ON T_o- ALEXANDROFF SPACES.

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ملخص: في هذا البحث قمنا بدراسة ووصف بعض المفاهيم التبولوجية على نوع خاص من الفضاءات التبولوجية، والتي تسمي فضاءات T_0 − الكساندروف . في هذا الصنف من الفضاءات التبولوجية تعرفنا علي قفل المجموعة، باطن المجموعة وعنقود المجموعة، ثم قمنا بدراسة المجموعات المفتوحة المعممة مثل المجموعة تمهيد مفتوحة، المجموعة شبه مفتوحة، والمجموعة ألفا− مفتوحة.

وجدنا أن فضاء T_0−اكساندرووف الذي يحقق خاصية السلسلة الصاعدة (ACC) له أهمية خاصة، أسميناه أرتينيان T_0−اكساندرووف، وقد حصلنا علي نتائج قوية علي هذا الفضاء مثل التعرف علي خاصية التقطع الشديد.

من النتائج الهامة التي حصلنا عليها إيجاد الشرط الضروري والكافي لجعل فضاء T_0−اكساندرووف شبه علوي.

ABSTRACT. In this paper, we study and describe some topological concepts on a special class of topological spaces called T_o- *Alexandroff spaces.* We characterize the closure, the interior, and the cluster points of a subset. We identify some of the generalized open sets as preopen, semi-open, $\alpha-$open and we study some related properties.

We focus on a special important type of T_o- Alexandroff spaces which satisfy the ACC. Since this is equivalent to DCC on certain ideals, we call such spaces *Artinian* T_o- *Alexandroff spaces.* We get strong results for this class, such as characterization of exteremally disconnectedness.

One of the important results is finding necessary and sufficient conditions for a T_o- Alexandroff space to be submaximal.

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1. Introduction.

In this paper, we study a class of topological spaces called T_o- Alexandroff spaces. In general, a topological space (X, τ) satisfies the property that a finite intersection of open sets is open and an arbitrary intersection of open sets need not be open. An *Alexandroff space* (or *minimal neighborhood space*) X is a space in which the arbitrary intersection of open sets is open. Equivalently, each point of X has a minimal neighborhood base. The class of Alexandroff spaces includes all finite topological spaces. In fact, each finite topological space is locally finite, and each locally finite topological space is Alexandroff.

This subject was first studied in 1937 by P. Alexandroff [28] under the name of Diskrete Räume (discrete space). The name is not valid now, since a discrete space is a space where the singletons are open. He gave an Example of a T_o- Alexandroff space on a poset $(P, \leq)$ taking $\mathcal{B} = \{\uparrow x : x \in P\}$ to be the unique minimal base. The induced topology on P, denoted by $\tau(\leq)$, is a T_o- Alexandroff space. If (X, τ) is an Alexandroff space, he defined its (Alexandroff) specialization order $\leq_\tau$ on X by that $a \leq_\tau b$ if $a \in \overline{\{b\}}$. The specialization order is reflexive and transitive. It is antisymmetric - and hence a partial order - if and only if X is T_o. Moreover, if $(X, \leq)$ is a poset and if $\tau(\leq)$ is its induced T_o- Alexandroff topology, then the specialization order of $\tau(\leq)$ is the order $\leq$ itself, i.e. $\leq_{\tau(\leq)} = \leq$. On the other hand, if (X, τ) is a T_o- Alexandroff space with specialization order $\leq_\tau$, then the induced topology by the specialization order is the original topology, i.e. $\tau(\leq_\tau) = \tau$ [10]. Therefore, T_o- Alexandroff spaces are completely determined by their specialization orders.

We focus on Alexandroff spaces that satisfy the separation axiom T_o. We use their specialization orders in proofs to illustrate the results and the concepts. The importance of this study comes from the fact that we can characterize topological properties just by looking at its specializing order (poset). For example, we know that a topological space X is a submaximal if each dense subset is open. For a T_o-Alexandroff space X, X is submaximal if each element in the corresponding poset is either maximal or minimal, i.e. the graph of its corresponding poset contains two rows, the row of maximal elements and the row of minimal elements.

A T_o- Alexandroff space whose corresponding posets satisfy the ACC is considered in this paper. We call such spaces *Artinian* T_o- *Alexandroff spaces*. The ACC on a T_o- Alexandroff space is weaker than the locally finite condition. Thus, each locally finite T_o- Alexandroff space is Artinian, while the converse is not true. So, the class of Artinian T_o- Alexandroff spaces is larger than the class of locally finite T_o- Alexandroff spaces, which is turn is larger than the class of finite T_o spaces. Therefore, our results hold for finite and locally finite T_o- Alexandroff spaces.

We get strong results for an Artinian T_o- Alexandroff space X such as:

(1) X is hereditarily irresolvable, scattered and $\alpha-$scattered.
(2) $PO(X) = \tau_\alpha$ and $PO(X) \subseteq SO(X)$.
(3) $PO(X) = SO(X)$ if and only if X is extremelly disconnected if and only if for each $x \in X$, $|\hat{x}| = 1$, i.e. there is exactly one element $y \in M$ with $x \leq y$, where M is the set of all maximal elements.
(4) The subset A is dense if $M \subseteq A$ and nowhere dense if $M \cap A = \emptyset$.
(5) For a set A, we characterize $pint(A), sint(A), pCl(A)$ and $sCl(A)$.

The dual notion of ACC is descending chain condition (DCC). A T_o- Alexandroff space in which the corresponding poset satisfies the DCC is called a *Noetherian* T_o- *Alexandroff space.*

Francisco G. Arenas studied Alexandroff spaces in [10]. For the class of Alexandroff spaces, he studied pseudo-metrizablity, regularity and other concepts. For the class of T_o- Alexandroff spaces, he studied connectedness, compactness, separability and others.

In a similar study [1], A. El-Fattah et al. considered only the finite case. Our results here hold for a larger class of topologies.

Throughout this paper, the symbol $(X, \tau(\leq))$ denotes a T_o- Alexandroff space where $\leq$ is its (Alexandroff) specialization order. For each element $x \in X$, $\uparrow x$ or $V(x)$ denotes the minimal neighborhood. For each subset A of X, the interior (resp. the closure, the derive, the boundary, the semi-interior, the semi-closure, the preinterior, the preclosure, the

$\alpha-$interior, the $\alpha-$closure) will be denoted by A^{o} (resp. $\overline{A}$, A', $bd(A)$, $sint(A)$, $sCl(A)$, $pint(A)$, $pCl(A)$, $int_{\alpha}(A)$, $Cl_{\alpha}(A)$). The notation := means equal by definition.

2. Preliminaries and Definitions

A partially ordered set (poset) [5] is a set P with a binary relation $\leq$ that satisfies the following:

i) $x \leq x$ (reflexivity).

ii) $x \leq y$ and $y \leq x$ imply $x = y$ (anti-symmetry).

iii) $x \leq y$ and $y \leq z$ imply $x \leq z$ (transitivity).

