

NONLINEAR TM SURFACE WAVES OF A THREE ANTIFERROMAGNET – SUPERCONDUCTOR STRUCTURE

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ABSTRACT: The characteristics features of TM nonlinear surface waves at infrared frequencies in a layered structure of an antiferromagnet, superconductor and linear dielectric have been performed. The complex wave number of TM surface wave is computed by solving the dispersion equation in order to find out the reduced phase and reduced attenuation index. The effects of the operating angular frequency and temperature of superconductor on the reduced phase and attenuation index have been examined. The power flow has also been studied as a function of the reduced phase and attenuation index for different values of the temperature of superconductor.

Keyword: Dispersion relation; Phase index; Attenuation index; Power flow.

1. Introduction

The investigation of the propagation of electromagnetic surface and guided waves in various planar waveguide structures containing gyromagnetic and gyrodielectric media [1-3] is very important for the modeling a future development opto-microwave electronics devices, such as isolators, switches, circulators and signal processing devices. The nonreciprocal behavior of the above-mentioned devices is based on the magneto-optic effect of the gyromagnetic and gyrodielectric media. Recently, many researchers [4,5,6] are interested in the propagation of electromagnetic surface waves in superconductor's media in view of their wide applications in waveguide structures.

In the last few years, nonlinear behavior of electromagnetic waves in antiferromagnetic films has attracted significant attention [7,8]. Many investigations have been carried out on nonlinear guided waves, including the stability [9], self-phase modulation, and dispersion properties [10]. The developments in signal processing and nonlinear optics make the possibility of performing all-optical switching using waveguide structures, and leading to analysis of waveguide problems in which one or more of the waveguiding

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media exhibit an intensity-dependent refractive index [11,12]. Almieda and Mills [13] derived the nonlinear susceptibility χ_{NL} for the first time in the study of the nonlinear infrared responses of the antiferromagnetics, and employed the χ_{NL} to explore the power-dependent transmission of electromagnetic field through thin antiferromagnetic films. Wang *et al* [10] proved theoretically the existence of spatial magnet solitons on the surface of an antiferromagnet.

Due to the discovery of the superconductors, an increasing applications of high-temperature superconductors to microwave and millimeter wave devices and circuits have been investigated. Yu. Y. Gulyaev *et al* [4] have reported the first experimental of the temperature dependence on the absorption of volume spin waves in composite structure of ferrite and superconductors. Tsutsumi *et al* [5] have studied both the phase and attenuation indexes of magnetostatic surface waves in the YBCO-YIG multilayered structure. Chien-Jang Wu [14] has studied the combined effect of the nonlinear dielectric permittivity as well as of vortices inside the superconductors on the attenuation and phase index. Hamada *et al* [15] have investigated the propagation characteristics of TM surface waves in a nonlinear Antiferromagnetic-Semiconductor-Superconductor waveguide structure. Abbas *et al* [16] have studied microwave component based of a dielectric nonlinearity by using superconductor -ferroelectric thin films, and they computed the effect of the temperature and the dc bias electric field on the relative phase velocity and phase index. Superconducting planar transmission line can be made tunable by using high T_c superconductors (HTS) and antiferromagnet. This means that adjusting the nonlinearity of antiferromagnet and temperature of superconductors can vary its infrared characteristics such as propagation and attenuation index.

In this communication, a theoretical study of the propagation of magnetic surface waves TM in a multilayered structure consisting of $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ superconductors film bounded by a nonlinear antiferromagnet (FeF_2) cover and a linear dielectric substrate is presented, taking into account the effect of the temperature of the superconductors and the nonlinearity of antiferromagnet. We found that the frequency band switching effect of the TM surface waves is actually caused by the fact that the nonlinear permeability is not only power dependent but also temperature dependent. Dispersion equation has been solved in order to compute the complex effective wave number of the TM surface waves and find out the propagation characteristics, such as the reduced phase and attenuation index. The power flow has also been studied against the reduced propagation and attenuation index.

2. Basic equations

Consider the waveguide structure as shown in Fig.1. A superconductor film (medium 2) of finite thickness (t) characterized by a negative dielectric function $\varepsilon_2(\omega)$ is sandwiched between a semi-infinite dielectric substrate medium (1) and a two-sublattice uniaxial antiferromagnet cover (medium 3) in the region $x > t$, and its permeability function being μ^{NL} .

