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S-WAVES IN A NONLINEAR, LEFT- HANDED MATERIALS AND FERRITE LAYERED STRUCTURE

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Abstract. Nonlinear magnetostatic surface wave in a slab waveguide structure has been investigated. The behavior of TE nonlinear waves in an optically nonlinear film is given. Electromagnetic surface waves propagating in a nonlinear dielectric film bounded by a ferrite cover and left handed material substrate (a medium with both negative dielectric permittivity and negative magnetic permeability-LHM) are examined theoretically. The new mathematical results are expressed in terms of physical parameters of the system. A dispersion relation based on Jacobian Elliptic Functions is derived. The general dispersion relation is derived and analyzed numerically. Dispersion curves labeled with optical power density at the lower film boundary, detailed plots of the variation of electric field amplitude as the wave number changes It is shown that the proposed waveguide structure depends on the refractive index efficiently controlled by varying the frequency.

Keywords: Left-handed materials, gyromagnetic ferrite, dispersion relation Nonlinear surface waves.

1. Introduction

The propagation characteristics of nonlinear electromagnetic and magnetostatic surface waves in gyromagnetic (Ferrite materials) wave guide structure have been studied [1-7]. Strongly Nonlinear Magnetostatic Surface Waves In a Ground Ferrite

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Film has been studied[8]. Materials with negative index of refraction: materials which exhibited both negative permeability and permittivity over a certain range of frequencies have received much attention [9] . Thus, they were called left-handed materials (LHMs) or double negative material, left-handed material (LHM), and so on[10]. Materials with negative effective permittivity (ϵ) and permeability (μ) in a certain band of frequency, referred to as metamaterial.

[11] studied electromagnetic fields propagating through left-handed material slabs.

Ziolkowsky et al has been studied left-handed materials both analytically and numerically[12]. Ilya V. Shadrivov, Andrey A. Sukhorukov, and Yuri S. Kivshar study both linear and nonlinear surface waves localized at the interface separating a left-handed medium and a conventional or right-handed dielectric medium. They demonstrate that the interface can support both TE- and TM-polarized surface waves—surface polaritons[13]. multilayer structures as negative refractive index and left-handed materials, and find that for one polarization there is a wide range ($\approx 90^\circ$) of incident angle within which negative refraction will occur. This comes about because the group velocity and the Poynting vector have a large component parallel to the layers, no matter what the angle of incidence of the incoming radiation is. This behavior in turn comes from the large anisotropy of the phase velocities. If one of the components is a ferromagnetic metal, the system can be a left-handed material above the ferromagnetic resonance frequency has been studied [14]. In this paper, we investigate theoretically the behavior of TE surface waves in a nonlinear dielectric film bounded by a gyromagnetic ferrite cover and left handed material (LHM) dielectric substrate. We derive an exact analytical dispersion equation. The obtained numerical results of the dispersion relation are presented and then discussed.

2. Theoretical Model of TE Surface Waves

We investigated and computed the dispersion relation of nonlinear electromagnetic surface waves in a waveguide structure consisting of dielectric film that bounded by a Ferrite cover and left handed material (LHM) substrate. The proposed waveguide structure is considered as a simple one of multi waveguide structure. The propagation characteristics in the above three layers can be tuned and controlled by selecting the different optically frequency .

Fig. 1 shows the coordinate system used, we shall assume that the nonlinear film occupies the region $0 < z < d$, bounded by the ferrite cover of space $z \geq d$ and bounded by LHM substrate. This model shows that the structured LHM have a range of frequencies over which the index of refraction is negative. In such a model ϵ_2 and μ_2 of the LHM take the form

$$\epsilon_2 = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu_2 = 1 - \frac{F \omega_0^2}{\omega^2 - \omega_0^2} \quad (1)$$

where ω is the frequency of the incident light, ω_p and ω_0 are the electronic and magnetic plasma frequencies.

The nonlinear film dielectric is assumed to be kerr-like and isotropic, where the index of refraction will depend linearly on the field intensity satisfying the Eq. (3), so that TE waves will be only considered here. The Kerr-effect is produced via application of an electric field to a material. The molecular structure of the material changes and the bulk properties of the material also change. In a Kerr-effect the materials become doubly refracting when placed in regions of strong electric fields. The nonlinear dielectric function is in the form:

$$\epsilon^{NL} = \epsilon_2 + \alpha \mathbf{E}_y^2, \quad (2)$$

where ϵ_2 is a frequency-dependent linear part, α is a nonlinear coefficient and E_y is real since the waves are guided.

For the nonlinear medium, occupying the film ($d > z > 0$), the field amplitude $E_y(\omega, z) \equiv E_y$, is a solution of the nonlinear equation.

$$\frac{d^2 E_y}{dz^2} + \left(\frac{\omega^2}{c^2} \epsilon_2 - k^2 \right) E_y + \frac{\omega^2}{c^2} \alpha E_y^3 = 0 \quad (3)$$

The cladding is assumed to be a gyromagnetic ferrite layer which is described by a magnetic permeability tensor as

$$\mu(\omega) = \begin{vmatrix} \mu_{xx} & 0 & \mu_{xz} \\ 0 & \mu_B & 0 \\ -\mu_{xz} & 0 & -\mu_{xx} \end{vmatrix} \quad (4)$$

where

$$\mu_{xx} = \mu_B \left(\frac{\omega_0(\omega_0 + \omega_m) - \omega^2}{\omega_0^2 - \omega^2} \right), \quad \mu_{xz} = i\mu_B \frac{\omega\omega_m}{\omega_0^2 - \omega^2}$$

and μ_B are the usual polder tensor elements, ω is the angular frequency of the supported wave, $\omega_0 = \gamma\mu_0\mathbf{H}_0$, $\omega_m = \gamma\mu_0M_0$, \mathbf{H}_0 the applied magnetic field, $\gamma = 1.76 \times 10^{11} s^{-1}T^{-1}$ the gyromagnetic ratio, M_0 considered as the dc saturation magnetization of the magnetic insulator and μ_B has been introduced as the background, optical magnon permeability

The electric and magnetic field vectors for TE waves propagating along the x- axis with angular frequency ω and wave number k are as follows;

$$\mathbf{E} = [0, \mathbf{E}_y(\omega, z), 0] e^{i(kx - \omega t)} \quad (5)$$

$$\mathbf{H} = [\mathbf{H}_x(\omega, z), 0, \mathbf{H}_z(\omega, z)] e^{i(kx - \omega t)} \quad (6)$$

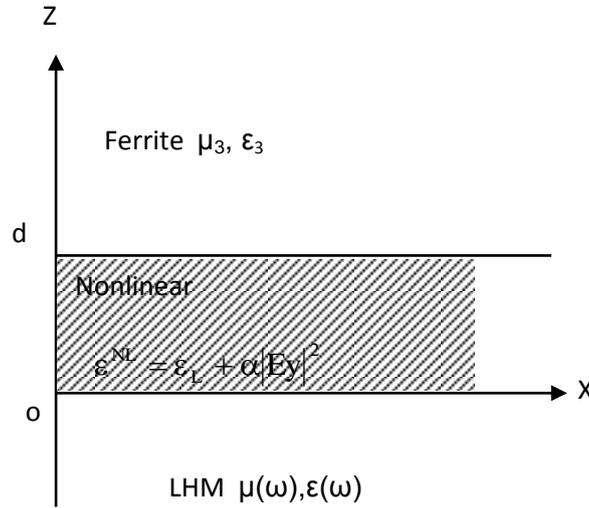


Fig. 1 nonlinear waveguide bounded by Ferrite cover and LHM substrate, $\epsilon_1 = 2.45$, $\epsilon_2 = 2.3$

The traverse electric field modes are going to be considered and propagate along x-axis with wave number k and angular frequency ω . The gyromagnetic ferrite cover has magnetic permeability tensor as in Eq. (3), the dielectric Function of the nonlinear dielectric medium is assumed to be Kerr-like, isotropic and depends on the electric field .

$$\ddot{E}_1 - k_1^2 E_1 = 0 \quad (7)$$

$$\ddot{E}_2 - (k_2^2 - 2\Lambda_2 E_2^2) E_2 = 0 \quad (8)$$

$$\ddot{E}_3 - k_3^2 E_3 = 0 \quad (9)$$

A dot denotes a differentiation with respect to z

The electric fields in cover and substrate for guided or surface waves must decay exponentially towards infinity

(I) In the Ferrite cover:

The electric and magnetic fields component in Ferrite structure are:

$$E_y^{(3)} = E_d e^{k_3(d-z)} \quad (10)$$

and

$$H_x^{(3)}(z) = \left(\frac{\mu_{xx} k_3 - ik \mu_{xz}}{i \omega \mu_o \mu_{xx} \mu_v} \right) E_y \quad (11)$$

$$\mu_{xx} \mu_v = \mu_{xx}^2 + \mu_{xz}^2$$

$$H_z^{(3)}(z) = \left(\frac{\mu_{xz} k_3 + ik \mu_{xx}}{i \omega \mu_o \mu_{xx} \mu_v} \right) E_y \quad (12)$$