A poset P is *a lattice* if $\forall x, y \in P$, $x \vee y$=sup$\{x, y\}$ and $x \wedge y$=inf$\{x, y\}$ exist, and P is *a complete lattice* if $\vee S$ and $\wedge S$ exist $\forall S \subseteq P$. It is well known that a complete lattice has *a top element* $\top$ (maximum) and *a bottom element* $\bot$ (minimum) such that $\forall x \in P, \bot \leq x \leq \top$. An element $x \in P$ is *a maximal element* if whenever $x \leq z$ then $x = z$ and y is *a minimal element* if whenever $z \leq y$ then $y = z$. Two elements x, y in P are *comparable* if either $x \leq y$ or $y \leq x$, otherwise they are *incomparable*. A subset $C \subseteq P$ is *a chain* if any two elements of C are comparable. Alternative names for a chain are *linearly ordered set and totaly ordered set*. A subset $A \subseteq P$ is *convex* if whenever $x, y \in A$ and $x \leq z \leq y$ we have $z \in A$. We say x is *covered* by y or y *covers* x and we write $x \rightarrowtail y$ or $y \leftarrowtail x$ if $x < y$ and when $x \leq z < y$ then $z = x$, where $z < y$ means $z \leq y$ and $z \neq y$. A set $O \subseteq P$ is *a down set* if whenever $x \in O$ and $y \leq x$, we have $y \in O$. A set $U \subseteq P$ is an *up set* if whenever $x \in U$ and $y \geq x$ we have $y \in U$. For $x \in P$ we define the down set $\downarrow x := \{y \in P : y \leq x\}$ and the up set $\uparrow x := \{y \in P : y \geq x\}$, and for a set $A \subseteq P$, we define the down set $\downarrow A := \{y \in P : (\exists x \in A) y \leq x\}$ and the up set $\uparrow A := \{y \in P : (\exists x \in A) y \geq x\}$. If $C = \{c_0, c_1, ..., c_n\}$ is a finite chain in P with $|C| = n + 1$, then we say that *the length* of C is equal to n. *The length* of the poset P (denoted by $\ell(P)$) is the length of the longest chain in P. A poset P satisfies *the ascending chain conditions*, (ACC) if for any sequence $x_1 \leq x_2 \leq ... \leq x_n \leq ...$ in P there exist $k \in \mathbb{N}$ such that $x_k = x_{k+1} = ...$, and the dual of the ACC is the *descending chain condition* (DCC).

If $(X, \tau(\leq))$ is a T_o- Alexandroff space then a subset A of X is open if and only if it is an up set with respect to the specialization order ($A = \uparrow A$), and A is closed if and only if it is a down set ($A = \downarrow A$).

Suppose that A is a subset of a T_o- Alexandroff space $(X, \tau(\leq))$. Then two types of topologies are induced on A. One is the T_o- Alexandroff Space on A with respect to the induced order $\leq$, and the other is the induced topology $\tau(\leq)|_A$ which makes A as a subspace. It is not difficult to see that the two types coincide (see Theorem 2.2 of [10]).

In any poset, $\uparrow x$ is usually called an ideal and $\downarrow x$ is called a filter. If $x_1 \leq x_2 \leq \cdots \leq x_n \leq \cdots$ then $\uparrow x_1 \supseteq \uparrow x_2 \supseteq \cdots \supseteq \uparrow x_n \supseteq \cdots$, so ACC is equivalent to saying that any sequence of descending ideals is finally constant, i.e. DCC on ideals. But this situation happens in rings and the ring in this case is called an Artinian ring. So I suggest that we call a T_o- Alexandroff space that satisfies ACC an "*Artinian* T_o- Alexandroff space". Dually, the T_o- Alexandroff space with DCC shoud be called *Noetherian* T_o- Alexandroff space.

If $(X, \tau(\leq))$ is an Artinian T_o- Alexandroff space, we define M to be the set of all maximal elements of X. For a point $x \in X$, we define $\hat{x} = \uparrow x \cap M$. The point x is *isolated* in X if $\{x\}$ is an open set, and hence maximal in X. So M is the set of all isolated points in X. If A is a subset of an Artinian T_o-Alexandroff space, then we define $M(A)$ to be the set of all maximal elements of A under the induced order. It is easy to see that if A is open then $\hat{x} \subseteq A \ \forall x \in A$ and if $M(A) \nsubseteq M$ then A is not open. Dually, if X is a Noetherian T_o- Alexandroff space, we define m to be the set of all minimal elements of X. For the point $x \in X$, we define $\check{x} = \downarrow x \cap m$. If A is a subset of a Noetherian T_o-Alexandroff space, then we define $m(A)$ to be the set of all minimal elements of A under the induced order. It is easy to see that if A is closed then $\check{x} \subseteq A \ \forall x \in A$ and if $m(A) \nsubseteq m$ then A is not closed.

