuring the sensitivity and extract the maximum value of sensitivity due to changing waveguide width (h). We then plot the results and propose the charts to help designing the sensor and bring it into the required working point. Although our expressions are valid for any order of TE modes, our discussion is restricted only to the fundamental mode ($m=0$) since that mode has the highest sensitivity [1, 5]. Plotting h and S against asymmetry parameters, we end up with the charts for each mode (due to $m=0,1,2,\dots$) that enable us to construct (design) our sensor. The cover material is chosen according to the proposed usage of the sensor thus giving the refractive index n_c . The choice of substrate (consequently n_s) is controlled by cost requirements, mechanical stability and temperature. As for the guiding material, chemical and optical stabilities are considered. However, there is a relative freedom in choosing n_g [2]. On the chart, maximum achievable sensitivity versus a_c and a_s , the point (a_c, a_s, S_{max}) is determined. Here, S_{max} is the highest sensitivity fitting the kind of usage and cost requirement. We then turn to the chart of $h(a_c, a_s)$ and determine h corresponding to the just determined values. To construct a more efficient or universal chart, one can make use of the fact that n_g and λ are arbitrary and plot $n_g h / \lambda$ rather than the mere h so that the relative waveguide thickness is retrieved.

Fig.2 illustrates the behavior of sensitivity with the wave guide width (h) for three configurations. It is ascertained in the figure that there exists a maximum in the sensitivity values at some value of wave guide width. These maxima increase with increasing a_c and are shown to be shifted towards lower values of h . In the three cases of a_c in the figure ($a_c = 0.3, 0.5, 0.65$) with a_s fixed to 0.7, the corresponding maximum sensitivities appeared at $h = 390, 320$ and 225 nm ; respectively. This behavior can be justified through cut-off and energy considerations. At cut-off, all power flow is restricted to the substrate region [1] so that sensitivity approaches zero. For thick films, however, good guiding is achieved and the whole power is now due to film. This means sensitivity is again zero. Between these extremes, sensitivity increases, reaches a maximum and then decreases again till vanishing at sufficiently high widths. For the sake of comparison, we plotted sensitivity (S_{TE}) against waveguide width for both linear and nonlinear cases. The results are shown in Fig.3. In this figure, sensitivities are shown to be higher valued than those of the linear case though appearing at lower values of waveguide widths. Thus if cost doesn't matter, sensors with nonlinear cladding are recommended. In Fig.4, sensitivity is plotted against h for different values of wave numbers (different wavelengths). Maximum sensitivity remains the same ($S_{max}=0.33$) for the three given configurations but are shifted with λ to the

region of higher waveguide width. This means that there is a wide tolerance choosing the wave length region at which the sensor is set to exhibit specific sensitivity. The maximum obtainable sensitivities are calculated through substituting the solutions of Eq.(14) in the sensitivity expression Eq.(12) and then plotted against a_c and a_s resulting in the chart shown in Fig.5. Having assigned S_{max} in this figure, the designer determines the corresponding a_c and a_s and then moves to Fig.6 when he finds the waveguide width (or $ng.h/\lambda$) needed to bring his sensor to its maximum sensitivity. The cut-off chart is displayed on the same figure in order to compare between the optimum and the cut-off conditions. It is obvious from the figures that all h 's are above the cut-off length as mentioned above.

4. Conclusion

The presented work reveals some promising properties of the nonlinear wave guided sensors. It was shown that sensitivity is enhanced by using nonlinear clad although the waveguide width making the maximum sensitivity is smaller in the nonlinear case. If cost is not the determinant aspect, the presented sensors are advantageous. Sensitivity is not affected by the change in wavelength but they are shifted towards the higher values of wavelengths giving rise to considerable freedom in choosing the optical region used. To the best of our knowledge, we are the first to work on the design of nonlinear optical wave guided sensors and we *do* believe that our work is worth being carried out experimentally in future.

