J Mar Sci Technol (2008) 13:40–49
DOI 10.1007/s00773-007-0257-6

ORIGINAL ARTICLE

Three-dimensional hydrostatic modeling of a bay coastal area

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Received: February 3, 2007 / Accepted: July 24, 2007

Abstract This article describes the development of a three-dimensional (3D) multilayer hydrostatic model of tidal motions in the Ariake Sea and its application. The governing equations were derived from 3D Navier–Stokes equations and were solved using the fractional step method, which combines the finite difference method in the horizontal plane and the finite element method in the vertical plane. This study introduced a 3D, time-dependent, hydrostatic, tidal current model that can compute wetting and drying in tidal flats due to tidal motion. The 3D model was first tested against analytical solutions for three standard cases in a rectangular basin in order to investigate the performance of the model. Then, the model was applied to Saigo fishery port and the Ariake Sea. For standard cases, the numerical solutions were almost identical to the analytical solutions. Finally, the model results for Saigo port and the Ariake Sea show good agreement with the field observations.

Key words Tidal currents · Wetting and drying · Fractional step method · Ariake Sea

Introduction

With the rapid increase in computer power in recent years, it is not surprising to note that three-dimensional (3D) numerical simulation of tidal currents has also shifted from academic research to practical applications, particularly for larger estuarine and coastal waters where larger grid sizes are commonplace. During the past few decades, several 3D tidal circulation models have been developed, e.g., those of Heaps,[1] Davies,[2] Oey and Mellor,[3] Stephens,[4] Shen,[5] Casulli and Cheng[6] Jin and Kranenburg,[7] Zhang and Gin,[8] Ozawa et al.,[9] Yu and Kyozuka,[10] and Oey.[11]

Vertical finite difference models with a fixed grid or with a sigma coordinate transformation are the most popular models currently being used. The limitation of using a regular fixed grid is the difficulty in resolving complicated topographic features, with limited vertical resolution in shallow areas where large bathymetric irregularities often exist (Lin and Falconer).[12]

There are two advantages of using our regular fixed grid model, which uses the Cartesian coordinate system. First, it is possible to resolve complicated topographic features, with high vertical resolution in shallow areas, by dividing the water depth into equal layers. It is also possible to finely divide the near sea-bottom layer. Therefore, the vertical flow distribution can be calculated even in extremely shallow sea areas. Second, this model uses 3D equations in the calculation of wetting and drying areas. However, the Oey[11] model, for example, which uses the sigma-coordinate system vertically and the orthogonal curvilinear coordinate system horizontally, uses depth-averaged equations in the calculation of wetting and drying.

The main goal of this study was to formulate and develop a 3D numerical model to clarify morphodynamic processes in the sea due to tidal motions. The authors have already developed the first phase of this model,[13,14] and this article presents the second phase.

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Mathematical formulation

Governing equations

Assuming the hydrostatic pressure distribution and applying the Boussinesq approximation, the governing equations of continuity and momentum for incompressible fluid flow in a Cartesian coordinate system, with the x-axis and y-axis set on the still water level and the z-axis taken upward from the still water level, are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}=fv-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial y}-w\frac{\partial u}{\partial z}-g\frac{\partial \eta}{\partial x}+\frac{\partial}{\partial x}\left(\varepsilon_h\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon_h\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon_v\frac{\partial u}{\partial z}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}=-fu-u\frac{\partial v}{\partial x}-v\frac{\partial v}{\partial y}-w\frac{\partial v}{\partial z}-g\frac{\partial \eta}{\partial y}+\frac{\partial}{\partial x}\left(\varepsilon_h\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon_h\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\varepsilon_v\frac{\partial v}{\partial z}\right) \tag{3}$$

$$w=w_{z=-h}-\frac{\partial}{\partial x}\int_{-h}^{z}u\,dz-\frac{\partial}{\partial y}\int_{-h}^{z}v\,dz \tag{4}$$

