

## Magnetostatic Surface Waves Propagation at the interface between Ferrite and MTMs Parallel Plate Waveguide structure

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**Abstract:** This work considers magnetostatic surface waves propagation in parallel structure composed of ferrite film bounded from below by metamaterial (MTM) placed on metal substrate and from top by air. The Maxwell equations are used to analyse propagation of slow (magnetostatic) surface electromagnetic wave in this structure. Results show that magnetostatic surface wave (MSSW) propagates at certain range of frequencies and varies as both MTM layer and ferrite layer thicknesses change. The new behaviour of the proposed waveguide in the presence of MTM in particular the appearance of the unidirectional MSSW is promising for practical applications such as isolators.

**Keywords:** Metamaterials, Ferrite, magnetostatic surface waves, surface waves.

### 1. Introduction

Metamaterials (MTMs) are media with simultaneous negative permittivity and permeability at a certain frequency. In these MTMs, the directions of the electric field, the magnetic field, and the wave propagation vector obey the left-hand rule, thus MTMs are also known as left-handed materials (LHMs). Veselago (1968) theoretically predicted numerous unusual electromagnetic phenomena of these MTMs, such as a sign variation of group velocity, negative refraction, and perfect lensing. Attempts to achieve negative permittivity by using metallic wires in the GHz band were made by Pendry et al. (1996), which was like the plasmons of metals in optical frequency regions. Further, Pendry et al. (1999) suggested that an effective negative permeability could be attained in the GHz band

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from a periodic arrangement of split ring resonators (SRRs). Pendry et al. (1996) showed that a combination of “magnetic atoms” and “electric atoms” with negative permeability and permittivity, respectively, can lead to materials with a negative index of refraction. These materials open a new part of photonics associated with new concepts and potential applications (Pendry, 2000; Smith, 2004).

Smith et al. (2000) assembled metallic wires and SRRs to create simultaneously negative permittivity and permeability in the desired frequency range. Kim et. al. (2006) investigated the guided dispersion characteristics of grounded slab waveguide structures which have practical applications in the area of microwave and millimeter wave circuits and antennas. A substantial amount of research has been conducted to study MTMs application in communication application such as isolators (El-Khozondar, H., 2008) and sensors (El-Khozondar, R., 2008).

Ferrites are magnetic materials characterized by anisotropic properties and various energetic interactions, such as dipole, exchange magnetoelastic and magneto-optical. The crystal structures of ferrites are the body-centered cubic lattice. Their electrical properties are semiconductors. Their low conductivity combined with a high magnetic permeability made them a most valuable material for transformer cores (Snoek, 1947). Rogers (1952) discovered that ferrites have an extremely high Faraday's effect, a feature fitting them for several applications in the microwave area. When ferrites are employed in microwave devices, they are operated in the saturated state. The permeability of magnetically saturated ferrites in the microwave range varies with the saturation magnetization, the microwave frequency, and the outside magnetic field. Consequently, ferrites make tuneable photonic band gaps probable with functional fields. Kee et al. (2000) investigated theoretically the properties of two-dimensional ferrite photonic crystals for two independent polarizations.

The propagation of magnetostatic waves in layered structures consisting of ferrite materials has attracted much attention (Damon, 1961; Bongianni, 1972; Bestler, 1959; Courtois, 1970; El-Khozondar, H. 2010), owing

to various applications of ferrite in the microwave devices and extremely important for designing integrated devices such as narrow-frequency optical or microwave filters and high-speed switches (Vasseur, 1996; Al-Wahsh, 1999; Gulyaev, 2001).

The type of surrounding materials affects the properties of the propagating modes. Therefore, it is interesting to see the combined effect of these materials on the surface wave propagation. In this paper, the transverse electric (TE) wave propagation in a structure consists of MTM film surrounded by a metal substrate and a magnetized ferrite cover bounded by air is investigated.

The paper is organized as follows: In the following section, the theoretical steps are summarized. Next, the numerical results are discussed. Last section is dedicated for conclusion.

## 2. Theoretical Analysis

Figure 1 exhibits the configuration of the proposed waveguide which is an asymmetric planar waveguide. It consists of MTMs film having width  $w$  placed on a metal substrate and bounded from above by a Ferrite film with width  $s$  covered by air.