Acknowledgments

This work was done within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

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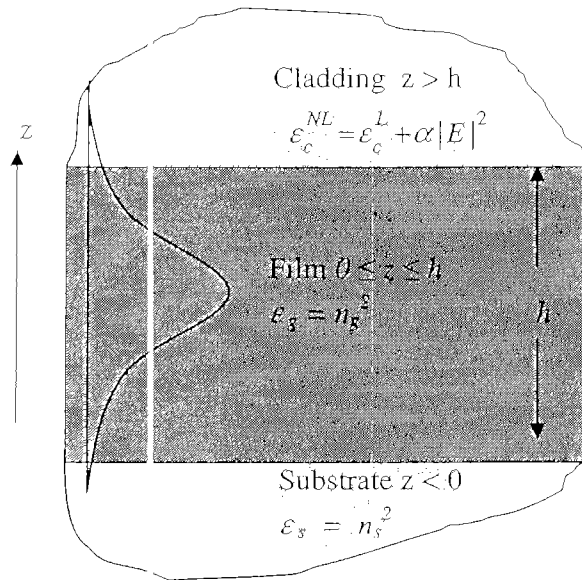


Fig. 1: Schematic diagram of the waveguide structure under consideration.

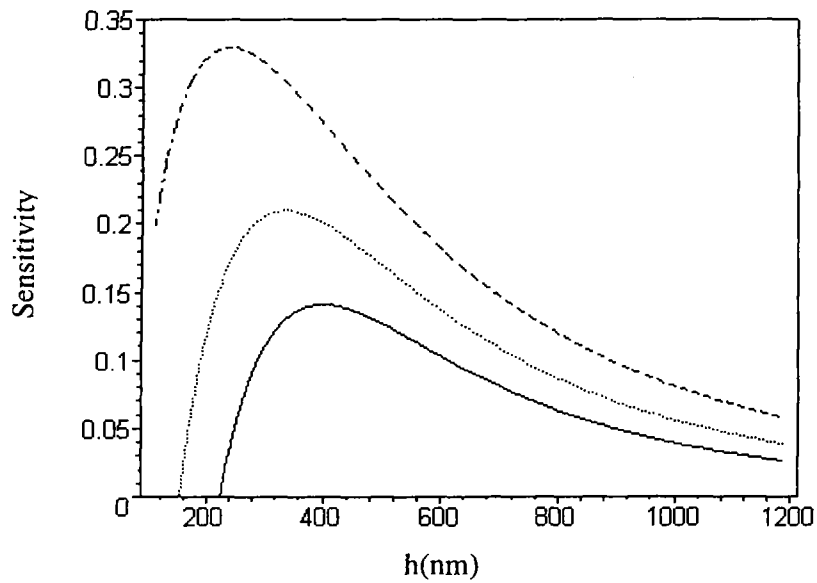


Fig. 2: Sensitivity vs. guide width (h) : a) $a_c = 0.35$ b) $a_c = 0.5$ c) $a_c = 0.65$
($a_s = 0.7$, $\epsilon_g = 4$, $\lambda = 1550$ nm).
Notice that maxima appear at waveguide widths : 390 , 320 and 225 nm; respectively.
Notice also that maximum sensitivity increases with increasing a_c .

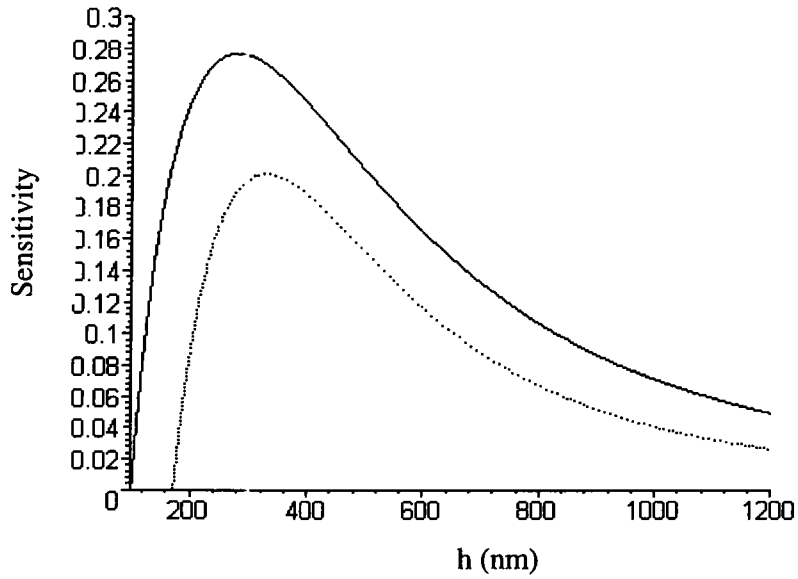


Fig. 3: Sensitivity vs. waveguide width h for a) linear (dotted line) and b) nonlinear (solid line).
 $(\epsilon_p = 4, a_s = 0.7, a_c = 0.65, \lambda = 1550 \text{ nm})$.
 Maximum values are higher valued but appear at smaller values of waveguide widths.