$$\frac{\partial \eta}{\partial t}=-\frac{\partial}{\partial x}\int_{-h}^{\eta}u\,dz-\frac{\partial}{\partial y}\int_{-h}^{\eta}v\,dz \tag{5}$$

where u, v, and w are Cartesian velocity components along axes x, y, and z, respectively; t is time; f is the Coriolis force coefficient calculated as $2\Omega \sin \varphi$ (Ω is the rotational angular velocity of the Earth); φ is the latitude; η is the water surface displacement; h is the water depth below the reference plane; g is the acceleration of gravity; and ε_h and ε_v are the horizontal and vertical eddy viscosity coefficients, which are estimated using the Smagorinsky model:

$$\varepsilon_h=C_h\Delta x\Delta y\left[\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]^{0.5} \tag{6}$$

$$\varepsilon_v=C_v\Delta z^2\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]^2 \tag{7}$$

where, C_h and C_v are dimensionless coefficients in the horizontal and vertical direction, respectively.

Numerical methods

The governing equations for the tidal currents were solved using the fractional step method (FSM), originally suggested by Koutitas and O'Connor;[15] this method combines the finite difference method in the horizontal plane and a Galerkin finite element method (FEM) in the vertical direction. This hybrid method was used to solve the equation of motion by dividing it into two differential sections and integrating separately for the two stages. Because the FEM was used in the vertical direction, the water depth could be divided into equal layer thicknesses, and near the sea-bottom layer, a fine division was possible. In this computation, the position of variables was defined using a space-staggered grid system (Fig. 1). Kuroiwa et al.[16] and Kuchiishi et al.[17] have applied this system to the quasi-3D analysis of nearshore currents.

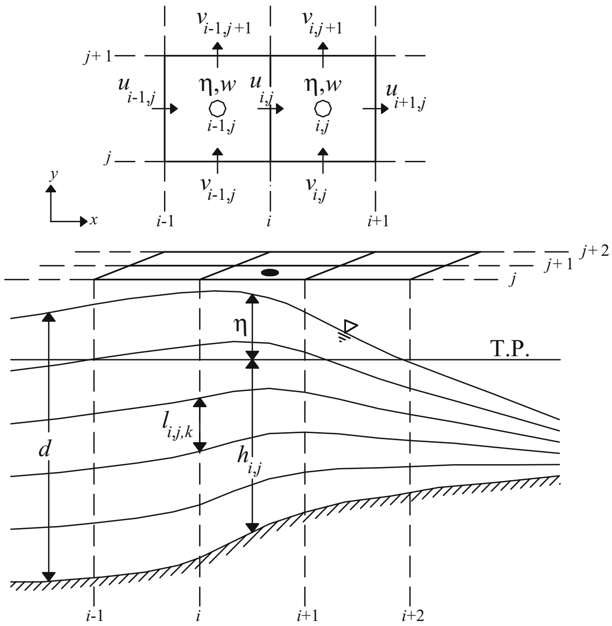

Fig. 1. Definition of the space-staggered grid system. *TP*, Tokyo Pail

The computational domain in the x-y plane was divided into a rectangular grid ($\Delta x = \Delta y$), as shown in Fig. 1. The vertical component of the current velocity (w) and the mean water level (η) were computed at the center of each grid cell, whereas the horizontal velocity components u and v were corrected midway along the cell faces. The mean water surface level for the next time step (η^{n+1}) was evaluated by using the previous time step (η^{n-1}). Figure 2 shows a flowchart of the numerical computations.

Boundary conditions

The shear stresses due to wind at the sea surface are considered as follows:

$$\varepsilon_z\frac{\partial u}{\partial z}=\frac{\tau_{sx}}{\rho},\qquad \varepsilon_z\frac{\partial v}{\partial z}=\frac{\tau_{sy}}{\rho}\qquad (z=\eta) \tag{8}$$

where τ_{sx} and τ_{sy} are: