Characteristics of Si-Solar Cell (PV) Waveguide Structure Using Transfer Matrix Method

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Characteristics of Si-Solar cell (PV) Waveguide Structure Using Transfer Matrix Method

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***Abstract*— In this work, the characteristics of a *Si*-solar cell (PV) model is studied. The proposed modelis a four layer system with an ultra- thin film of *Fe-InGaAsP*laid above Silicon *(Si)* substrate and covered by *AlON* layer that is exposed to air directly. The efficiency is measured by the reflectance power (*R*) and transmittance power (*T*). *R* and *T* are derived by using the transfer matrix method for both TE and TM modes. The total reflectance (R_{avg}) and the total transmittance (T_{avg}) are taken as the average value for the TE and TM modes. R_{avg} and T_{avg} are plotted versus the wavelength at different values for *AlON* layer thickness l_1 and *Fe-InGaAsP* layer thickness l_2 using Maple. The result shows that the minimum value of R_{avg} is shifted toward higher wavelengths with increasing l_1. The minimum of R_{avg} is almost zero while the rest of the spectrum is less than 0.25% which is lower than any previous results. We also noticed that the maximum transmittance power in visible light range is decreasing whenever the *AlON* thickness increased. In addition, the average reflectance power hardly changes as values of l_2 varies at the minimum.**

Keywords— Photovoltaic, Light trapping, Transfer Matrix Method, Efficiency, Reflection Power.

I. Introduction

Environment friendly systems for extracting electrical energy has attracted a lot of attention, especially that using photovoltaic (PV) cells. PV cells are recognized as a means for providing power and increased quality of life to those who do not have grid access.

PV captures and converts sunlight into electricity. The first PV cell was developed in 1954 at Bell laboratories. However, the cost of electricity produced by the first PV was much higher than the cost of electricity production using fossil fuels.

In the 1980s, silicon solar cells are researched leading to an increase in solar cells efficiency followed by zillions of researches on modifying solar cell efficiency. Currently, The fast growing and increasingly use of PV energy these years is related to the increasing efficiency reduced cost and use of non-hazardous materials of solar cells [1- 2].

The efficiency can be easily obtained once the reflection coefficient and transmission coefficient are calculated. There are many methods for calculating the reflection coefficients such as the Recursive method [3-4] and the transfer matrix method (TMM) [5-6]. The transfer matrix can be defined as a matrix relate between the amplitudes of the waves on both sides of a film. The derivation displayed in our work is taken from [7-8].

A variety of materials and processes can potentially satisfy the requirements for photovoltaic energy conversion. A review reported for cells and modules made from different semiconductors and for sub-categories within each semiconductor grouping (e.g. crystalline, polycrystalline and thin film) in [9].

PV is an optical device consists of multilayer structures. It uses material layers such as semiconductors [10], graphene-Metmaterials[11-12], and nanoparticles [13].

Next section will cover the model and theory. Numerical results will be given in section III followed by conclusion.

II. Model and Theory

The proposed solar cell is assumed to have two layers sandwiched between silicon (*Si*) substrate and air as in Fig. 1. The layers stratified in z direction. All media are considered to be nonmagnetic media such that μ = 1. Air has a refractive index $n_0 \approx 1$[14]. The first layer is made of Aluminum Oxynitride (*AlON*) with $n_1 \approx 1.7632$ [15] and thickness l_1. The second layer is Iron- indium Gallium arsenide phosphide (*Fe-InGaAsP*) has $n_2 \approx 3.3723\text{-}j0.0021$ [16] and thickness l_2. The *Si* substrate has $n_3 \approx 3.673$[17].

For each films (S = 0, 1, 2, 3) of thickness l_s, its upper and lower bounding interfaces are denoted, respectively by [illegible] and

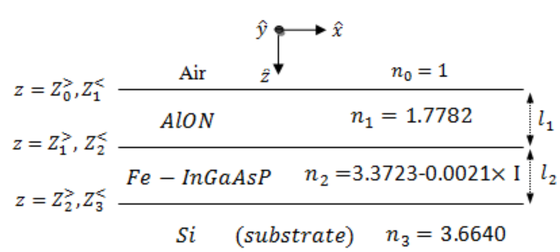

Fig. 1. The four layers PV cell proposed structure. The upper and lower interfaces of each layer are denoted by, [illegible] respectively.