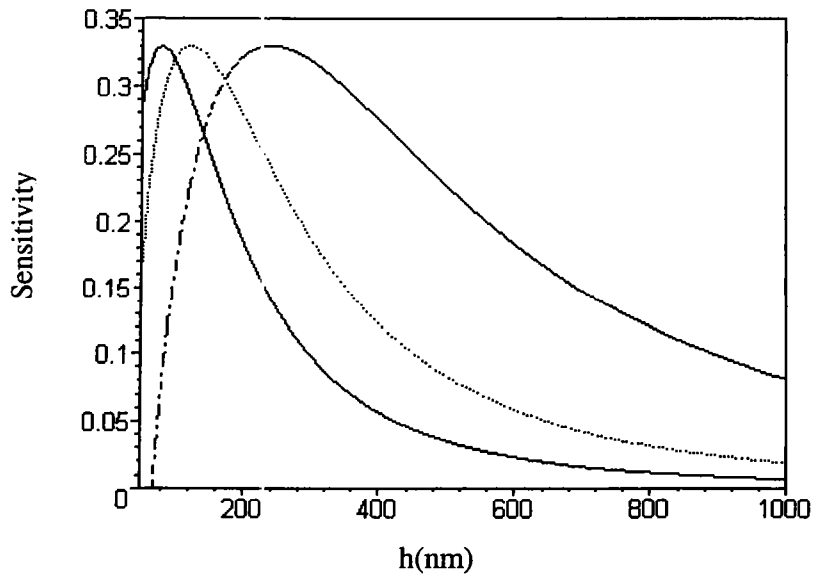


Fig. 4: Sensitivity vs. h for different wavelengths:
 $(\lambda = 515 \text{ nm (solid), } 785 \text{ nm (dotted), } 1550 \text{ nm (dashed))}$.
 Notice that maxima are equal (0.33) but are shifted with wavelength.

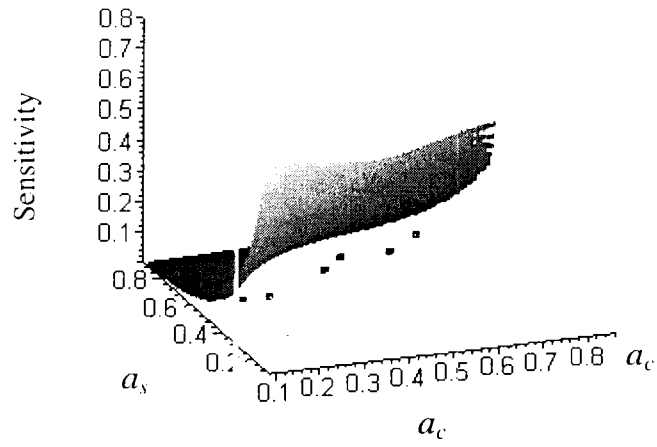


Fig. 5: Maximum achievable sensitivity (S_{\max}) vs. a_s and a_c ($a_s > a_c$)

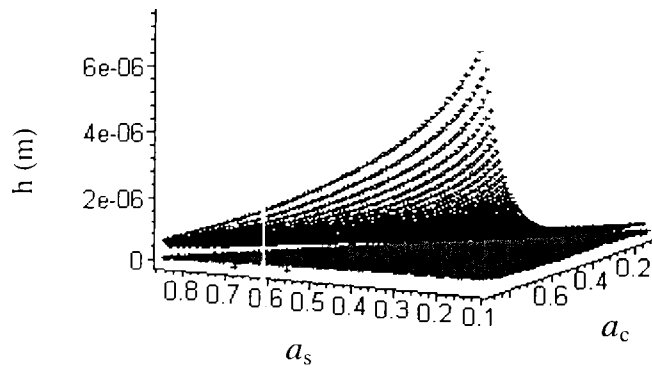


Fig. 6: Wave guide width (h) achieving maximum vs. a_c and a_s