The permeability tensor for the absence of an applied Zeeman field, describing the nonlinear response of the crystal to the intense rf field is a diagonal one is [7]:

$$\mu_{yy}(\omega) = \mu^{NL}(\omega) = \mu^L(\omega) + \chi_{NL}(\omega)|h|^2, \quad (1)$$

where $\mu^L(\omega)$ is the linear permeability has the form

$$\mu^L(\omega) = 1 + \frac{2\omega_M\omega_A}{\omega_c^2 - \omega^2}, \quad (2)$$

where $\omega_M = \gamma\mu_o M_S$, $\omega_A = \gamma\mu_o H_A$, $\omega_E = \gamma\mu_o H_E$, and $\omega_C = \sqrt{\omega_A^2 + 2\omega_A\omega_E}$

is the resonance frequency of the system. M_s is the saturation magnetization field, H_A is the anisotropy field, H_E is the exchange field of the crystal and γ is the gyromagnetic ratio.

The nonlinear part $\chi_{NL}(\omega)|h|^2$ is always positive for a linearly polarized electromagnetic wave, which mean that the crystal is a self-focus crystal. We consider χ_{NL} is constant, because $|\partial\chi_{NL}(\omega)/\partial\omega| \ll \partial\mu^L/\partial\omega$ is satisfied.

The negative relative dielectric constant(ε_2) of the superconductors can be approximated in two-fluid model as [17]:

$$\varepsilon_2(\omega) = \left[1 - \frac{1}{\omega^2 \mu_o \lambda_L^2 \varepsilon_o} \right] - i \frac{\sigma}{\omega \varepsilon_o}, \quad (3)$$

where $\lambda_L^2 = \frac{\lambda_o^2}{\left[1 - \left(\frac{T}{T_c} \right)^4 \right]}$, λ_o is the field penetration depth at temperature

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$T=0$ K, $\sigma=\sigma_0[T/T_c]^4$ and T_c is the critical temperature of the superconductor. The electric and magnetic field of TM wave propagating in the z-direction can be written as:

$$\vec{E} = (e_x, 0, e_z) \exp[ik_0(\beta z - ct)], \quad (4a)$$

$$\vec{H} = (0, h_y, 0) \exp[ik_0(\beta z - ct)], \quad (4b)$$

where $\beta = \frac{k}{k_0}$ is the complex effective wave index, k is the complex wave number, and k_0 is the wave number of the free space
 $\beta = \text{Re}(\beta) + i \text{Im}(\beta), \quad (5)$

where $\text{Re}(\beta)$ is the reduced phase index and $\text{Im}(\beta)$ is the reduced attenuation index, k_0 is the wave number of the free space, and c is the velocity of light in free space.

Substitution of equations (3a) and (3b) into Maxwell's equations, yields the following three differential equations in the three layers:

$$\frac{\partial^2 h_y}{\partial x^2} - k_0^2(\beta^2 - \epsilon_1)h_y = 0, \quad x < 0 \quad (6)$$

$$\frac{\partial^2 h_y}{\partial x^2} - k_0^2(\beta^2 - \epsilon_2)h_y = 0, \quad 0 \leq x \leq t \quad (7)$$

$$\frac{\partial^2 h_y}{\partial x^2} - k_0^2(\beta^2 - \epsilon_3 \chi_{NL} |h_y|^2)h_y = 0, \quad x > t \quad (8)$$

where $k_3 = k_0 \sqrt{\beta^2 - \epsilon_3 \mu^L}$, ϵ_3 is the relative dielectric constant. The exact solution of equations (5-7) has the form:

1- In nonlinear antiferromagnetic layer:

$$h_y = \frac{k_3}{k_0} \sqrt{\frac{2}{\chi_{NL} \epsilon_3}} \sec h[k_3(x - x_0)], \quad (9)$$

where $k_3 = k_0 \sqrt{\beta^2 - \epsilon_3 \mu^L}$, and x_0 is the peak of the field.

2- In superconductor layer, and for $\beta^2 > \epsilon_2$

$$h_y = A \sinh(k_2 x) + B \cosh(k_2 x), \quad (10)$$

where $k_2 = k_o \sqrt{\beta^2 - \varepsilon_2}$.

3- In dielectric region:

$$h_y = C e^{k_1 x}, \quad x < 0 \quad (11)$$

where $k_1 = k_o \sqrt{\beta^2 - \varepsilon_1}$, and A, B and C are amplitude coefficients, which

can be determined by the boundary conditions.

By requiring the tangential components of the electromagnetic field to be continuous at the boundaries and eliminating the constants A, B and C of equations (9) and (11), the dispersion equation is then obtained:

$$\tanh(k_2 t) = \frac{k_2(k_3 \varepsilon_1 \varepsilon_2 \nu - k_1 \varepsilon_2 \varepsilon_3)}{(k_2^2 \varepsilon_1 \varepsilon_3 - k_1 k_3 \varepsilon_2^2 \nu)}, \quad (12)$$

where $\nu = \tanh[k_3(x_o - t)]$.

Eq. (12) is similar to the dispersion equation of nonlinear TE in a Three layered dielectric waveguide [21].

In the linear limit, $\chi_{NL} = 0, \mu_L = 1$, then z_o should go to infinity, so $\nu = \tanh[k_3(x_o - t)] = 1$, and then the dispersion equation can be the similar equation of TE waves in a linear waveguide [18].

The total Power flux (P) of the wave propagation in the z-direction can be written as[6]:

$$P = \frac{1}{2} \int (\vec{E} \times \vec{H}^*) dx = \frac{1}{2} \int_{-\infty}^{\infty} e_x h_y^* dx = P_{Diel} + P_{Sup} + P_{NI}, \quad (13)$$

where P_{Diel} , P_{Sup} and P_{NI} are respectively the power fluxes in the dielectric, superconductor, and nonlinear antiferromagnet media, given by:

$$P_{Diel} = \frac{1}{2} \frac{p_o \beta B^2}{\varepsilon_1 \varepsilon_3 q_1} \quad (13a)$$

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$$P_{Syp} = \frac{1}{2} \frac{p_o \beta B^2}{\varepsilon_2 \varepsilon_3} \left\{ k_o t \left[1 - \left(\frac{q_1 \varepsilon_2}{q_2 \varepsilon_1} \right)^2 \right] + \frac{\sinh(k_o t)}{q_2} \times \left[\left(1 + \left(\frac{q_1 \varepsilon_2}{q_2 \varepsilon_1} \right)^2 \right) \cosh(k_2 t) + 2 \frac{q_1 \varepsilon_2}{q_2 \varepsilon_1} \sinh(k_2 t) \right] \right\} \quad (13b)$$

$$P_{NI} = \frac{2 p_o q_3}{\varepsilon_3^2} (1 + \nu) \quad (13c)$$

where $q_1 = \sqrt{\beta^2 - \varepsilon_1}$, $q_2 = \sqrt{\beta^2 - \varepsilon_2}$, $q_3 = \sqrt{\beta^2 - \varepsilon_3 \mu^L}$ and

$$B = \left[2(1 - \nu^2) \right]^{\frac{1}{2}} q_3 \left[\cosh(k_2 t) + \frac{q_1 \varepsilon_2}{q_2 \varepsilon_1} \sinh(k_2 t) \right]^{-1}. \quad (14)$$

3. Results and discussion

In our discussion, we have used the data parameters of antiferromagnetic material Ferrous Florid (FeF_2) as in[7], where the values of the anisotropy field H_A , exchange field H_E , saturation magnetization M_s , and the relative dielectric constant ε_3 of FeF_2 crystal are 1.59×10^4 A/m, 4.3×10^4 A/m, 4.46×10^4 A/m and 4 respectively.

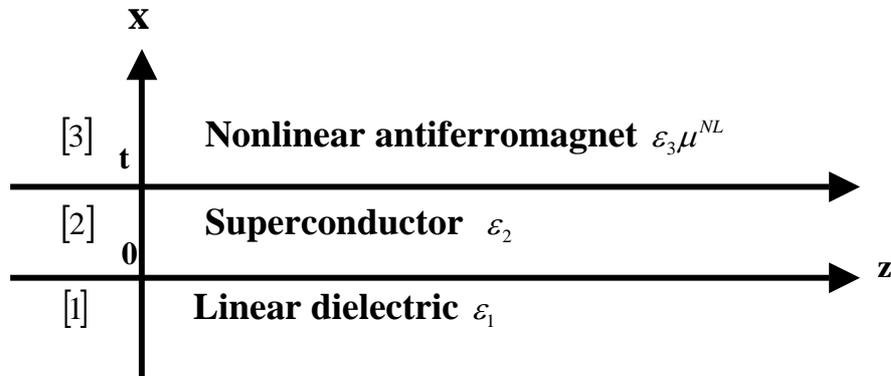


Fig.1. TM surface waveguide composed from a antiferromagnet-YBCO layered structure.

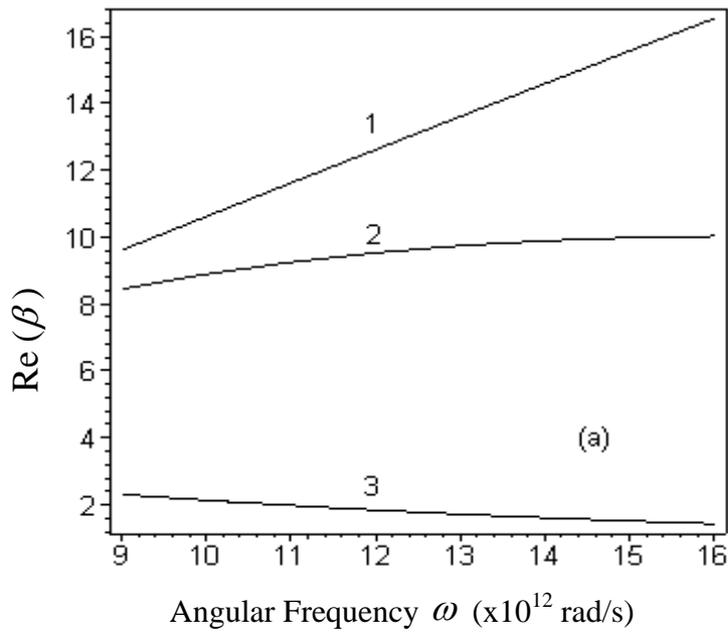


Fig. (2a) shows the phase index $\text{Re}(\beta)$ against the angular frequency (ω) at different values of reduced temperature (1) $T/T_c = 0.4$; (2) $T/T_c = 0.6$; (3) $T/T_c = 0.8, t = 0.44 \times 10^{-8}$ (m) and $v = -0.16$.

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The gyromagnetic ratio γ is $1.7 \times 10^{11} \text{ (Ts)}^{-1}$. The relative dielectric constant of linear dielectric substrate $\epsilon_1 = 3$. The parameters for superconductors ($\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$) [5] are T_c , λ_o and σ_o are respectively 86 K, $0.22 \text{ } \mu\text{m}$ and $6.56 \times 10^6 \text{ s/m}$.

Eq.(12) has been solved to calculate the reduced phase and attenuation constant as shown in Fig. (2) and Fig. (3).

Figs. (2a) and (2b), respectively, demonstrate angular frequency dependence of the reduced phase index $\text{Re}(\beta)$, and reduced attenuation index $\text{Im}(\beta)$ for different values of T/T_c (0.4, 0.6, and 0.8) where $\nu = -0.16$ and $t = 0.44 \times 10^{-8} \text{ m}$.

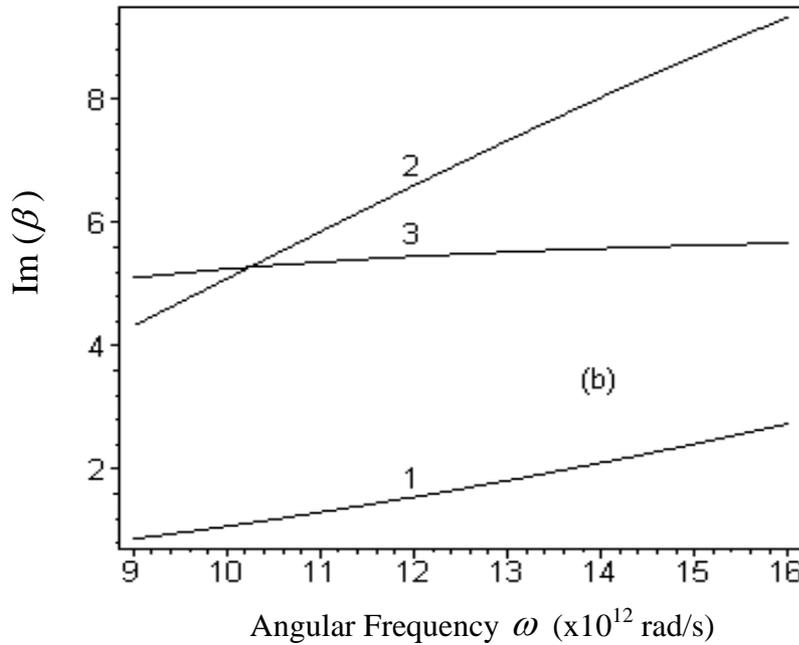


Fig. (2b). The attenuation index [$\text{Im}(\beta)$] against the angular frequency (ω) at different values of reduced temperature (1) $T/T_c = 0.4$; (2) $T/T_c = 0.6$; (3) $T/T_c = 0.8$, $t = 0.44 \times 10^{-8} \text{ (m)}$ and $\nu = -0.16$.

In Fig. (2a), we notice that, at $T/T_c=0.4$, $\text{Re}(\beta)$ increases with increasing the operating frequency. While at T/T_c equals 0.6 the reduced phase index is increasing slowly on the angular frequency. But at higher temperature where $T/T_c = 0.8$, $\text{Re}(\beta)$ is weakly dependent on the angular frequency as shown in Fig. (3a). Fig. (2b) shows that, at T/T_c equals 0.4 and 0.6, the reduced attenuation index increases by increasing the angular frequency ω . The nonlinearity of antiferromagnet here increases the range of tunability of $\text{Im}(\beta)$ as the case of the linear dielectric [14] where the attenuation index increases with increasing frequency. We also see that at higher T/T_c (0.8), the reduced attenuation index [$\text{Im}(\beta)$] is decreasing and nearly constant with ω as shown in Fig. (3b).

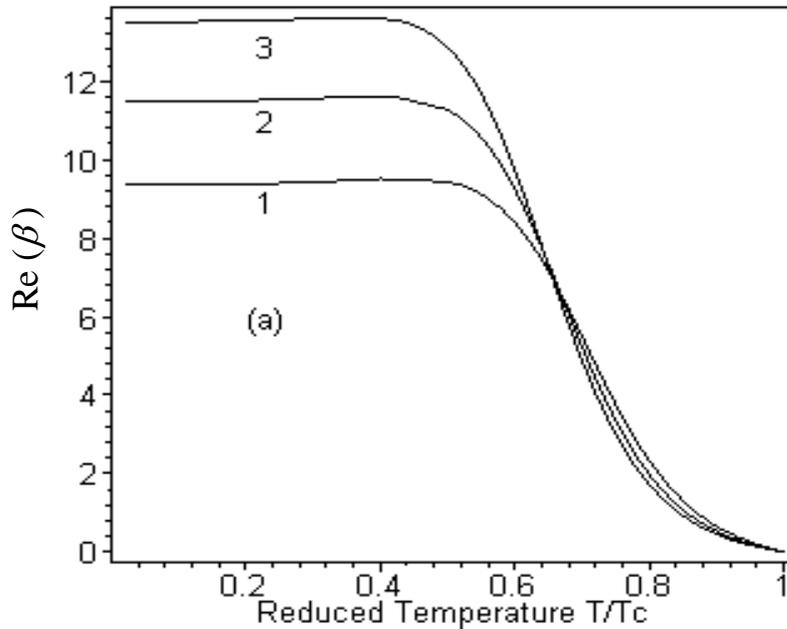


Fig (3a). Calculated the phase index $\text{Re}(\beta)$ against the reduced temperature T/T_c at different values of angular frequency (ω) (1) 9×10^{12} ; (2) 11×10^{12} ; (3) 13×10^{12} (rad/s), $t=0.44 \times 10^{-8}$ (m) and $v = -0.16$.

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In Fig. 3 we plot in (a) the reduced phase index $\text{Re}(\beta)$, and (b) the reduced attenuation index $\text{Im}(\beta)$ as a function of reduced temperature for various values of angular frequency (ω) (9×10^{12} , 11×10^{12} , 13×10^{12}) rad/s where

$$\nu = -0.16 \text{ and } t = 0.44 \times 10^{-8} \text{ m.}$$

In Fig. (3a) we notice that, at T/T_c below 0.5, the $\text{Re}(\beta)$ is nearly independent to reduced temperature. But at high T/T_c , above 0.6, the reduced phase index $\text{Re}(\beta)$ decreases with increasing T/T_c , and goes to zero but is nearly independent to the angular frequency as shown in Fig. (2a). This result is the same as result had been obtained by Wu *et al* [14] at higher frequency.

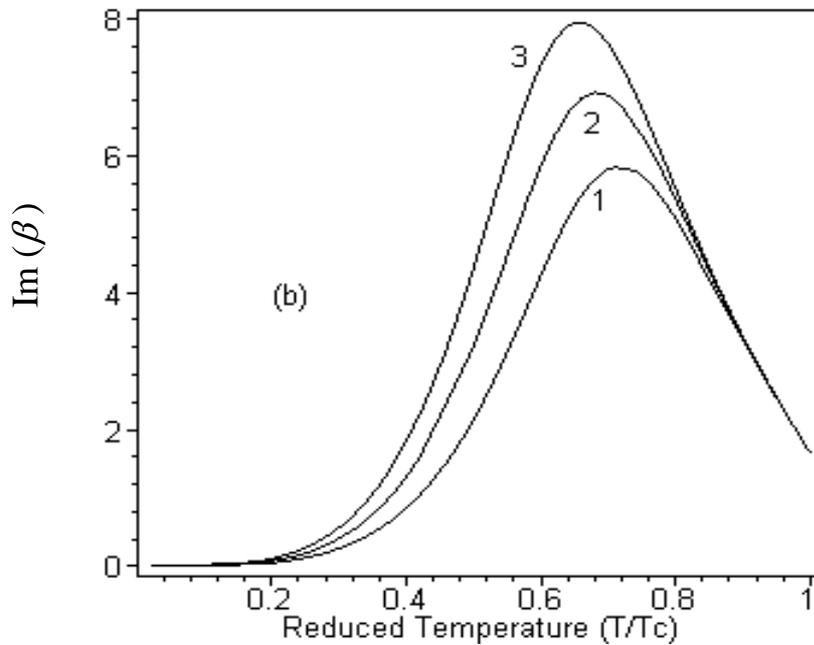


Fig (3b). The attenuation index $\text{Im}(\beta)$ against the reduced temperature T/T_c at different values of angular frequency (ω) (1) 9×10^{12} ; (2) 11×10^{12} ; (3) 13×10^{12} (rad/s), $t=0.44 \times 10^{-8}$ (m) and $\nu = -0.16$.

In Fig. (3b), we see that, as T/T_c increases, the reduced attenuation index $\text{Im}(\beta)$ increases up to some critical value, which is nearly equals 0.7, and then

decreases. But we also see that at T/T_c above 0.7 the reduced attenuation index is independent to the angular frequency (ω), while at $T/T_c < 0.7$, $\text{Im}(\beta)$ depends on ω . However, near zero absolute temperature, λ_L is practically independent of T and the temperature dependence of $\text{Im}(\beta)$ results from the temperature dependence of σ . So, at low temperature σ goes to zero at 0 K, since the number of normal electrons goes to zero.

We have solved Eq. (12) and Eq. (13) to plot the normalized power P/P_o versus the reduced phase index $\text{Re}(\beta)$ as shown in Fig.(2a) and the reduced attenuation index [the positive $\text{Im}(\beta)$] as shown in fig.(2b) for a fixed angular frequency $\omega = 8.899 \times 10^{12}$ rad/s, the superconductor thickness $t = 0.44 \times 10^{-8}$ m and different values of reduced temperature T/T_c (0.25, 0.5, and 0.55). The normalized power has been computed, which equals the real power divided by P_o . P_o is equal to $\frac{1}{2\chi_{NL}\omega\epsilon_o} \approx 0.43$ MW/m. It is quite high but since it is inverse $\chi_{NL}\omega$ working at higher frequencies requires a smaller χ_{NL} to offset this fact.

Figs. (4a) and (4b) illustrate the dependence of the dimensionless normalized power P/P_o on the reduced phase index $\text{Re}(\beta)$, and the reduced attenuation index $\text{Im}(\beta)$, respectively, for different values of temperature of the superconductor. We notice in Fig.(4a) that the reduced phase index $\text{Re}(\beta)$ increases, as the normalized power P/P_o increases. We also see, that the normalized power P/P_o is independent on the temperature of superconductor at lower $\text{Re}(\beta)$, that is, when the velocity of guided waved is increasing the power flow is no longer sensitive to the temperature.

Figs. 4(a-b) also exhibit a different type of optical limiter action under appropriate conditions, where the TM surface wave cuts off at a finite guided wave power.

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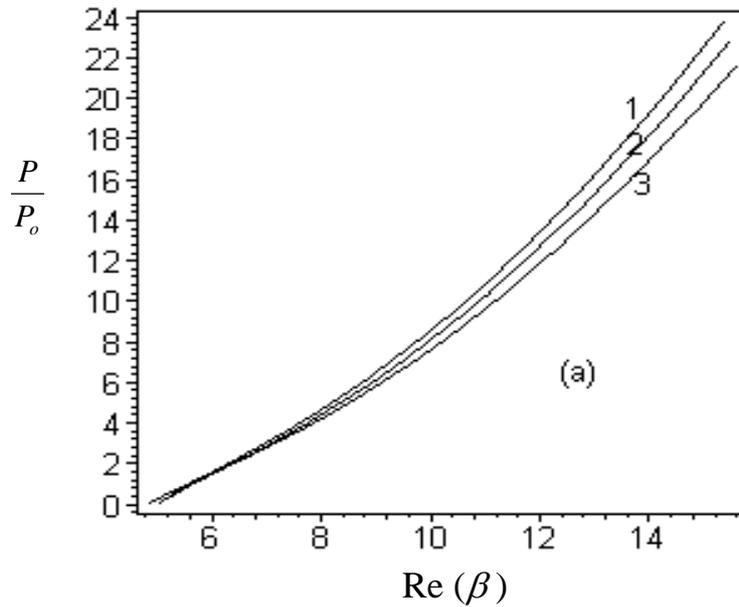


Fig.(4a). Calculated normalized total power P/P_0 vs. the reduced phase index $[\text{Re}(\beta)]$ for different values of reduced temperature (1) $T/T_c = 0.25$; (2) $T/T_c = 0.5$; (3) $T/T_c = 0.55$ and $\omega = 8.899 \times 10^{12}$ rad/s.

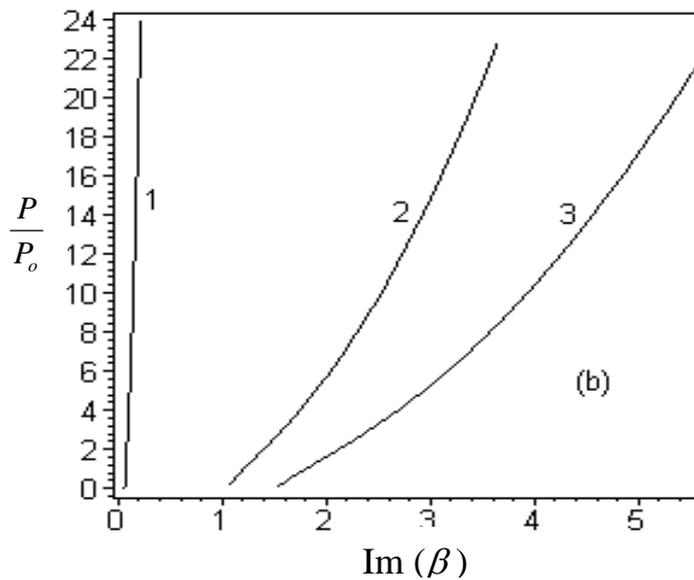


Fig. (4b) shows the normalized total power P/P_0 vs. the reduced attenuation index $[\text{Im}(\beta)]$ for different values of reduced temperature (1) $T/T_c = 0.25$; (2) $T/T_c = 0.5$; (3) $T/T_c = 0.55$ and $\omega = 8.899 \times 10^{12}$ rad/s.

4. Conclusions

We have studied the nonlinear behavior of TM surface waves in three layers of two-sublattice uniaxial antiferromagnet, superconductor and dielectric structures. The effect of the superconductors and nonlinear of antiferromagnet on TM surface waves has been examined as a function of the angular frequency and temperature. The nonlinear susceptibility is treated as constant throughout this article, since it is sensitive to frequencies compared to $\mu^L(\omega)$ in the off-resonance range. We have found that the loss versus the reduced temperature is slightly dependent on the operating frequency and the phase index depends strongly on the operating frequency for the higher values of the reduced temperature. Also, we have found that the magnetic surface waves have a power threshold and it is very sensitive to the temperature of superconductor. The variation of the $\text{Re}(\beta)$, $\text{Im}(\beta)$ with power at different values of T/T_c of superconductors is often regarded as implying the possibility of microwave switching.

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