(II) For Nonlinear medium:

The electric and magnetic waves are:

$$E_y^{(2)} = P \operatorname{cn} [q(z + z_o) | m], \quad (13)$$

$$H_x^{(2)} = \frac{-iq}{\omega \mu_o} P \operatorname{sn} [q(z + z_o)] \operatorname{dn} [q(z + z_o)], \quad (14)$$

and

$$H_z^{(2)} = \frac{k}{\omega \mu_o} [P \operatorname{cn} [q(z + z_o) | m]]. \quad (15)$$

Where $q = (k_2^4 + 4 \Lambda_2 C_2)^{1/4}$, $\Lambda_2 = \frac{\omega^2 \alpha_2}{2c^2}$ and $P^2 = (q^2 + k_2^2)/2\Lambda_2$,

(III) For Left Handed Material(LHM):

We have the other components of magnetic and electric field as

$$E_y^{(1)} = E_o \exp[k_1 z] \quad (16)$$

$$H_x^{(1)} = \frac{iE_y^{(1)} k_1}{\omega \mu_o \mu_1} \quad (17)$$

$$H_z^{(1)} = \frac{k_x}{\omega \mu_o \mu_1} E_y^{(1)} \quad (18)$$

Where E_o is the electric field at the lower boundary .

$$k_x^2 - \omega^2 \mu_o \varepsilon_o \varepsilon_1 \mu_1 = k_1^2 \quad \omega^2 \mu_o \varepsilon_o \varepsilon_2 \mu_2 - k_x^2 = k_2^2 \quad k_x^2 - \omega^2 \mu_o \varepsilon_o \varepsilon_3 \mu_3 = k_3^2$$

and $\mu_o \varepsilon_o = \frac{1}{c^2}$

Applying the boundary conditions on E and H at both $z = 0$ and $z=d$ we obtain :

$$\begin{aligned} E_y^{(1)} &= E_y^{(2)} \\ E_o \exp(k_1 z) &= P \operatorname{cn} [q(z_o) | m] \\ E_o &= P \operatorname{cn}(z_o q), \\ H_x^{(1)} &= H_x^{(2)} \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{iE_y^{(1)} k_1}{\omega \mu_o \mu_1} &= \frac{-i}{\omega \mu_o} P q \operatorname{sn} [qz_o] \operatorname{dn} [qz_o] \\ \operatorname{sn} [qz_o] \operatorname{dn} [qz_o] &= \frac{-E_o k_1}{P q \mu_1} \end{aligned} \quad (20)$$

From Eq's. (19) And (20) we have :

$$\frac{E_o}{P} = \operatorname{cn}(z_o q) \quad (21)$$

$$, 1 - \frac{E_o^2}{P^2} = \operatorname{sn}^2(z_o q) \quad (22)$$

where: $\operatorname{cn}^2 + \operatorname{sn}^2 = 1$

Similarly; for the upper boundary, we have:

$$\frac{E_d}{P} = \operatorname{cn} \left[q \left(d + \frac{1}{q} \operatorname{cn}^{-1}(E_o/P) \right) \right] \quad (23)$$

$$, 1 - \frac{E_d^2}{P^2} = \operatorname{sn}^2 \left[q \left(d + \frac{1}{q} \operatorname{cn}^{-1}(E_o/P) \right) \right] \quad (24)$$

Using Jacobian Functions formula [16]:

$$\operatorname{cn}[qd] = \frac{\operatorname{cn}[qz_o] \operatorname{cn} [q(d+z_o) + \operatorname{sn} [qz_o] \operatorname{dn} [qz_o] \operatorname{sn} [q(d+z_o)] \operatorname{dn} [(d+z_o)]q}{1 - m \operatorname{sn}^2[qz_o] \operatorname{sn}^2[q(d+z_o)]} \quad (25)$$

Then we have the dispersion relation as:

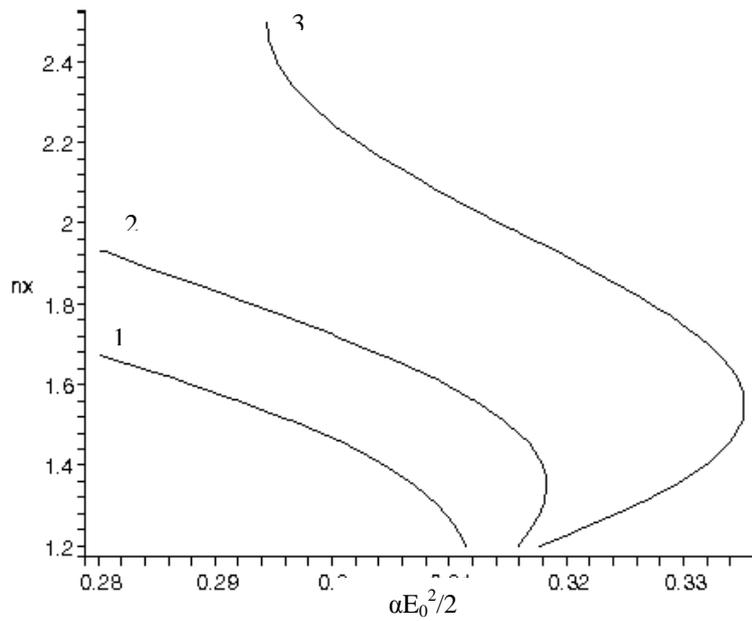
$$\operatorname{cn}[qd] = \frac{2 \left(\frac{\alpha_2 E_o^2}{2} \right) \left(\frac{\alpha_2 E_b^2}{2} \right) (Q^2 - n_1 n_3)}{\frac{\alpha_2 E_o E_b}{2} \left\{ (n_1^2 \alpha_2 E_o^2 / 2) + (n_3^2 \alpha_2 E_b^2 / 2) + Q^2 (\alpha_2 E_o^2 / 2 + \alpha_2 E_b^2 / 2) + \left(\frac{\alpha_2 E_o^2}{2} - \frac{\alpha_2 E_b^2}{2} \right) \right\}} \quad (26)$$

Equation (26) is the dispersion relation and $\alpha E_0^2/2$ is the optical power density .

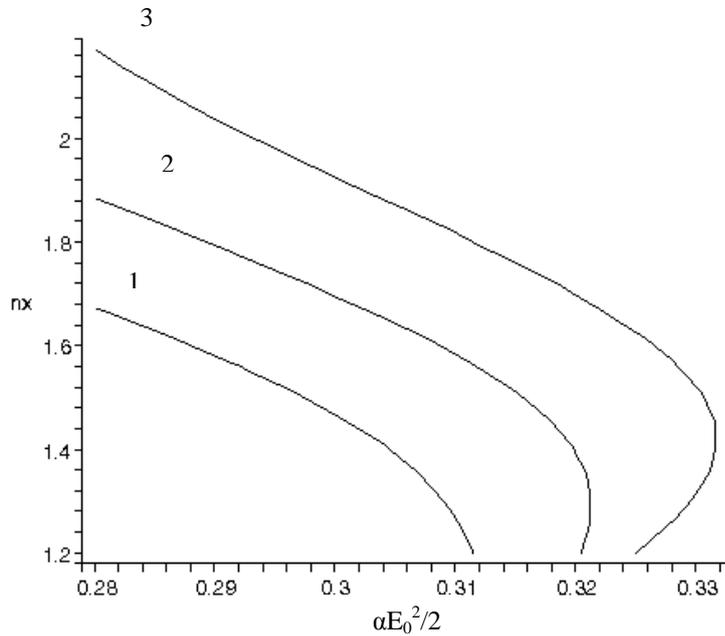
Numerical results and discussion:

We analytically studied the TE surface waves in a slab waveguide. Numerical computations is performed in order to calculate the propagation characteristics of the nonlinear dispersion equation. The dispersion relation, equation(26) has been solved numerically to compute the refractive index $n[x]$ as a function of power density $\alpha E_0^2/2$. The numerical calculations were carried out with the same parameters for the ferrite (YIG) substrate as[14]. the data parameters for the LHM as[15]: $F = 01.25$, $\omega_0 / 2\pi = 4$ GHz , and $\omega_p / 2\pi = 10$ GHz. fig(2) shows the refractive index of the structure versus the power density. In computation, the data is used as reported by shabat [15] $\mu_o H_o = 0.5T$, $\mu_B = 1.25$, $\mu_o M_o = 0.175T$ and $\gamma = 1.76 \times 10^{11} \text{ rads}^{-1}T^{-1}$. for $\mu_v \leq 0$, the frequency in the range from $\sqrt{f_o(f_o + f_m)}$ to $(f_o + f_m)$ is taken into consideration. It is noticed that the power density sensitivity to the change in frequency and decrease with the frequency increase. Fig(2) showing the bistable property of the waves ,the bistability appeared by decreasing the frequency. beside that ,when the nonlinear medium thickness is increase the power density and refractive index are increase as shown in fig(3). In fig(4) we plot the refractive index versus the power density with different thickness and $\alpha E_0^2/2 = .5$ instead of .4 as in fig(3). In fig(5),(6) we plot the refractive index versus the power density with different thickness and $(\alpha E_0^2)/2 = .5$ and different gyro magnetic ratio $\gamma = 1.77 \times 10^{11}$ and $\gamma = 1.78 \times 10^{11}$ respectively. increasing the gyromagnetic ratio occur increasing in power density and shift the curve towards increasing refractive index. showing the bistable property of the waves.

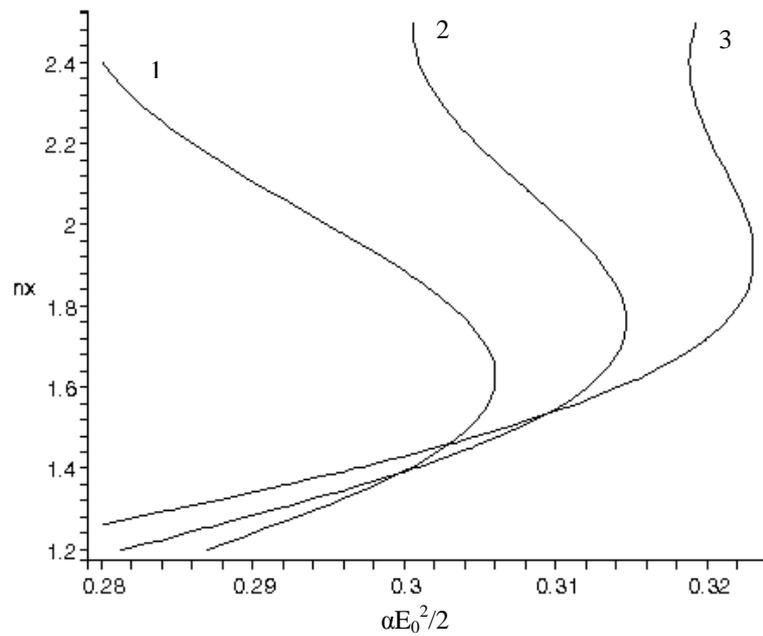
And the bistability appeared by decreasing the thickness of the film. We note that the structure of the dispersion curves for surface waves depends on the relation between the power density and refractive index for different value of gyromagnetic ratio. Fig(7).(8),(9), Dispersion curves of TE-polarized surface waves in three dimension the curve represents the relation between power density in the upper and lower boundary with refractive index for different frequencies = 104.3GHz , 104.4GHz, 104.5GHz respectively and $d = 3.15 \mu\text{m}$



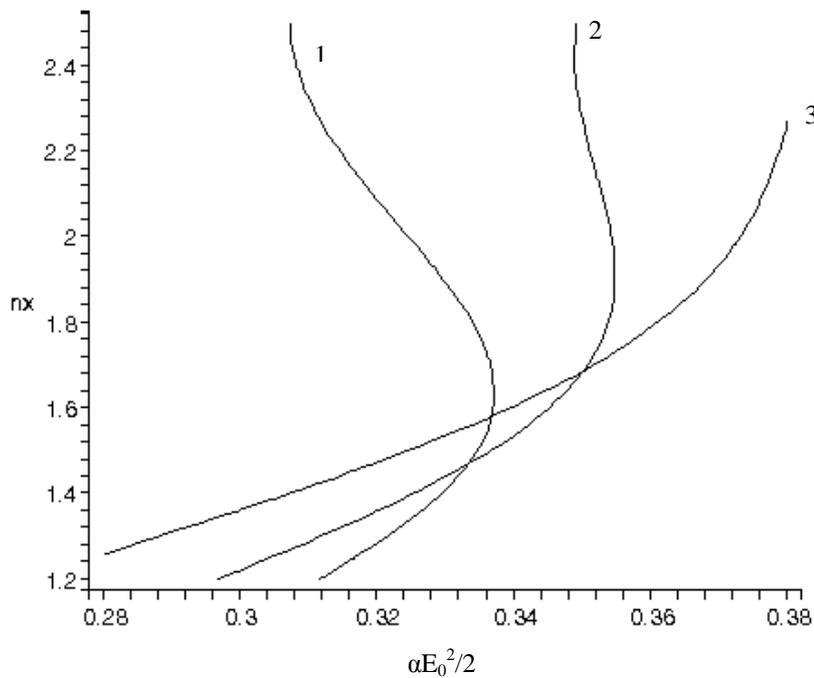
Fig(2) Dispersion curves of TE-polarized surface waves for different frequency curve(1)= 104.6×10^9 curve(2)= 104.4×10^9 curve(3)= 104.3×10^9 respectively for $b=4$, $\epsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $F = 1.25$, $d = 3\mu\text{m}$, $\epsilon_3 = 1$, $\gamma_f = 1.76 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}$, $\mu_0 M = 0.175 \text{ T}$, and $\mu_0 H_0 = 0.5$



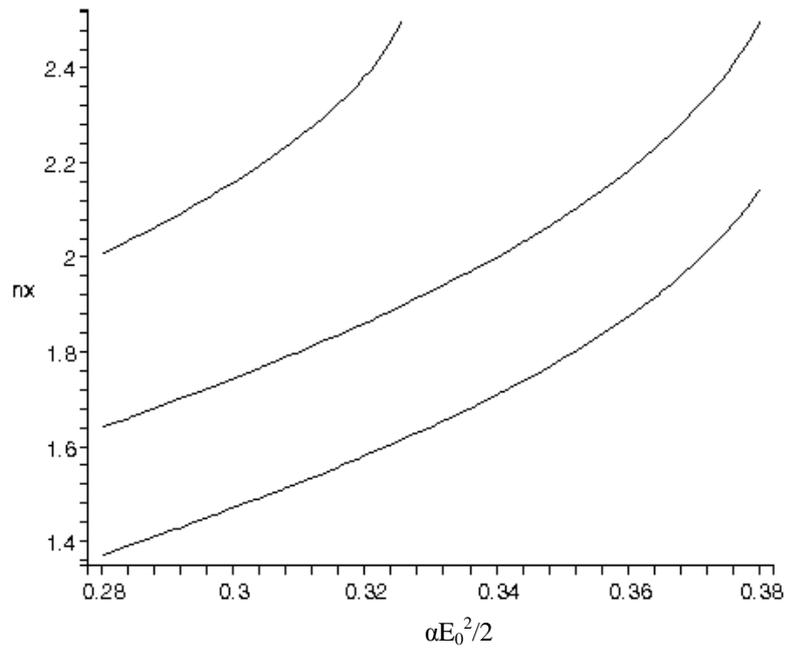
Fig(3) Dispersion curves of TE-polarized surface waves for different thickness curve(1): $d=3.15 \mu\text{m}$, curve(2): $d=3.2 \mu\text{m}$ and curve(3): $d=3.25 \mu\text{m}$ respectively for $b=4$, $\epsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $F = 1.25$, $\omega = 104.6$ GHz, $\epsilon_3 = 1$, $\gamma_f = 1.76 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}$, $\mu_0 M = 0.175 \text{ T}$, and $\mu_0 H_0 = 0.5$



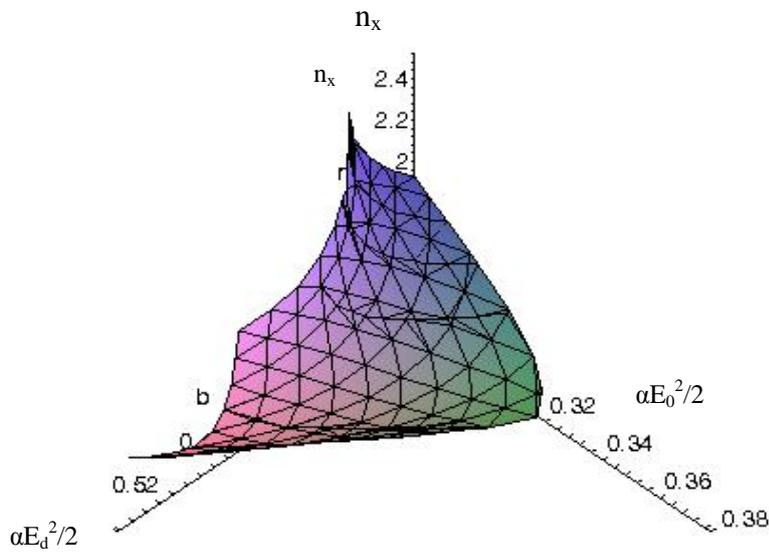
Fig(4) Dispersion curves of TE-polarized surface waves for different thickness curve(1): $d=3.15\mu\text{m}$,curve(2): $d=3.2\mu\text{m}$ and curve(3): $d=3.25\mu\text{m}$ respectively for $b=.5, \epsilon_1 = 2.45, \mu_1 = 1, \omega_0 = 4.0 \text{ GHz}, \gamma = 0.03\omega_p, F = 1.25, \omega=104.6 \text{ GHz}, \epsilon_3 = 1, \gamma_f = 1.76 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}, \mu_0 M = 0.175 \text{ T},$ and $\mu_0 H_0 = .5$



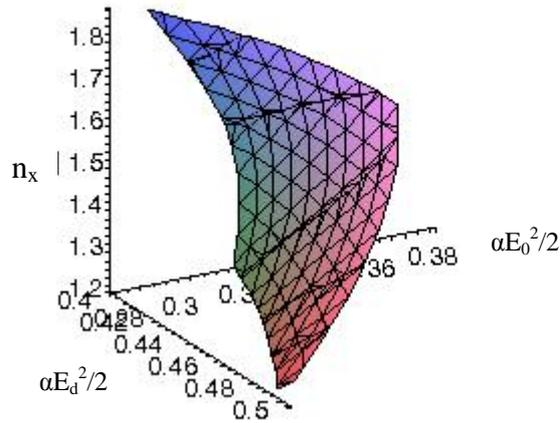
Fig(5) Dispersion curves of TE-polarized surface waves for different thickness curve(1): $d=3.15\mu\text{m}$,curve(2): $d=3.2\mu\text{m}$ and curve(3): $d=3.25\mu\text{m}$ respectively for $b=.5, \epsilon_1 = 2.45, \mu_1 = 1, \omega_0 = 4.0 \text{ GHz}, \gamma = 0.03\omega_p, F = 1.25, \omega=104.6 \text{ GHz}, \epsilon_3 = 1, \gamma_f = 1.77 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}, \mu_0 M = 0.175 \text{ T},$ and $\mu_0 H_0 = .5$



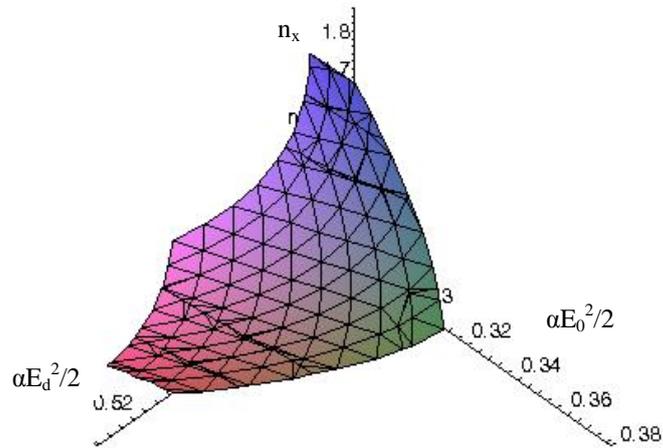
Fig(6) Dispersion curves of TE-polarized surface waves for different thickness curve(1): $d=3.15\mu\text{m}$,curve(2): $d=3.2\mu\text{m}$ and curve(3): $d=3.25\mu\text{m}$ respectively for $b=.5, \epsilon_1 = 2.45, \mu_1 = 1, \omega_0 = 4.0 \text{ GHz}, \gamma = 0.03\omega_p, F = 1.25, \omega=104.6 \text{ GHz}, \epsilon_3 = 1, \gamma_f = 1.78 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}, \mu_0 M = 0.175 \text{ T},$ and $\mu_0 H_0 = .5$



Fig(7) Dispersion curves of TE-polarized surface waves in three dimension for frequency= 104.3GHz , $d=3.15\mu\text{m}$, $b=.5, \epsilon_1 = 2.45, \mu_1 = 1, \omega_0 = 4.0 \text{ GHz}, \gamma = 0.03\omega_p, F = 1.25, \epsilon_3 = 1, \gamma_f = 1.78 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}, \mu_0 M = 0.175 \text{ T},$ and $\mu_0 H_0 = .5$



Fig(8) Dispersion curves of TE-polarized surface waves in three dimension , $d=3.15\mu\text{m}$, $\epsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $F = 1.25$, $\omega=104.4$ GHz, $\epsilon_3 = 1$, $\gamma_f = 1.78 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}$, $\mu_0 M = 0.175$ T, and $\mu_0 H_0 = .5$



Fig(9) Dispersion curves of TE-polarized surface waves in three dimension for frequency=104.5 GHz , $d=3.15\mu\text{m}$, $b=.5$, $\epsilon_1 = 2.45$, $\mu_1 = 1$, $\omega_0 = 4.0$ GHz, $\gamma = 0.03\omega_p$, $F = 1.25$, $\epsilon_3 = 1$, $\gamma_f = 1.78 \times 10^{11} \text{ s}^{-1}\text{T}^{-1}$, $\mu_0 M = 0.175$ T, and $\mu_0 H_0 = .5$

Conclusion

We proposed here an approach describing a new type behavior Surface -Waves in a Nonlinear, Left-Handed Material and Ferrite Layered Structure which is very promising for opt microwave electronic devices. We analytically studied the TE

surface waves in a slab waveguide. It contained nonlinear film between two ferrite cover and left handed material substrate. A dispersion relation for TE surface waves has been derived and numerically investigated. There found out that the wave effective refractive index was changing in value and sign depending on the operating frequency and the dielectric slab thickness. Also for different value of gyromagnetic ratio for ferrite cladding γ_f , the relation between power density and refractive index is changing and the curves shift towards increasing the refractive index n_x . And in some cases one guided density corresponds to two different effective refraction indexes showing the bitable property of the waves.

REFERENCES

- [1] A.D. Boardman and P. Egan, "Optically Nonlinear Waves in Thin Films, IEEE of Quant. Electronics. QE-22(2), February, (1986), 319.
- [2] A.D. Boardman, M.M. Shabat, and R.F. Wallis, "TE Waves at an Interface between Linear Gyromagnetic and Nonlinear Dielectric Media", J. Phys. D., 24, (1991), 1702-1707.
- [3] M.M. Shabat, and D. Jäger, Nonlinear Electromagnetic Surface Waves Guided by a Single Hexagonal Planar Ferrite,"Proceedings of XIVth International Conference on Microwave Ferrite", Eger, Hungary (1985), 127-130.
- [4] M.M. Shabat, D. Jäger, "Microwave Nonlinear Characteristics of TE Waves at Ferrite-Ferroelectric Interface,"Proceeding of 10th Microcoll", Budapest, Hungary, (1999), 343-346.
- [5] M.M. Shabat, "New Nonlinear Magnetostatic Surface Waves in a Metallised Ferromagnetic Film", Opt. Applicata, 43(4), (1994), 293-295.
- [6] Q. Wang, Z. Wu, S. Li., and L. Wang, "Nonlinear Magnetic Surface Waves On the Interface between Ferromagnet and antiferromagnet", J. App. Phy., 87(4), (2000), 1908-1913.
- [7] G.N. Burlak, V.V. Grimal'skii, and S.V. Koshevaya, "Nonlinear Waves In a Three Layer Ferroelectric-Ferrite- Ferroelectric System", Phy. Solid, 35(8), (1993), 1049- 1050.
- [8] M.M. Shabat, "Strongly Nonlinear Magnetostatic Surface Waves In a Ground Ferrite Film", phy. stat. sol.(a), 149, (1998), 691-696.
- [9] Xu,W.,L. W. Li,H. Y. Yao,and T. S. Yeo, "Left-handed material effects on waves modes and resonant frequencies: filled waveguide structures and substrate-loaded patch antennas," J. of Electromagn. Waves and Appl.,V ol. 19,2033 - 2047,2005.
- [10] Grzegorzcyk,T. M. and J. A. Kong, "Review of left-handed metamaterials: evolution from theoretical and numerical studies to potential applications," J. of Electromagn. Waves and Appl., Vol. 20,No. 14,2053 - 2064,2006.

- [11] Y. Zhang, T.M. Grzegorzcyk, and J.A. Kong, *Progr. Electromagn Res PIER* 35, 271-286 (2001).
- [12] R.W. Ziolkowsky and E. Heyman, " Wave propagation in media having. negative permittivity and permeability" *Phys Rev. E* 64, 056625-15 , Oct.2001.
- [13] Ilya V. Shadrivov, Andrey A. Sukhorukov, and Yuri S. Kivshar , " Nonlinear surface waves in left-handed materials." *PHYSICAL REVIEW E* 69, 016617 (2004).
- [14] S T Chui¹, C T Chan² and Z F Lin, " Multilayer structures as negative refractive and left-handed materials." *J. Phys.: Condens. Matter* 18 L89–L95,(2006).
- [15] M. M. Shabat, "Nonlinear Magnetostatic Surface Waves in a gyromagnetic film" *philosophical Magazine B*, Vol. 73, No. 4, 669 – 676, (1996).
- [16] Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat – Nasser and S. Schultz, "Composite Medium with Simultaneously Negative Permeability and Permittivity", *Phys. Rev. Lett.*, Vol. 84, No. 18, 4184, (2000).
- [17] M. Abramowitz and I. A Stegun. *Handbook of Mathematical Functions*, Appl. Math series, Washington, Dc: NBS, P 569, (1972).