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STABILITY OF NON LINEAR TE SURFACE WAVES ALONG THE INTERFACE OF NONLINEAR DIELECTRIC AND (LANS) SUPERLATTICES MEDIA

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استقرار الموجات السطحية غير الخطية TE في طبقة مادة غير خطية وشبكيات (LANS)

ملخص: لقد تم في هذا البحث دراسة الخواص الاستقرارية للموجات السطحيةغير الخطية المنتشرة بيـن شبكيات غير مغناطيسية (LAN.s) ووسف عازل غير خطي وذلك بطريقة عددية وتمثيلة ، حيـث مواد الشبكات غير المغناطيسية لها خواص تعتمد على تردد الموجات الكهرومغناطيسية وقد وضعت خاصيتها باستخدام نظرية الوسط الفعال، لقد أوجدنا معدل النمو لعملية الاضطراب الرياطية وذلـك بحل معادلة التشتت للحالة الاضطرابية ولقد وجدنا أن الموجات السطحية غير الخطية غير مسـتقرة وذلك لأن معدل النمو الاضطرابي حقيقي في وجود محدد النفاذية المغناطيسية (Me) هذا يعنـي ان اسـتقرار الموجـات السطحية غير الخطية يعتمد على نسبة المغناطيسية في الشبكيات المدروسة.

ولقد درسنا أيضاً إنسياب الطاقة في محور الانتشار كدالة في معامل الانتشار الفعال (Mx) لعدد من قيم المتطير المتشبع MP.

ولقد تبين أن طاقة الموجات السطحية غير الخطية تزداد بازدياد MP والقيمة المكانية لسعة المجال المسـتقر بينـت أن إزدياد عند ثبات MP =، وتوزيع الجمال يكون حاداً ومركزاً في الوسط غير الخطي.

Abstract: The stability characteristics of non-linear surface waves propagating at a lateral antiferromagnetic non magnetic superlattices (LANS) substrate and a non-linear dielectric cover have been simulated numerically by using the perturbation method . LANS are linear frequency- dependent gyromagnetic media, described with an effective- medium theory [1]. The growth rate of perturbation is computed by solving the dispersion equation of perturbation. We found that the nonlinear surface waves are unstable because their growth rate of perturbation δ is real with the existence of the magnetic matter of permeability tensor μ^e. This means that the stability of nonlinear surface waves is magnetic fractional dependent. The variation of the nonlinear surface waves energy flow [2,3] along the propagation axis as a function of the effective index n_x for different values of the saturation parameter μ_p has been studied. It illustrates that the energy of the non-linear surface waves increases with increasing μ_p . The spatial evolution [4] of the steady state field amplitude is determined by using computer simulation method [5,6]. The calculation shows that with increasing the n_x at fixed μ_p , the field distribution is sharpened and concentrated in the non linear medium.

1.INTRODUCTION

Nonlinear surface waves propagating along the interface of linear and nonlinear media have a number of novel extraordinary properties which attracted attention of many investigators[2-7] Understanding the stability of nonlinear surface waves is essential for the exploitation of these waves in optical devices. Up to date, this problem has not been solved completely, where there are numbers of approches to the problem by using both numerical simulations by Akhmediev et. al. [4] and Moloney et. al. [2] and analytical methods [3]. These methods have been based on steady-state solutions to a nonlinear wave equation which contains an intensity dependent refractive index. This paper concerns the stability of nonlinear surface waves propagate along the boundary of linear magnetic supper lattices, we solve this problem by using computer simulation method [8].

