

NONLINEAR MAGNETOSTATIC SURFACE WAVES IN A FERRITE-LEFT-HANDED WAVEGUIDE STRUCTURE

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Abstract

The paper presents a theoretical investigation of the propagation of strongly TE nonlinear magnetostatic surface waves propagating in a composite waveguide structure of a Ferrite (YIG) film, Left-handed substrate and nonlinear cover. The dispersion relation and the effect of nonlinearity on the propagation characteristics have been examined. It is found that the differential phase shift can be minimized for the smaller operating frequency and relatively thick film. The power flow has also been calculated for different values of operating frequency.

Keywords: Dispersion relation, Left-handed material, Phase shift, Power flow.

I. INTRODUCTION

The investigation of linear and nonlinear magnetostatic surface waves is very important because of their applications to delay lines in Microwaves Integrated Circuits (MIC), and Signal Processing, such as band-pass filter, resonators filter, delay microwave and millimeter wave frequencies due to their nonreciprocal characteristic [1-7]. Magnetostatic surface waves have been studied in the Voigt geometry by several workers. Some of them have addressed a weakly nonlinear magnetostatic surface waves on YIG [8, 9], and other [6-7] have studied the strong nonlinear magnetostatic surface waves in the Voigt structure for a YIG substrate and nonlinear dielectric cover. These studies were carried out in a media with positive refractive index. Such media are called right-handed material (RH).

Recently, much attention has been paid to study of electromagnetic waves in media of negative permittivity $\epsilon(\omega)$ and permeability $\mu(\omega)$. Such media are called left-handed (LH) media since the electric and magnetic fields form a left-handed set of vectors with the wave vector [10]. The theory of the propagation of the electromagnetic waves in such media was developed by Veselago [11]. These materials have been shown to exhibit unique properties, among which the reversals of Snell Law, the Doppler shift, and Cherenkov radiation. Such new artificial materials are recently found and examined and called left handed metamaterials (LHM) [12-14].

These materials have been built by using two dimensional arrays of splitting resonators and wires and are operating the microwave range.

In this paper, the dispersion relation for the strongly nonlinear magnetostatic surface waves in a ground ferrite film has been studied. Full details have been reported and the results of a new type of strongly nonlinear magnetostatic surface waves in a YIG film, bounded by a nonlinear cover and a LHM substrate. The numerical results are also presented and discussed, especially the dispersion characteristics, and the flow of power.

II. DERIVATION OF DISPERSION RELATION

The structure to be analyzed is shown in Fig. 1. It consists of a ferrite film adjacent to a semi-infinite of nonlinear dielectric cover and LHM substrate, each of which extend to infinity in the yz plane. A static magnetic field H_o is applied in z direction. We consider for simplicity that the wave propagates in the positive y direction and that all the field components are independent on z ($\partial/\partial z=0$). In the following analysis, we take only TE magnetostatic surface waves mode inherent to a ferrite, not TM surface wave modes, since we are interested only in the magnetostatic surface waves. All the components of the fields are taken to be proportional to $e^{i(\omega t-ky)}$, where k is the propagation constant. The dielectric function of the nonlinear dielectric cover is assumed to be Keer-like and isotropic and depends on the electric field and can be written[15] as $\varepsilon^{NL} = \varepsilon_1 + \alpha E_y^2$, where ε_1 is the linear part and α is the nonlinear coefficient.

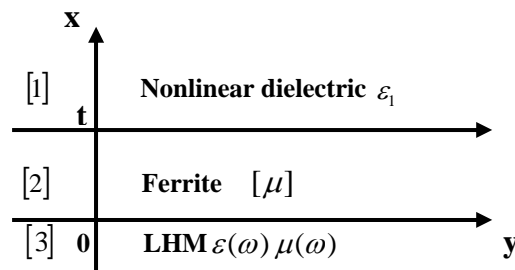


Fig.1. TE magnetostatic surface waveguide composed from a Ferrite-LHM layered structure

We seek the solutions of Maxwell's equations for the TE waves in the form:

$$\mathbf{H} = (H_x, H_y, 0)e^{i(\omega t-ky)}, \quad (1a)$$

$$\mathbf{E} = (0, 0, E_z)e^{i(\omega t-ky)}. \quad (1b)$$

i. In nonlinear dielectric cover ($x > 0$)

$$\nabla \times \mathbf{H}_1 = i\omega\varepsilon_o\varepsilon^{NL}\mathbf{E}_1, \quad (2)$$

$$\nabla \times \mathbf{E}_1 = -i\omega\mu_o\mathbf{H}_1. \quad (3)$$

Eqs. (2) and (3) are cast into the equation for E_{z1} :

$$\frac{\partial^2 E_{z1}}{\partial x^2} = (k^2 - k_o^2\varepsilon_1)E_{z1} - k_o^2\alpha E_{z1}^3, \quad (4)$$

where $k_o^2 = \frac{\omega^2}{c^2}$.

In the magnetostatic range, the wave-number to be as $k \gg (\omega/c)\varepsilon_1^{1/2}$; so eqn. (6) can be written as [7, 16]

$$\frac{\partial^2 E_{z1}}{\partial x^2} = k^2 E_{z1} - k_o^2\alpha E_{z1}^3. \quad (5)$$

The solution of eqn. (5), can be written as

$$E_{z1} = \frac{k}{k_o} \left(\frac{2}{\alpha} \right)^{1/2} \sec h[k(x - x_o)], \quad (6)$$

where x_o is the position of the maximum of the field component in the nonlinear cover and can be determined from the boundary condition.

The magnetic field components in the nonlinear cover are:

$$H_{x1} = \frac{k}{\omega\mu_o} E_{z1}, \quad (7)$$

$$H_{y1} = \frac{ik^2}{\omega\mu_o k_o} \left(\frac{2}{\alpha} \right)^{1/2} \sec h[k(x - x_o)] \tanh[k(x - x_o)], \quad (8)$$

ii. In ferrite film ($0 \leq x \leq t$)

$$\nabla \times \mathbf{H}_2 = 0, \quad (9)$$

$$\nabla \cdot \mathbf{H}_2 = 0, \quad (10)$$

$$\nabla \times \mathbf{E}_2 = -i\omega\mu_o [\mu] \mathbf{H}_2. \quad (11)$$

Where $[\mu]$ is the magnetic permeability tensor of the gyromagnetic ferrite (YIG) film is described as [16]:

$$[\mu] = \begin{bmatrix} \mu_{xx} & i\mu_{xy} & 0 \\ -i\mu_{xy} & \mu_{xx} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix}, \quad (12)$$

where $\mu_{xx} = \mu_B \left(1 + \frac{f_o f_m}{f_o^2 - f^2} \right)$, $\mu_{xy} = \mu_B \frac{f_o f_m}{f_o^2 - f^2}$, and $\mu_{zz} = \mu_B$.

The tensor are called the usual Polder tensor element, with $f_o = (1/2\pi)\gamma\mu_o H_o$ and $f_m = (1/2\pi)\gamma\mu_o M_o$. M_o is the dc saturation magnetization and μ_B is the background (optical - magnon) permeability.

By virtue of eqn. (9), magnetic potential Ψ exists and it satisfies the following equation:

$$\frac{\partial^2 \Psi}{\partial x^2} = k^2 \Psi. \quad (13)$$

The solution is written as [16]:

$$\Psi = -(Ae^{kx} - Be^{-kx})e^{i(\omega t - ky)}. \quad (14)$$

Each component of r.f. magnetic field is calculated using $\mathbf{H} = -\nabla\Psi$, and written as:

$$H_{x2} = k(Ae^{kx} - Be^{-kx})e^{i(\omega t - ky)}, \quad (15)$$

$$H_{y2} = -ik(Ae^{kx} + Be^{-kx})e^{i(\omega t - ky)}, \quad (16)$$

$$B_{x2} = \mu_0 k[(\mu_{xx} + s\mu_{xy})Ae^{kx} - (\mu_{xx} - s\mu_{xy})Be^{-kx}]e^{i(\omega t - ky)}, \quad (17)$$

$$E_{z2} = \frac{\omega}{k} B_{x2}, \quad (18)$$

where s takes +1 for the propagation of the waves in the forward direction, and s takes -1 for the propagation of the waves in the backward direction.

iii. In LHM substrate ($x < 0$)

$$\nabla \times \mathbf{H}_3 = i\omega \varepsilon_o \varepsilon(\omega) \mathbf{E}_3, \quad (19)$$

$$\nabla \times \mathbf{E}_3 = -i\omega \mu_o \mu(\omega) \mathbf{H}_3, \quad (20)$$

where $\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$, $\mu(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_r^2}$ the losses are neglected, and the values of the parameters ω_p , ω_r and F are chosen to fit approximately to the experimental data [10, 17]: $\omega_p / 2\pi = 10$ GHz, $\omega_r / 2\pi = 4$ GHz, and $F=0.56$. For this set of parameters, the region in which permeability and permittivity are simultaneously negative is from 4 GHz to 6 GHz.

From equations (19) and (20), we can get the differential equation as:

$$\frac{\partial^2 E_{z3}}{\partial x^2} = (k^2 - k_o^2 \varepsilon(\omega) \mu(\omega)) E_{z3}. \quad (21)$$

The solution of equation (21) can be written as

$$E_{z3} = C e^{k_3 x} e^{i(\omega t - ky)}, \quad (22)$$

where $k_3 = (k^2 - k_o^2 \varepsilon(\omega) \mu(\omega))^{1/2}$ and A , B , and C are amplitude coefficient determined from the boundary condition. From equations (20) and (22), we get the magnetic field components as follows,

$$H_{x3} = \frac{k}{\omega \mu_o \mu(\omega)} E_{z3}, \quad (23)$$

$$H_{y3} = \frac{-i}{\omega \mu_o \mu(\omega)} \frac{\partial}{\partial x} E_{z3}. \quad (24)$$

Matching the field components H_y and E_z at the boundaries $x = 0$ and $x = t$, the dispersion relation is found to be;

$$e^{-2kt} = \frac{[1 + u(\mu_{xx} + s\mu_{xy})] \left[\frac{k\mu(\omega)}{k_3} + (\mu_{xx} - s\mu_{xy}) \right]}{[1 - u(\mu_{xx} - s\mu_{xy})] \left[\frac{k(\omega)}{k_3} - (\mu_{xx} + s\mu_{xy}) \right]}, \quad (25)$$

where $u = \tanh[k(t - x_o)]$ is called the nonlinear term, and varies from -1 to 1, according to the values of the cover-film interface nonlinearity. Noting that the change of the sign s changes the dispersion relation, this implies that the nonlinear magneto static surface waves exhibit the non-

reciprocity phenomena. For the case, where the substrate is dielectric, $k_3 = k$, and $\mu(\omega) = 1$, so we can get the same dispersion relation for Shabat [7]:

$$e^{-2kt} = \frac{[1 + u(\mu_{xx} + s\mu_{xy})][1 + (\mu_{xx} - s\mu_{xy})]}{[1 - u(\mu_{xx} - s\mu_{xy})][1 - (\mu_{xx} + s\mu_{xy})]} \quad (26)$$

In the linear limit $u = 1$ or $\alpha = 0$, we get the dispersion relation:

$$e^{-2kt} = \frac{[1 + (\mu_{xx} + s\mu_{xy})][1 + (\mu_{xx} - s\mu_{xy})]}{[1 - (\mu_{xx} - s\mu_{xy})][1 - (\mu_{xx} + s\mu_{xy})]} \quad (27)$$

This is exactly the expression for the Doman-Eshbach surface wave mode [1, 2].

III. POWER FLOW

The power flux of the surface waves along the direction the propagation (y-direction) is calculated by integrating the Poynting vector over the variable x:

$$P = \frac{1}{2} \int (\mathbf{E} \times \mathbf{H}^*) dx = \frac{1}{2} \int_{-\infty}^{\infty} E_z H_x^* dx, \quad (28)$$

$$= P_{NL} + P_{Fer} + P_{LH},$$

where P_{NL} , P_{Fer} and P_{LH} are the power fluxes in the nonlinear dielectric media, Ferrite, and LHM media respectively and given by:

$$P_{NL} = \frac{2k^2 \varepsilon_o P_o}{\mu_o k_o^2} (1 + \nu), \quad (29-a)$$

$$P_{Fer} = \frac{k_o^2 B_o^2 \varepsilon_o P_o}{\mu_o k_o^2} [Z^2(\mu_{xx} + s\mu_{xy})e^{2kt} - (\mu_{xx} - s\mu_{xy})e^{-2kt} - Z^2(\mu_{xx} + s\mu_{xy}) + (\mu_{xx} - s\mu_{xy}) - 4Zkt\mu_{xx}], \quad (29-b)$$

$$P_{LH} = \frac{k^3 B_o^2 \varepsilon_o P_o [Z(\mu_{xx} + s\mu_{xy}) - (\mu_{xx} - s\mu_{xy})]^2}{\mu_o \mu_{eff} k_3 k_o^2}, \quad (29-c)$$

where $B = (1 - u^2)^{\frac{1}{2}} [Z(\mu_{xx} + s\mu_{xy})e^{kt} - (\mu_{xx} - s\mu_{xy})e^{-kt}]^{\frac{-1}{2}}$, $P_o = \frac{1}{2\omega\alpha\varepsilon_o}$,

and $Z = \frac{\frac{k\mu(\omega)}{k_3} + (\mu_{xx} - s\mu_{xy})}{(\mu_{xx} + s\mu_{xy}) - \frac{k\mu(\omega)}{k_3}}$.

IV. RESULTS AND DISCUSSION

The nonlinear dispersion relation (25) has been solved to find out the effective wave index (β_{eff}) as a function of the frequency (f), where $\beta_{eff} = ck / \omega$, for different values of nonlinearity (u) as shown in Fig. 2. The numerical computations were carried out with the same

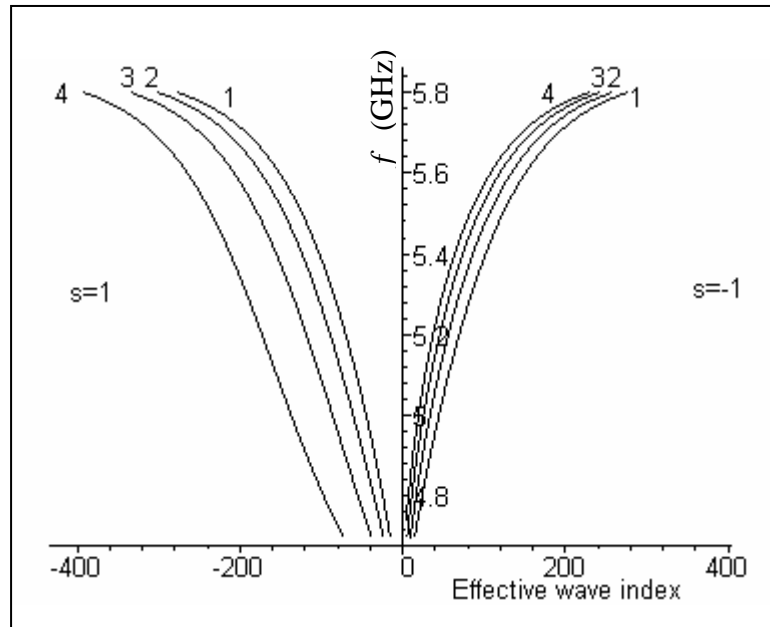


Fig. 2. Computed the effective wave index β_{eff} versus operating frequency (f) for different values of the nonlinear term u (curve 1, $u = 0$; curve 2, $u = -0.2$; curve 3, $u = -0.4$; curve 4, $u = -0.6$), and $t = 50 \mu\text{m}$

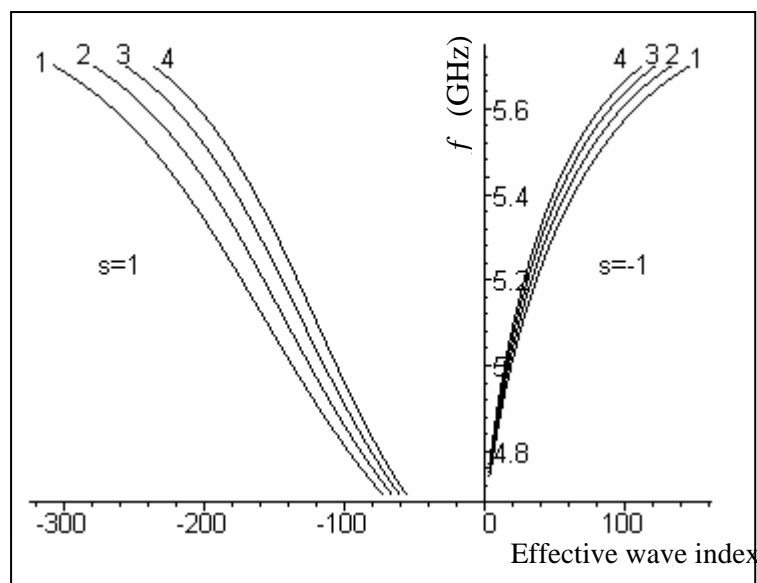


Fig. 3. Computed the effective wave index (β_{eff}) versus operating frequency (f) for different values of film thickness (t) (curve 1, $t = 50 \mu\text{m}$; curve 2, $t = 55 \mu\text{m}$; curve 3, $t = 60 \mu\text{m}$; curve 4, $t = 65 \mu\text{m}$), and $u = -0.6$.

Data parameters for the gyromagnetic ferrite film and nonlinear dielectric cover as used in papers [7,18]: $\mu_o H_o = 0.1\text{ T}$, $\mu_B = 1.25$, $\mu_o M_o = 0.1750\text{ T}$, $\alpha = 1.55 \times 10^{-10} \text{ m}^2\text{V}^{-2}$, $\varepsilon_f = 1$, $\gamma = 1.72 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}$. The region of interest is $\mu_v = \frac{\mu_{xx}^2 - \mu_{xy}^2}{\mu_{xx}} \leq 0$, so the solution should be limited to the range of the magnetostatic surface waves from $\sqrt{f_o(f_o + f_m)}$ to $(f_o + f_m)$.

Figure 2 represents the propagation characteristics of the strongly nonlinear magnetostatic surface waves guided by a fixed YIG film ($t = 50 \mu\text{m}$) and different values of the nonlinear terms u ($u = 0; -0.2; -0.4; -0.6$). It has been found that the nonlinear TE magnetostatic surface waves structure can be supported by the considered structure in the range is approximately from 4.75 GHz to 5.85 GHz. In the case, where the substrate is dielectric as in Eqn. (27), Sodha, Srivastava, [2] and Shabat [7], had found the reciprocal behavior in the linear case, where $u = 1$.

However, in our case, we have found that the reciprocal behavior is only in the nonlinear case, where $u = 0$, as shown in fig. 2 (Curve 1). It is also found that the propagation constant can be made different or tuned for the wave propagation in the forward and backward directions, for a thin film. This results, is in contrast with the results which has been obtained by Shabat[7]. It is also seen that effective propagation constant (β_{eff}) is more sensitive to the nonlinearity for the forward than the backward direction. The dispersion relation is also been solved at a fixed value of the cover-film interface nonlinearity ($u = -0.6$).

Figure 3 shows the dispersion curves (β_{eff} against f) for different values of the film thickness ($t = 50; 55; 60; \text{ and } 65 \mu\text{m}$). It can be seen that the effective propagation constant (β_{eff}) is more sensitive to the change of the film thickness in the forward ($s = 1$) than in the backward direction ($s = -1$).

The differential phase constant or the phase shift $\Delta\beta$ between the counter-propagating waves is calculated from equation (25) as:

$$\beta_{\pm} = \frac{1}{2k_o t} \ln \frac{[1 + u(\mu_{xx} + s\mu_{xy})] \left[\frac{k\mu_{eff}}{k_3} + (\mu_{xx} - s\mu_{xy}) \right]}{[1 - u(\mu_{xx} - s\mu_{xy})] \left[\frac{k\mu_{eff}}{k_3} - (\mu_{xx} + s\mu_{xy}) \right]}, \quad (30)$$

where β_+ is the effective propagation constant for $s = 1$, β_- is the effective propagation constant for $s = -1$, and $\Delta\beta = \beta_+ - \beta_-$.

Figure 4, illustrates the phase shift $\Delta\beta$ for forward and backward directions, as a function of the film thickness for different values of signal operating frequency and a fixed value of nonlinearity. It is shown that, the guiding structure exhibits minimum non-reciprocity, when the structure has a thicker film and a smaller operating frequency, which is similar as in the case when the substrate is dielectric [7]. This means that the values of non-reciprocity can be controlled by adjusting the signal operating frequency and the YIG film thickness.

Equation 29 with the help of the dispersion relation (25), has been calculated to compute the normalized power (P/P_o) as a function of effective wave index (β_{eff}), for different values of frequency ($f = 5.5; 5.7; \text{ and } 5.75 \text{ GHz}$) as shown in Figure 5. Several main points of interest have been found. The expected nonreciprocal behavior shows a strong nonlinear dependence upon power.

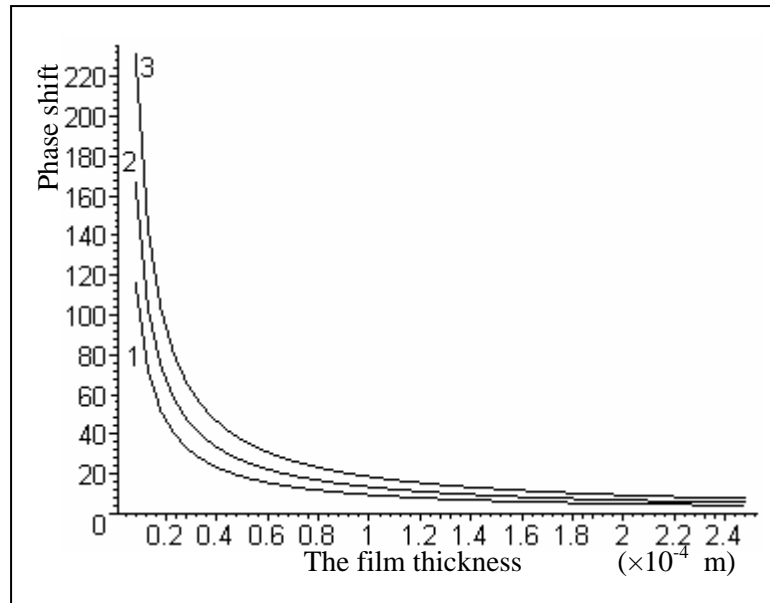


Fig. 4. Computed the phase shift $\Delta\beta$ for wave propagating in the two directions for different values of frequency (f): (curve 1, $f = 4.8$ GHz; curve 2, $f = 5$ GHz; curve 3, $f = 4.8$ GHz), and $u = 0.2$.

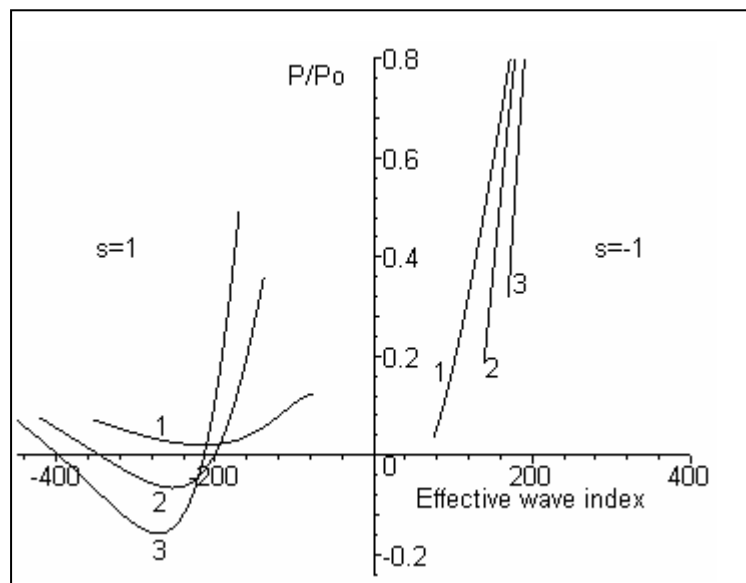


Fig. 5. Normalized total power (P/P_0) versus effective wave index (β_{eff}), for different values of frequency (f): (1) $f = 5.5$ GHz, (2) $f = 5.7$ GHz, (3) $f = 5.75$ GHz and $t = 50 \mu\text{m}$.

For a given frequency, the power can be significantly greater in the backward direction ($s=-1$) than in the forward direction ($s=1$). This could lead to some interesting experimental possibilities involving nonreciprocal nonlinear power transfer. In forward direction we see that the power is strongly dependent upon the frequency, while in the backward direction, the power-frequency dependence is decreased by increasing β_{eff} . In high frequency (curves labelled 1, 2), the power shows an inverted behavior, as it has a negative magnitude. This behavior is in contrast to the case of a right-handed material (RHM) substrate. We also notice some kinds of bistability in high frequency, where one level of power can excite two nonlinear magnetostatic surface waves with different speeds, in the

forward direction and it has not been found in the other direction. Finally, figure 5, shows a definite end or cut-off end. These behaviors might be used in the design and construction of microwave devices such as switches, filters, isolators, phase shifter, filters and circulators.

V. CONCLUSION

The dispersion characteristics of strongly nonlinear magnetostatic surface waves propagating in a three layered structure of nonlinear dielectric cover, a YIG film and LHM substrate have been investigated theoretically. We have found that the reciprocal behavior is only found in a special case where the nonlinearity $u = 0$; which is in contrast for the RH material. We have seen some kind of bistability in forward direction and it is not found the other. We also seen the non-reciprocity can be minimized by the signal operating frequency and the YIG film thickness.

REFERENCES

- [1] Damon R. J., Esbach J.: Bragg diffraction of surface magnetostatic waves from magnetic lattices. *Phys. Chem. Solids*: 19 (1961), 308.
- [2] Sodha M. S., and Srivastava N. C.: *Microwave Propagation in Ferromagnetic*, Plenum Press, New York, London, 1981.
- [3] Boardman A. D., Shabat M. M., and Wallis R. F.: Nonlinear magnetodynamic waves on magnetic materials. *Phys. Rev.:* B 41, (1990), 717.
- [4] Boardman A. D., Shabat M. M., and Wallis R. F.: TE waves at an interface between linear gyromagnetic and nonlinear dielectric media. *J. Phys. D: Appl. Phys.:* 24, (1991), 1702.
- [5] Shabat M. M.: New nonlinear magnetostatic surface waves in a metallised ferromagnetic film. *Optica Applicata*: 24 (1994) 293.
- [6] Shabat M. M.: Strongly nonlinear magnetostatic surface waves in a grounded ferrite film. *Phys. Status Sol.:* (a), 149 (1995) 691.
- [7] Shabat M. M.: Nonlinear magnetostatic surface waves in a gyromagnetic film. *Philosophical Magazine*: B, 73 (4), (1996) 664.
- [8] Zvedin A. K., and Popkov A. F.: Three- and four-magnon decay of nonlinear surface magnetostatic waves in thin ferromagnetic films. *Soviet Phys. JETP*: 57 (1983) 350.
- [9] Boardman A. D., Nikitov S. A., and Wang Q.: Theory of multistable magnetostatic waves. *IEEE Trans. Magn.:* 30 (1994)1.
- [10] Shadrivov I. V., Sukhorukov A. A., Kivshar Y. S., Zharov A. A., Boardman A. D., and Egan P.: Nonlinear surface waves in left-handed material. *Phys. Rev.:* E 69, (2004) 016617.
- [11] V. G. Veselago: The electrodynamics of substances with simultaneously negative values of ϵ and μ . *Sov. Phys.: Usp.* 10 , (1968) 509[*Usp. Fiz. Nauk* 92,(1967) 517].
- [12] Smith D. R., Padilla W., Vier D. C., Nasser N., and Shultz S.: Guided modes in negative-refractive-index waveguide. *Phys. Rev. Lett.:* 85, (2000) 4184.
- [13] Shelby R. A., Smith D. R., Shultz S.: Experimental verification of a negative index of refraction. *Science*: 292, (2001) 77.
- [14] Shelby R. A., Smith D. R., Nasser N., and Shultz S.: Microwave transmission through a two-dimensional, isotropic, left-handed metamaterial. *Appl. Phys. Lett.:* 78 (4) (2001) 489.
- [15] Mihalache D., Nazmitdinov R. G., and Fedyanin V. K.: Nonlinear optical waves in layered structure. *Sov. J. Nucl.:* 20(1) (1989) 86.
- [16] Awai I., Ohtsuki K.,and Ikenoue J.: Interaction of magnetostatic waves with drifting carriers. *Japanese Journ. of Appl. Phys.:* 15 (7) (1976) 1305.

- [17] Ruppin R.: Surface polaritons of a left-handed material slab. *J. Phys: Condens. Matter:* 13 (2001) 1811.
- [18] Shabat M. M., and Pelzl J.: Nonlinear electromagnetic surface waves in magnetic structure. *infrared Phys. & Tech.:* 37 (1996) 265.