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$z_s^<$ such that $z_s^> \ge z$ and $z_s^< \le z$ for z within the layer. The homogeneous medium of each layer is characterized by permittivity ε_s and permeability μ_s. For substrate layer, it assumed to have an infinite thickness in the positive z direction that is it has upper boundary with layer S=2. The same apply for air layer which assumed to have infinite thickness in the negative z direction; thus, it has lower interface with S=1.

The tangential component of the electric and magnetic field for *AlON* layer (S=1) can be written as:

$$E_t\left(z_0^>\right) = E_t\left(z_1^<\right), \quad H_t\left(z_0^>\right) = H_t\left(z_1^<\right) \tag{1}$$

$$E_t\left(z_1^>\right) = E_t\left(z_2^<\right), \quad H_t\left(z_1^>\right) = H_t\left(z_2^<\right) \tag{2}$$

Where (t) denote the tangential components. Also, we can get the tangential component of the electric and magnetic field for the third layers (*Fe-InGaAsP*) and get a similar analysis. The total transfer matrix of two thin films tangled between semi-infinite mediums is product of the individual transfer matrices as:

$$\begin{bmatrix} E_x(z_1^{\langle}) \\ E_y(z_1^{\langle}) \\ H_x(z_1^{\langle}) \\ H_y(z_1^{\langle}) \end{bmatrix} = M \begin{bmatrix} E_x(z_2^{\rangle}) \\ E_y(z_2^{\rangle}) \\ H_x(z_2^{\rangle}) \\ H_y(z_2^{\rangle}) \end{bmatrix} \tag{3}$$

$$M = M_1 M_2 = \begin{bmatrix} m_{11} & 0 & 0 & m_{14} \\ 0 & m_{22} & m_{23} & 0 \\ 0 & m_{32} & m_{33} & 0 \\ m_{41} & 0 & 0 & m_{44} \end{bmatrix} \tag{4}$$

Such that

$$M_i = \begin{bmatrix} \cos(\delta_i) & 0 & 0 & \mathrm{i}\gamma_i \sin(\delta_i) \\ 0 & \cos(\delta_i) & \dfrac{\mathrm{i}\sin(\delta_i)}{\beta_i} & 0 \\ 0 & \mathrm{i}\beta_i \sin(\delta_i) & \cos(\delta_i) & 0 \\ \dfrac{\mathrm{i}\sin(\delta_i)}{\gamma_i} & 0 & 0 & \cos(\delta_i) \end{bmatrix} \tag{5}$$

Where i stands for 1 and 2,

$$m_{11} = \cos(\delta_1)\cos(\delta_2) - \frac{a_1\cos(\theta_1)}{a_2\cos(\theta_2)}\sin(\delta_1)\sin(\delta_2)$$

$$m_{14} = ia_2\cos(\theta_2)\cos(\delta_1)\sin(\delta_2) + ia_1\cos(\theta_1)\cos(\delta_2)\sin(\delta_1)$$

$$m_{22} = \cos(\delta_1)\cos(\delta_2) - \frac{b_2\cos(\theta_2)}{b_1\cos(\theta_1)}\sin(\delta_1)\sin(\delta_2)$$

$$m_{23} = \frac{i\cos(\delta_1)\sin(\delta_2)}{b_2\cos(\theta_2)} + \frac{i\sin(\delta_1)\cos(\delta_2)}{b_1\cos(\theta_1)}$$

$$m_{32} = ib_1\cos(\theta_1)\sin(\delta_1)\cos(\delta_2) + ib_2\cos(\theta_2)\cos(\delta_1)\sin(\delta_2)$$

$$m_{33} = \cos(\delta_1)\cos(\delta_2) - \frac{b_1\cos(\theta_1)}{b_2\cos(\theta_2)}\sin(\delta_1)\sin(\delta_2)$$

$$m_{41} = \frac{i\sin(\delta_1)\cos(\delta_2)}{a_1\cos(\theta_1)} + \frac{i\cos(\delta_1)\sin(\delta_2)}{a_2\cos(\theta_2)}$$

$$m_{44} = \cos(\delta_1)\cos(\delta_2) - \frac{a_2\cos(\theta_2)}{a_1\cos(\theta_1)}\sin(\delta_1)\sin(\delta_2)$$

$$\delta_i = k_0\Delta_i = \left(\frac{2\pi}{\lambda_0}\right) n_i h_i \cos\left(\theta_i\right)$$

To find the transfer matrix for TE mode from the Hybrid mode, we choice that $E_x(z_1^{<>}) = 0$ and $H_y(z_1^{<>}) = 0$ and get on:

$$\begin{bmatrix} E_y(z_1^{\langle}) \\ H_x(z_1^{\langle}) \end{bmatrix} = \begin{bmatrix} m_{22} & m_{23} \\ m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} E_y(z_1^{\rangle}) \\ H_x(z_1^{\rangle}) \end{bmatrix} \tag{6}$$

The reflection coefficient (r_{TE}) and the transmission coefficient (t_{TE}) for TE mode respectively are:

$$r_{TE} = \frac{m_{22}\beta_0 + m_{23}\beta_0\beta_i - m_{32} - m_{33}\beta_i}{m_{22}\beta_0 + m_{23}\beta_0\beta_i + m_{32} + m_{33}\beta_i} \tag{7}$$

$$t_{TE} = \frac{2\beta_0}{m_{22}\beta_0 + m_{23}\beta_0\beta_i + m_{32} + m_{33}\beta_i} \tag{8}$$

where $\beta_i = \dfrac{k_{iz}}{\mu_i} = \dfrac{k_i}{\mu_i}\cos(\theta_i)$.

The reflectance power (R_{TE}) and the transmittance power (T_{TE}) for TE mode respectively are

$$R_{TE} = |r|^2 = r \cdot r^* \tag{9}$$

$$T_{TE} = \frac{\beta_S}{\beta_0}|t|^2 = \frac{\beta_i}{\beta_0}(t \cdot t^*) \tag{10}$$

Also, we can do a similar steps to get the reflectance coefficient (r_{TM}) and the transmittance coefficient (t_{TM}) for TM.

$$r_{TM} = \frac{m_{41}\gamma_0\gamma_s + m_{44}\gamma_0 - m_{11}\gamma_i - m_{14}}{m_{41}\gamma_0\gamma_s + m_{44}\gamma_0 + m_{11}\gamma_i + m_{14}} \tag{11}$$

$$t_{TM} = \frac{2\gamma_0}{m_{41}\gamma_0\gamma_i + m_{44}\gamma_0 + m_{11}\gamma_i + m_{14}} \tag{12}$$

where $\gamma_i = \dfrac{k_{iz}}{\varepsilon_i} = \dfrac{k_i}{\varepsilon_i}\cos(\theta_i)$.

The reflectance power (R_{TM}) and transmission power (T_{TM}) respectively are

$$R_{TM} = |r|^2 = r \cdot r^* \tag{13}$$

$$T_{TM} = \frac{\gamma_i}{\gamma_0}|t|^2 = \frac{\gamma_i}{\gamma_0}(t \cdot t^*) \tag{14}$$

The probability that the sunlight consists of TE modes and TM modes is fifty-fifty. Thus, the reflectance will have equal contributions from the complex Fresnel coefficient of reflection for TE and TM modes which can be represented as

$$R_{average} = \frac{R_{TE} + R_{TM}}{2} \tag{15}$$

$$T_{average} = \frac{T_{TE} + T_{TM}}{2} \tag{16}$$

III. Numerical Results and Discussion

In the solar cell model, we interested in getting a minimum reflectance and maximum transmittance in the visible light region. In Fig. 2, the average reflectance power (R_{avg}) from (23) at normal incidence is plotted versus the operating wavelength at different values of l_1. The other parameters are ε_0 *(air)*≈1, ε_1 (AlON) ≈*3.16199,* ε_2 (Fe-InGaAsP) ≈ (3.3723-*i*0.0021)^2,ε_3 (Si) ≈ *13.4909,* $\theta_0=0°$

The values of l_1 are 68 nm,82 nm and 90nm. It can be seen that R_{avg} increases when λ is around 300nm and decreases to reach a minimum value then it increases again. The minimum value of R_{avg} has red shift as the value of l_1 increases. We notice that R_{avg} approaches zero value at the minimum and the rest of the values are less than 0.25% which makes this results much better than our previous work which maintain a reflectance lower than 1% by applying MTM layer and one ARC layer [6] and even better than the calculation by Beye who used one or two RAC layers [18].

Similarly, Fig. 3 displays the average transmittance (T_{avg}) as function of the operating wavelength at l_1 equals 68 nm,82 nm and 90nm for same parameters. We noticed that the transmittance power has opposite values of the reflectance.

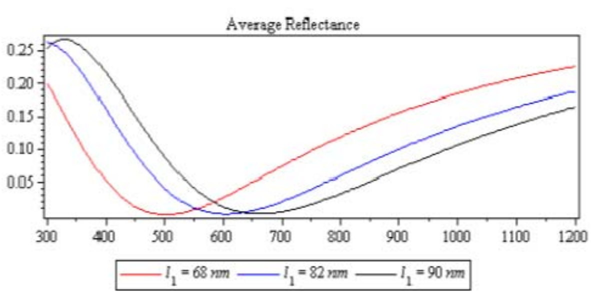

Fig.2.Average reflectance (R_{avg}) versus the wavelength (λ) for different values of l_1 (thickness of AlON), l_2 = 20 nm (thickness of Fe-InGaAsP), ε_0 *(air)*≈1, ε_1 (AlON) ≈*3.16199,* ε_2 (Fe-InGaAsP) ≈ (3.3723-*i*0.0021)^2,ε_3 (Si) ≈ *13.4909,* $\theta_0=0°$

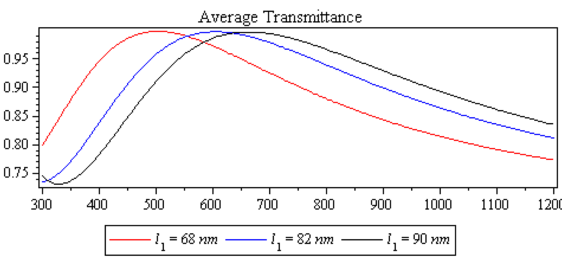

Fig. 3. Average transmittance (T_{avg}) versus the wavelength (λ) for different values of l_1 (thickness of AlON).

Fig. 4 presents the average reflectance power as function of wavelength at l_1=82 nm and various values of l_2; e.g., 15 nm, 20 nm and 30 nm. The other parameters are kept the same. The minimum which occurs at λ=600 nm is hardly changes by changing l_2. The same values are taken to calculate the average transmittance power as function of λ and plotted in Fig. 5.

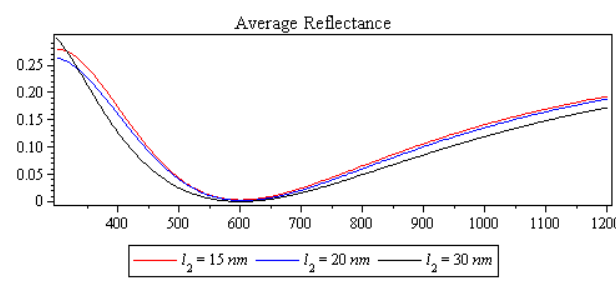

Fig. 4.Average reflectance (R_{avg}) versus the wavelength (λ) for different values ofl_2 and l_1 =82 nm.

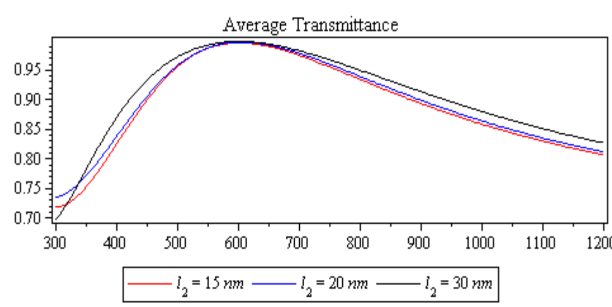

Fig. 5.Average transmittance (T_{avg}) versus the wavelength (λ) for different values of l_2.

IV. Conclusion

The four layers solar cell model has been designed and investigated. The solar cell structure under consideration has an ultra- thin film nonmagnetic metal (μ = 1) on (Si) substrate and covered by *Fe-InGaAsP* layer, *AlON* layer that is exposed to air directly. The reflectance and transmittance have been derived by using the transfer matrix method for both TE and

TM modes. The total reflectance transmittance are considered the average value for the obtained values from TE and TM modes. Then, they are solved numerically by Maple. The reflectance (*R*), the transmittance (*T*) are plotted versus the wavelength at different values for *AlON* layer thickness l_1 and *Fe-InGaAsP* layer thickness l_2. The minimum value of reflectance and the maximum value of transmittance are shifted toward higher wavelengths with increasing l_1. The minimum almost zero while the rest of the spectrum is less than 0.25% which is lower than previous results. We also noticed that the maximum transmittance power in visible light range is decreasing whenever the *AlON* thickness increased. In addition, the average reflectance power hardly changes as values of l_2 varies at the minimum.

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