2.BASIC EQUATIONS

The guiding structure to be considered is shown in figure (1). It consists of a non-linear semi-infinite cladding contacts everywhere to a linear, semi-infinite superlattice at y = 0 planar interface. The coordinate system is such that, the *y* axis is normal to the interface and the wave vector is directed along the x axis. The magnetic field H_o is applied parallel to the interface and perpendicular to the direction of propagation of the waves. H_o and magnetization (m_0) of the superlattices are in the direction of the z-axis. Since the wave propagation is in x-direction then, the Maxwell equations for S-polarized wave (TE) are reduced to the following equation [4]

$$\nabla^2 E + \varepsilon\left(y, |E|^2\right)E = 0 . \qquad (1)$$

The dielectric constant of the linear medium in the region $y < 0$ is $\varepsilon_\perp$, while that in the region $y > 0$ is

$$\varepsilon^{\mu l} = \varepsilon_3 + \alpha|E|^2 . \qquad (2)$$

Assuming that the nonlinear medium is self-focusing the solution of the wave equation which is polarized along the z-axis is

$$E_z(x, y) = \alpha^{1/2} A(x, y) e^{i(\vec{n}.\vec{x} - \varpi t)} \qquad (3)$$

Where A (x, y) is a slowly varying field envelope, n_x is the effective refractive index.
Substituting Eq.(3) into Eq. (1), the equation for the slowly varying amplitude $A(x, y)$ is then [2]

$$2i\, n_x \frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial y^2} - k_2^2(y)A + \frac{\alpha}{\alpha_0}|A|^2 A = 0\,, \tag{4}$$

i denote to $\sqrt{-1}$ and $k_2^2(y) = n_x^2 - \varepsilon_3$, is the decay constant of the nonlinear medium, ε_3 is the linear dielectric constant of the non linear medium, the coordinates x and y are normalized by the factor ω/c, and the fields are normalized by the factor $\alpha_0^{1/2}$ where ω is the wave angular frequency, c is the light velocity in free space and $\alpha_\circ$ is the non-linearity coefficient.
The investigation of the stability of nonlinear surface wave (NSW) propagation along the interface between the linear and non linear medium has been focused in looking for the steady-state solution $A(x, y) = A_O(y)$ of equation (4) for our structure as:

$$A_O(y) = \begin{cases} 2^{\frac{1}{2}}\left(k_2^2 - \beta^2\right)^{1/2} e^{\beta y}\,,\ y < 0,\ \text{for linear medium} \\ 2^{\frac{1}{2}} k_2\ \operatorname{sec}h\left(k_2\left(y - y_\circ\right)\right), y > 0, \text{for nonlinear medium} \end{cases} \tag{5}$$

β is the decay constant of the linear medium, it is given by

$$\beta = \sqrt{(\mu_{xx} / \mu_{yy})n_x^2 - \varepsilon_\perp \mu_v} \tag{6a}$$

with

$$\begin{aligned} \mu_{xx} &= \left[1 / \left(f_1 + f_2\mu\right)\right]\left[\left(f_1^2 + f_2^2\right)\mu + f_1\ f_2\left(1 + \mu^2 - \mu_\perp^2\right)\right] \\ \mu_{yy} &= \mu / \left(f_1 + f_2\ \mu\right) \end{aligned} \tag{6b}$$

where the expressions of μ and $\mu_\perp$ are:

$$\begin{aligned} \mu &= 1 + \omega_a\omega_m\left\{\left[\omega_r^2 - (\omega_0 - \omega)^2\right]^{-1} + \left[\omega_r^2 - (\omega_0 + \omega)^2\right]^{-1}\right\}, \\ \mu_\perp &= \omega_a\omega_m\left\{\left[\omega_r^2 - (\omega_0 - \omega)^2\right]^{-1} + \left[\omega_r^2 - (\omega_0 + \omega)^2\right]^{-1}\right\} \end{aligned} \tag{6c}$$

with

$$\omega_m = 4\,\pi\,\gamma\ m_0\,, \omega_a = \gamma\ H_a\,, \omega_0 = \gamma\ H_0\,,$$

and

$$\omega_r = \gamma\ \sqrt{2\ H_a H_e + H_a^2}$$

where, H_a represents the an isotropy field, H_e the exchange field, and γ the gyromagnetic ratio, m_0 is the sublattice magnetization, ε_1 is the dielectric constant of the layers, ω_r is the resonance frequency. At the same time, the effective dielectric constant is expressed [1] by

$$\varepsilon_{\perp} = f_1\,\varepsilon_1 + f_2\,\varepsilon_2 \tag{6d}$$

and y_0 is the integration constant, that actually gives the position of the maximum E_Z in the upper nonlinear half space.
At the interface between the two media $y = 0$, we assume the condition that the dielectric constant of the linear medium $\varepsilon_{\perp} > \varepsilon_3$
To determine the stability criterion for (NSW)s, we numerically stimulated the steady state solution of Eq. (4) with small perturbation as in[4] :

$$A(x, y) = A_O(y) + \mu_p\; f\,(x, y), \tag{7}$$

where $f\,(x, y)$ is a perturbation function of the steady state solution, μ_p is the saturation parameter.
Substituting Eq. (7) into Eq.(4), we can obtain

$$2i\,n_x\,\frac{\partial f}{\partial x} + \frac{\partial^2 f}{\partial y^2} - k_2^2(y) f + \frac{\alpha}{\alpha_\circ} A_\circ^2(y)\left(2f + f^*\right) = 0\,, \tag{8}$$

we shall consider the z dependence of the perturbation function, so that the function can be written in the form [5]

$$f\,(x, y, z) = \frac{1}{2}\left[(u + v)e^{(\delta x + i r z)} + \left(u^* - v^*\right)e^{\left(\delta^* x - i r z\right)}\right] \tag{9}$$

where u and v are functions of y only, we take the case $r^2 = \delta^2$ for nonlinear medium.
Substituting Eq. (9) into Eq. (8), we obtain the set of differential equations of eigenvalue δ. δ is either real or imaginary. The solutions of differential equations decay as $\bar{y} \to \infty$ for self focused waves in nonlinear medium of the form

$$\begin{aligned} u &= c_1\, e^{-p\bar{y}}\left[-i\,\xi + 2p\tanh\bar{y} + 2\tanh^2\bar{y}\right] \\ &\quad + c_2\, e^{-p^*\bar{y}}\left[+i\,\xi + 2p^*\tanh\bar{y} + 2\tanh^2\bar{y}\right], \\ v &= c_1\, e^{-p\bar{y}}\left[2 + i\,\xi - 2p\tanh\bar{y}\right] \; - c_2\, e^{-p^*\bar{y}}\left[2 - i\,\xi - 2p^*\tanh\bar{y}\right] \end{aligned} \tag{10}$$

where $\xi' = 2n_x\delta$, $\bar{y} = k_2(y - y_o)$, $\xi = \xi'/k_2^2$,

where $p=(1+i\xi)^{1/2}$, c_1, c_2 are constants to be determined from the boundary conditions
and primes denote the derivatives with respect to $\bar{y}$.
In a linear medium, the solutions decaying as $\bar{y}\rightarrow -\infty$, are

$$u = A_1 e^{s\bar{y}} + A_2 e^{s^*\bar{y}}, \quad v = A_1 e^{s\bar{y}} - A_2 e^{s^*\bar{y}}, \tag{11}$$

where $s=\left(\frac{\beta^2}{k_2^2}-i\xi\right)^{1/2}$, and $\bar{y}=k_2\, y$, A_1, A_2 are constants to be determined (12) from the boundary conditions. For a surface wave ξ is either real or imaginary, suppose that $\xi=\xi_r$ is real thus by a bit of algebra we can obtain a dispersion relation for determining ξ of the form [4]

$$\left|p(1+i\xi)-2i\xi t-3pt^2+2t^3-s(p-t)^2\right|^2-(1-t^2)^2\left|p-2t+\bar{s}\right|^2=0,$$

where $t=\tanh(k_2\, y_\circ)$ which implies $0<t<1$ and $\tanh(k_0 k_z y_0)=\dfrac{-\beta\,\mu_{yy}+n_x\,\mu_{xy}}{\mu_{yy}\,\mu_v\,k_2}$.

If the applied magnetic field H_O equal to zero, the dispersion relation for NSW is $\beta=\mu_{xx}k_2\tanh(k_2\, y_\circ)$ leads to $t=\left(\dfrac{\beta}{\mu_{xx}\,k_2}\right)$ (13)

Eq. (12) may be solved analytically by expanding each of the two expressions under the absolute value in terms of ξ up to the fourth order and calculate the absolute values of these expressions, one obtains that [5]

$$\xi_r^2=0.533(1-2t), \tag{14}$$

when $t<\frac{1}{2}$, $\xi_r^2>0 \Rightarrow \xi_r$ is real, the growth rate δ is related with ξ_r by [4] $\delta=\dfrac{\xi_r\, k_2^2}{2n_x}$ which cause the NSW to be unstable. When $t>\frac{1}{2}, \xi_r^2<0$ *leads to* ξ_r is imaginary where δ becomes imaginary and NSW is stable. At $t=\frac{1}{2}$, n_x in this case is the critical refractive index. The evolution of the perturbed field amplitude $A(y)$ at the propagation distance

(15)

x, is calculated by determination the constants c_1, c_2, A_1, A_2 through applying the boundary conditions at y = 0 as [4]:

1) $E_{z_l} = E_{z_{Nl}}$,it is found by substituting Eq.(5)and Eq.(7) into Eq.(3)

2) $B_{x_l} = B_{x_{Nl}}$, B is the magnetic induction of the system[1] ,

where

$$B_{x_{Nl}} = \frac{\mu_{x_{Nl}}}{\omega \mu_\circ} \frac{\partial E_{z_{Nl}}}{\partial y}, \quad B_{x_l} = \mu_{xx} H_{x_l} + i \mu_{xy} H_{y_l} , \qquad (16a)$$

$$H_{x_l} = \frac{-\beta\mu_{yy} + n_x \mu_{xy}}{\omega \mu_\circ \mu_v \mu_{yy}} E_z, \quad H_{y_l} = \frac{n_x \mu_{yy} - \beta\mu_{xy}}{\omega \mu_\circ \mu_v \mu_{yy}} E_z .$$

Since the wave function u vanishes at the boundary, say y =10 then

3) $u_{N_l} = 0$ at $y = 10$ 4) $u_l = 0$ at $y = -10$. (16b)

At the initial perturbation where $x = 0$. Its convenient to take $c_2 = c_1^*$ and $A_2 = A_1^*$, then by solving the two equations (15), (16) , we can obtain the values of the constants c_1, c_2, A_1, A_2 By numerical simulation method its easy to study the evolution of the steady state field amplitude $A(x, y, z)$, *at* $x = 0$, $x_1 = 2.9$, *and* $x = 3$.

The variation of the energy integral of the nonlinear surface waves versus the mode index n_x is also calculated analytically for different values of the saturation parameter μ_p , and different values of the magnetic fraction f_1 through the integral of square perturbed field amplitude in linear and nonlinear medium as [4]:

$$I = \int_{-\infty}^{0} |A_l(x, y)|^2 \, dy + \int_{0}^{\infty} |A_{Nl}(x, y)|^2 \, dy \qquad (17)$$

where $A_l(x, y)$, $A_{Nl}(x, y)$ are the perturbed field amplitude in linear and nonlinear medium respectively.

3.COMPUTER SIMULATION AND DISCUSSION:

In this paper some numerical calculations are presented for the simulation of the stability equation (7) of the proposed structure which consists of a lateral $f_e f_2$ /$z_n f_2$ superlattice and a nonlinear dielectric cover. $f_e f_2$ is ferrous floride which is antiferromagnetic film and $z_n f_2$ is zinc floride which is nonmagnetic film. The parameters are [1] as follows: the applied field H_0 =

0 G, $m_0 = 0.56\ kG$, $H_a = 200\ kG$, $H_e = 540\ kG$, and $\varepsilon_1 = 5.5$ for antiferromagnetic layers. Fig.(2).shows the nonlinear surface waves energy flow versus the n_x for various values of μ_p . For propagation length x = 0 and applied magnetic field H_0 = 0, the nonlinear surface waves are always unstable and its nonlinear energy increases by μ_p . Fig.(3) illustrates the variation of the nonlinear surface waves energy flow with n_x for different values of the magnetic fraction f_1. It shows that as the power varies with n_x also varies with t which is less than 0.5. These values are strongly dependent on magnetic fraction f_1, they are shifted to higher wave index as the magnetic fraction is increased. This dependence is shown as a shift in the curves. The increasing of f_1 leads to a decreasing of growth rate of perturbation and nonlinear waves of higher energy have low unstability.
For increasing values of n_x Fig.(4b-a) displays the spatial evolution of steady state field amplitude $A_l(x, y)$ as a function of the normal to the interface y for increasing the propagation distance x . This spatial evolution of perturbations leads to a shift of the field energy into the nonlinear medium, with the subsequent excitation of the nonlinear surface waves of the unstable branch. Also the calculation shows that with increasing n_x at fixed μ_P, the field distribution is sharpened and shifted in nonlinear medium as shown in Figs (5a-b-c) at $x = 3$..

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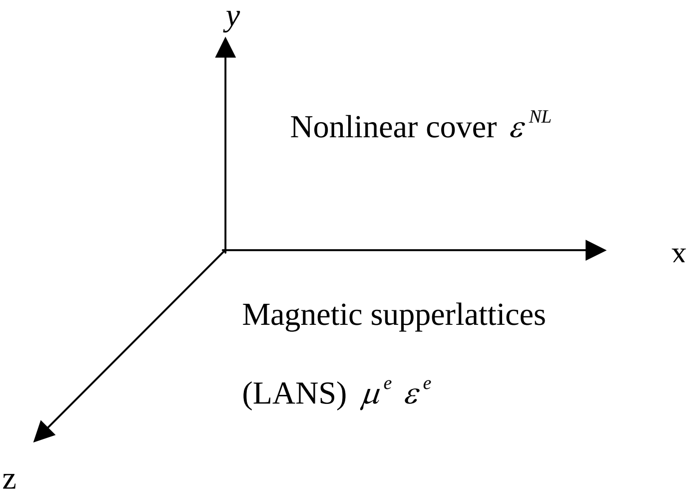

Fig.(1) Coordinate system guiding TE surface waves along the interface.

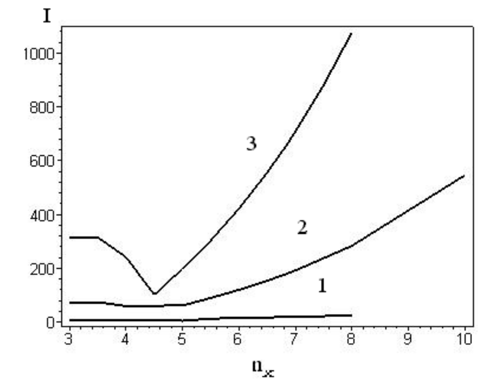

Fig.(2) The energy flow of the nonlinear surface waves as a function of n_x **for(1)** $\mu_p = 0$ **, (2)** $\mu_p = 0.4$ (3) $\mu_p = 0.8$, **x =0, f_1 =0.8**

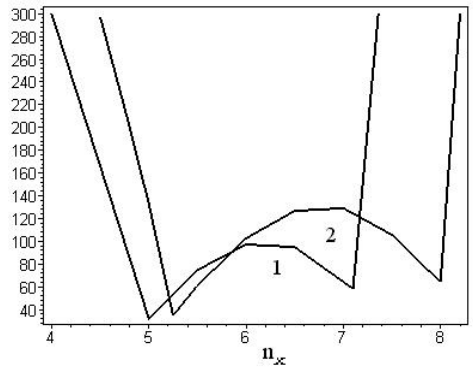

Fig.(3) The energy flow of the nonlinear surface waves as a function n_x **for f_1 =0.8 ,(2) f_1 =0.82,** [illegible]$_p$ **=997.3x10^{10}rad/sec, H $_o$=0 , x = 0.**

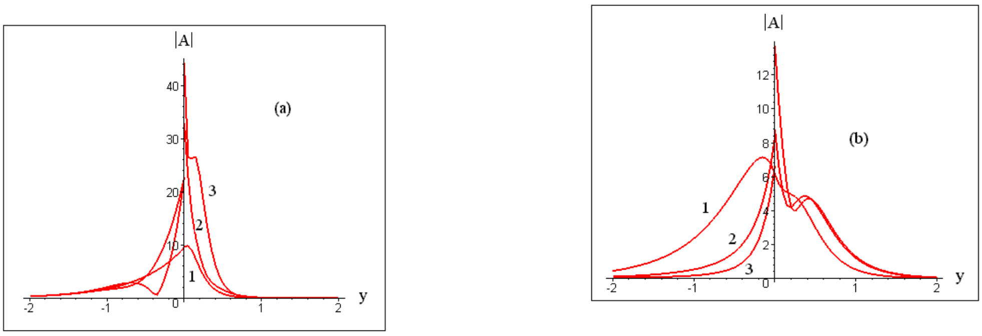

Fig.(4a-b) The unstable branch of nonlinear surface waves (a) n_x = 7.1, ▯ =2.38, and for (b) n_x = 3.5,▯ =.725 , ▯ =997.3 x 10^{10} rad / sec , μ_p = 0.3 and (1) x = 0, (2) x =2.9, (3) x =3.

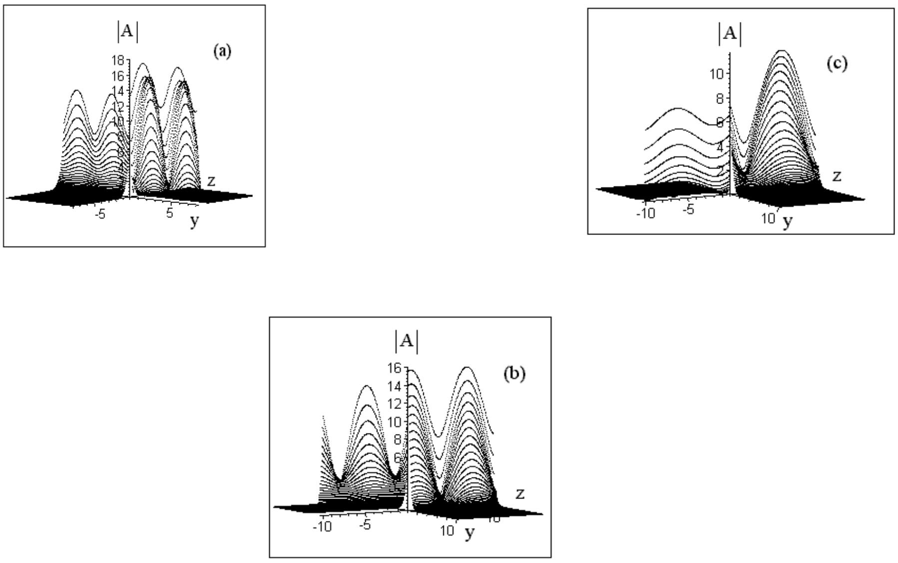

Fig.(5a-b-c).The field distribution A(y,z) for(a) n_x = 5.2 , □ =1.493 (b)) n_x = 4 , □ =.9554, (c) n_x =3.5,□ =.725 for μ_p = 0.3, f_1 = 0.8, □ = 997.3x10^{10} rad / sec , x = 3.