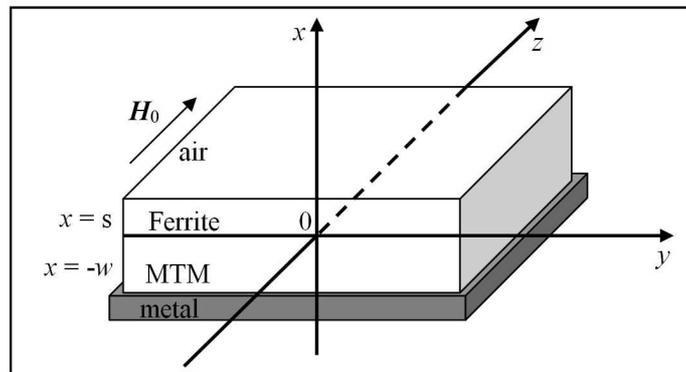


Figure 1. Proposed asymmetric waveguide structure

The slab has infinite extent in the  $z$  and  $y$  directions. We assumed lossless MTM film with both permittivity  $\epsilon_M$  and permeability  $\mu_M$  are function of the frequency ( $\omega$ ) as follows:

$$\epsilon_M = 1 - \frac{\omega_p^2}{\omega^2}, \quad (1)$$

$$\mu_M = 1 - \frac{F \omega^2}{\omega^2 - \omega_r^2}, \quad (2)$$

where  $\omega_r$  is the resonance frequency and  $\omega_p$  is the plasma frequency. The values of the parameters  $\omega_r$ ,  $\omega_p$  and  $F$  are chosen to fit the experimental data (Shebly, 2001):  $\omega_p = 26.6\pi$  GHz, and  $F = 0.37$ .

In the ferrite slab a static magnetic field is applied in the  $z$  direction, resulting in a uniform intensity  $H_0$  within the ferrite. The ferrite slab has positive permittivity  $\epsilon_f$  and permeability  $\mu_f$  defined as (Bestler, 1959; Courtois, 1970; El-Khozondar, H., 2010, Vasseur, 1996).

$$\|\mu_f\| = \begin{bmatrix} \mu_{11} & i \mu_{12} & 0 \\ -i \mu_{12} & \mu_{11} & 0 \\ 0 & 0 & \mu_z \end{bmatrix}, \quad (3)$$

where  $\mu_{11} = 1 + \frac{\omega_M \omega_H}{\omega_H^2 - \omega^2}$ ,  $\mu_{12} = \frac{\omega \omega_M}{\omega_H^2 - \omega^2} \mu_z = 1$ ,  $\omega$  is surface wave frequency,  $\omega_M = 4\pi\gamma M_0$  is the magnetic frequency,  $\omega_H = \gamma H_0$  is the Larmor frequency,  $\gamma$  is the electromagnetic oscillation frequency,  $4\pi M_0$  is the ferrite saturation magnetization, and  $H_0$  is the applied magnetic field (Bespyatykh, 2001).

In this study, we only considered transverse electric fields (TE). The TE fields are assumed to be  $\mathbf{E} = (0, 0, E_z) e^{j\omega t}$  and  $\mathbf{H} = (H_x, H_y, 0) e^{j\omega t}$ . Waves are assumed to propagate in the  $y$  direction. The variation in the  $z$ -direction is assumed to be zero. Applying the defined fields into Maxwell's equations gives the following field equation,

$$\frac{\partial^2 E_{zi}}{\partial x^2} + \frac{\partial^2 E_{zi}}{\partial y^2} + q_i^2 E_{zi} = 0, \quad (4)$$

where  $i$  indicates  $f$  for Ferrite layer,  $M$  for MTMs layer, and  $l$  for the linear cladding layer (air),  $q_l = k_0$ ,  $q_f = k_0 \sqrt{\epsilon_f \mu_v}$ ,  $q_M = k_0 \epsilon_M$ ,  $k_0 = \omega/c$  and  $\mu_v = \frac{\mu_{11}^2 - \mu_{12}^2}{\mu_{11}}$ , which is called the Voigt permeability.

Solving above equations for each layer and implementing boundary conditions, we get the following dispersion equation,

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$$\left(k_{xf} + \frac{\nu}{\mu}k_y - \mu_v k_{xl}\right) \left( \left( \frac{\nu}{\mu}k_y - k_{xf} \right) \tanh(k_{xM}w) + \mu_v k_{xM} \right) e^{-(2k_{fs} s)} +$$

$$\left(k_{xf} + \frac{\nu}{\mu}k_y + \mu_v k_{xl}\right) \left( \left( \frac{\nu}{\mu}k_y + k_{xf} \right) \tanh(k_{xM}w) + \mu_v k_{xM} \right) = 0, \quad (5)$$

where  $k_y^2 - k_{xi}^2 = q_i^2$ . The dispersion relation equation (5) relates the transverse wave numbers for each media. It is an implicit equation that gives the surface wave dispersion relation. When  $w \rightarrow \infty$ , equation (5) becomes,

$$\left(k_{xf} + \frac{\nu}{\mu}k_y - \mu_v k_{xl}\right) \left( \left( \frac{\nu}{\mu}k_y - k_{xf} \right) + \mu_v k_{xM} \right) e^{-(2k_{fs} s)} +$$

$$\left(k_{xf} + \frac{\nu}{\mu}k_y + \mu_v k_{xl}\right) \left( \left( \frac{\nu}{\mu}k_y + k_{xf} \right) + \mu_v k_{xM} \right) = 0. \quad (6)$$

In the limit  $w = 0$ , the structure reduces to ferrite-metal structure and equation (5) reduces to the following form,

$$k_{xf} \coth(k_{xf} s) - \frac{\nu}{\mu}k_y + \mu_v k_{xl} = 0. \quad (7)$$

Equation (5) to equation (7) can only be solved numerically. The solutions of these equations give the MSSW at the different limits. The limiting frequencies for MSSW free ferrite film are as follows (Damon, 1961):

$$\omega_s = \sqrt{\omega_H^2 + \omega_H \omega_M} \quad (8)$$

$$\omega_{fin} = \omega_H + \omega_M / 2 \quad (9)$$

where  $\omega_s$  is the starting frequency and  $\omega_{fin}$  is the final frequency.

### 3. Numerical Calculation

In all the calculations, we chose  $\omega_H = 1.76 \times 10^7 H_0$  rad/s,  $\omega_M = 1.76 \times 1870 H_0$  rad/s,  $H_0 = 3670$  Oe,  $\epsilon_2 = 15$ , and  $\omega_p = 83.56 \times 10^9$  rad/s. The value of frequency  $\omega$  is chosen such that  $\epsilon_M$ ,  $\mu_M$ , and  $\mu_v$  are negative. Accordingly,  $\omega_s / \omega_p = 0.19$ ,  $\omega_{fin} / \omega_p = 0.27$  and  $\omega_{finFM} / \omega_p = 0.47$ .

The normalized MSSW frequency with respect to plasma frequency ( $\omega / \omega_p$ ) is plotted as a function of propagation constant ( $k_y$ ) in Figure 2. The parameter values are: MTM-layer thickness  $w = 79 \times 10^{-4}$  cm and the ferrite thickness  $s$  varies as follows:  $s_1 = 6.2$   $\mu\text{m}$ ,  $s_2 = 7.2$   $\mu\text{m}$ ,  $s_3 = 8.2 \times 10^{-4}$  cm and  $s_4 = 9.2$   $\mu\text{m}$ . One can see that the curve

is not distinct from the one for the free ferrite where MSSW appears at almost the same range of frequency interval, 0.1965 to 0.2333. However, there is not a second curve that exist for free ferrite film and would be symmetric to the existing curve regarding to the frequency axis. That is the MSSW is always directed oppositely to y-axis, i.e. this MSSW is unidirectional and backward.

Figure 3 illustrates the dependence of the normalized MSSW frequency on  $k_y$  at  $w=7.9 \mu\text{m}$  for the same values of  $s$  as mentioned above. We noticed that the MSSW can travel in both  $\pm y$ -axis. However, the symmetry that exists in the free ferrite is lost. The forward MSSW takes place for the range of frequencies from 0.206 to 0.219 and backward frequencies appears at the other frequencies. It is obvious from figure 2 and figure 3 that the behaviour of the MSSW varies with the changes of thicknesses of both the MTM layer and the ferrite layer.

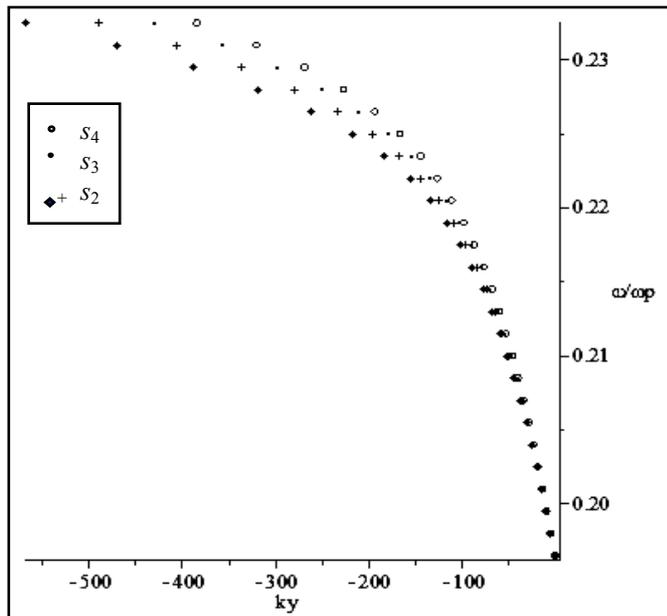


Figure 2. The normalized frequency versus  $k_y$  at  $w=79 \mu\text{m}$  for different values of  $s$ .

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Similar calculation is done for  $s=7.9 \mu\text{m}$  and variant values of  $w$  as follows:  $w_1=6.2 \mu\text{m}$ ,  $w_2=7.2 \mu\text{m}$ ,  $w_3=8.2 \mu\text{m}$ , and  $w_4=9.2 \mu\text{m}$ . The normalized frequency versus  $k_y$  is shown in figure 4. It is shown in the figure that both forward and backward MSSW exist and the symmetry regarding frequency axis is lost. The forward MSSW occurs at frequencies ranges from 0.206 to 0.219. Looking at figure 3 and figure 4, we notice that the backward MSSW is clearly diverse for different values of  $s$ ; on the other hand, the forward MSSW differs for different values of  $w$ .

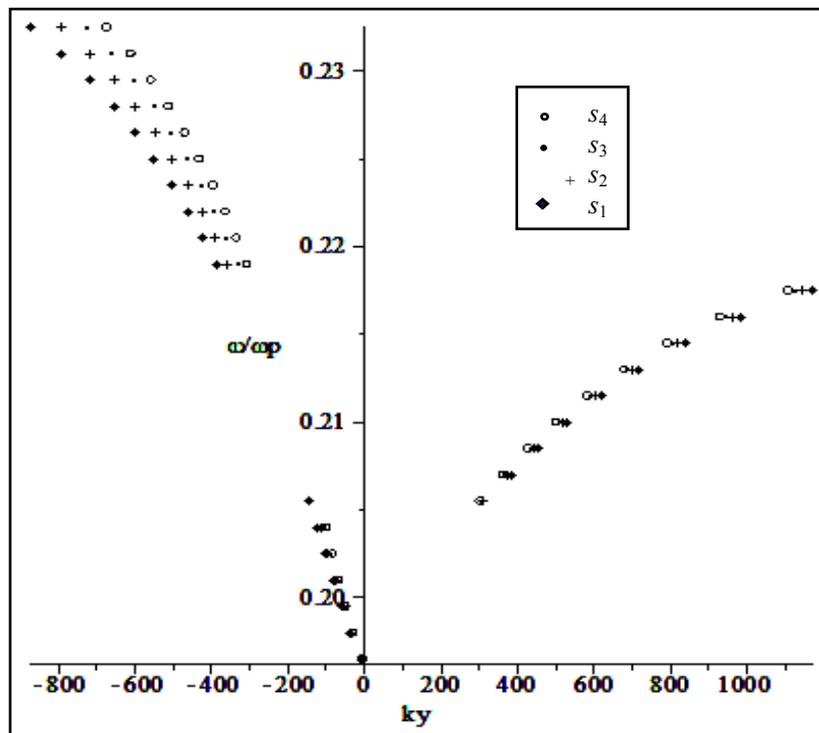


Figure 3. The normalized frequency versus  $k_y$ , at  $w=7.9 \mu\text{m}$  for different values of  $s$ .

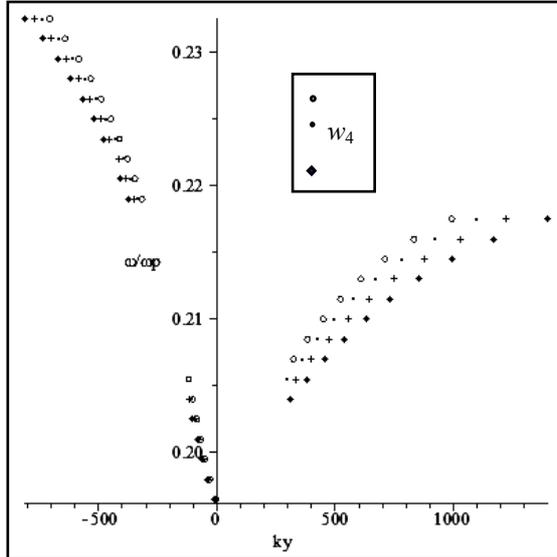


Figure 4. The normalized frequency versus  $k_y$ , at  $s=7.9 \mu\text{m}$  for different values of  $w$ .

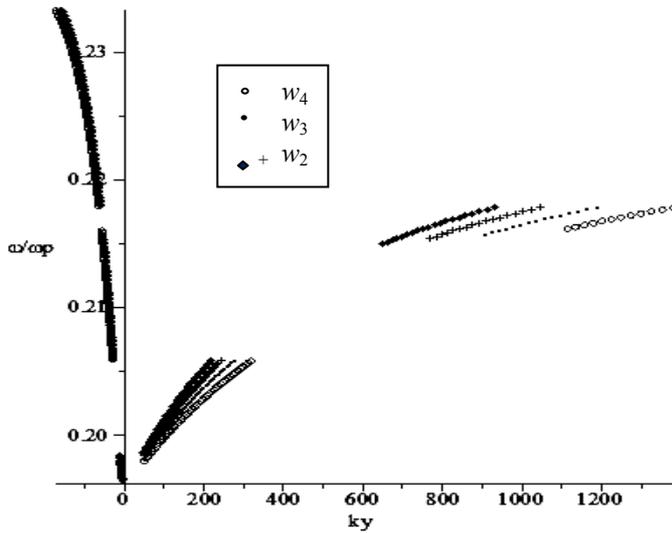


Figure 5. The normalized frequency versus  $k_y$ , at  $s=79 \mu\text{m}$  and different values of  $w$ .

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The MSSW normalized frequency dependence on propagation constant is plotted in figure 5 for constant value of  $s=79 \mu\text{m}$  and the values of  $w$  vary as follows:  $w_1=6.2 \mu\text{m}$ ,  $w_2=7.2 \mu\text{m}$ ,  $w_3=8.2 \mu\text{m}$ , and  $w_4=9.2 \mu\text{m}$ . It is observed from figure 5 that the MSSW changes as  $w$  changes and has different values for the forward modes. It is also found that the forward MSSW appears in two regions: in the first region the values of  $\omega/\omega_p$  change from 0.198 to 0.205 and in the second region the values of  $\omega/\omega_p$  vary from 0.217 to 0.218.

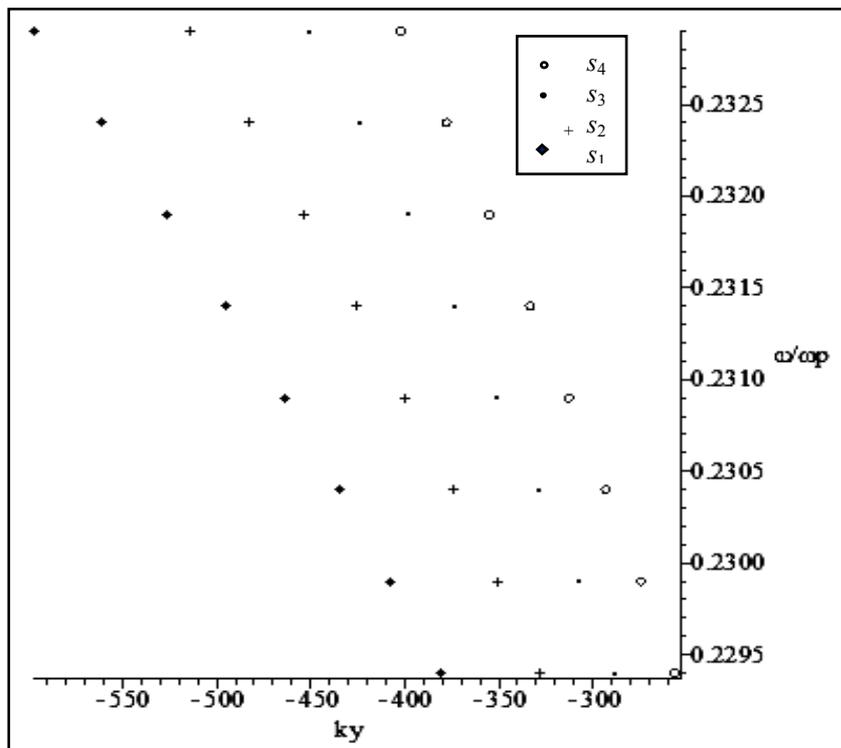


Figure 6. The normalized frequency as a function of propagation constant when  $w \rightarrow \infty$  for different values of  $s$ .

Figure 6 exhibits the relation between  $\omega/\omega_p$  and  $k_y$  when  $w \rightarrow \infty$  for the following values of  $s$ :  $s_1=6.2 \mu\text{m}$ ,  $s_2=7.2 \mu\text{m}$ ,  $s_3=8.2 \mu\text{m}$ , and  $s_4=9.2 \mu\text{m}$ . It is shown that only backward MSSW appears at limited range of frequency ranges from 0.2294 to 0.2329. This behaviour

agrees with the results we obtained in figure 2 and disagrees with the results in figure 3 where  $w$  decreases one order of magnitude. That is for small values of  $w$  both forward and backward MSSW exists while for large values of  $w$  only backward MSSW exists.

The normalized magnetostatic waves frequency is plotted as a function of propagation constant at  $w = 0$  for  $s$  varies as follows:  $s_1=6.2 \mu\text{m}$ ,  $s_2=7.2 \mu\text{m}$ ,  $s_3=8.2 \mu\text{m}$  and  $s_4=9.2 \mu\text{m}$  in figure 7. In this case only backward MSSW appears for the same range of frequencies from 0.1965 to 0.2333 as shown in figure 2 and changes as  $s$  varies.

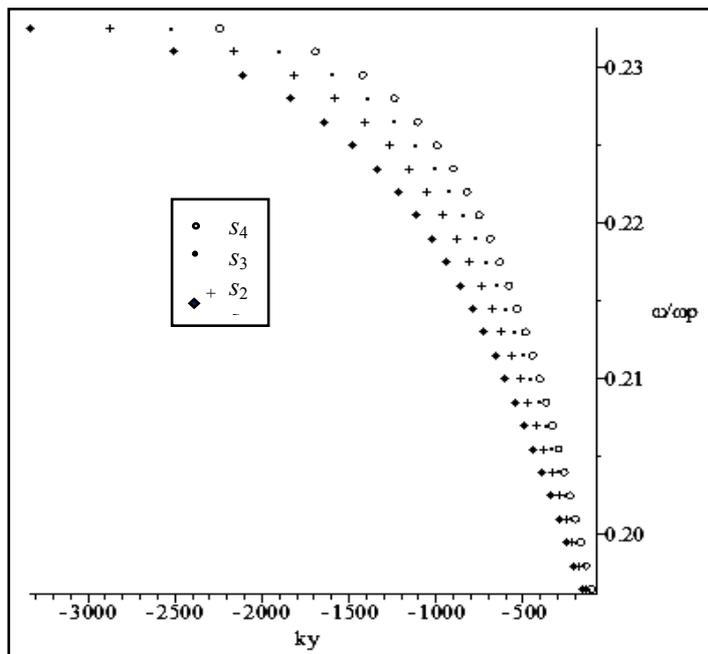


Figure 7. The normalized frequency as a function of propagation constant for  $w = 0$  and different values of  $s$ .

#### 4. Conclusion

The considered configuration is an asymmetric slab waveguide consisting of MTM film placed on a metal substrate and bounded from above by a Ferrite film covered by air. The dispersion relation for the magnetostatic waves are studied at three different values of the

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MTM layer thickness. The three resulted dispersion equations are numerically solved. Results are presented by plotting the MSSW frequency as a function of the propagation constant. Results demonstrate that MSSW frequency depends on both MTM layer and ferrite layer thicknesses. It is found that the range of the allowed frequency changes; moreover, the dependency of the positive and negative propagation constants change with changing MTMs thickness and /or the ferrite thickness. This result is promising in improving the waveguide performance. It also has industrial applications; *i. e.* isolators.

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