Some of T_o- Alexandroff spaces are given in the following examples:

Example 2.1.

(1) Let $X = \{a, b, c, d\}$ with the partial order $a \leq b,\ a \leq c$ and $d \leq c$ as shown in Figure 1 below

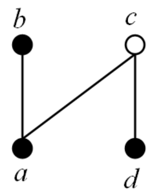

FIGURE 1

then the T_o- Alexandroff topology is

$$\tau = \{\emptyset, X, \{a, b, c\}, \{b\}, \{c\}, \{d, c\}, \{b, c, d\}, \{b, c\}\}$$

with the minimal base $\mathcal{B} = \{\{a, b, c\}, \{b\}, \{c\}, \{d, c\}\}$. The set $A = \{a, b, d\}$ is down set which is not up, so it is closed and not open. Note that $A^c = \{c\} \in \tau$.

(2) Let $X = \mathbb{R}$ with the usual order, so for $x \in \mathbb{R},\ V(x) = [x, \infty)$ is the minimal basic neighborhood of x and the T_o- Alexandroff topology on $\mathbb{R}$ is the usual right ray topology.

(3) Let $X = \{a, b, c, d\}$, with the T_o-Alexandroff topology

$$\tau = \{\emptyset, X, \{a, b, c\}, \{b\}, \{c\}, \{b, d, c\}, \{b, c\}\}.$$

We can find the specialization order as follows: $\overline{\{a\}} = \{a\}$, $\overline{\{d\}} = \{d\}$, $\overline{\{b\}} = \{b, a, d\}$, and $\overline{\{c\}} = \{c, a, d\}$, so $a \leq b,\ d \leq b,\ a \leq c$, and $d \leq c$ and hence the figure of the poset X is

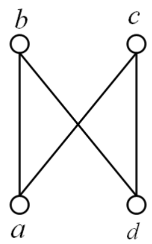

FIGURE 2

3. Identification of Basic Topological Concepts

Let A be a subset of a T_o- Alexandroff space X. A point x is *a cluster point* of A if $V(x)$ intersects $A\backslash\{x\}$. The point $x \in A$ is *isolated* if $\{x\}$ is open in the subspace A, i.e. if $x \notin A^{'}$. A set A is *dense-in-itself* if A contains no isolated points. A is *perfect* if A is closed and dense-in-itself. The space X is *scattered* if no subset of X is dense-in-itself, and $\alpha-$*scattered* [20] if it has a dense set of isolated points. A point $x \in X$ is called *pure* if $\{x\}$ is either open or closed. Otherwise it is called *mixed*.

Proposition 3.1. *Let $(X, \tau(\leq))$ be a T_o-Alexandroff space, and let $A \subseteq X$.*

1. *For $x \in X$, $\overline{\{x\}} = \downarrow x$.*
2. $A^{o} = \{x \in A : \uparrow x \subseteq A\} = \cup\{\uparrow x : \uparrow x \subseteq A\}$.
3. $\overline{A} = \cup\{\downarrow x : x \in A\}$.
4. $A^{'} = \overline{A}\backslash\{x : x$ *is maximal in* $A\}$.

Proof. (1) $\downarrow x$ is a down set and hence a closed set containing x. So $\overline{\{x\}} \subseteq \downarrow x$. Now let $y \in \downarrow x$, so $y \leq x$. If $y \in \overline{\{x\}}^{c}$, which is an open set then $\uparrow y \cap \overline{\{x\}} = \emptyset$. Since $x \in \uparrow y$, we get that $x \notin \overline{\{x\}}$, a contradiction.

(2) Straightforward.

(3) If $x \in A$, then $\overline{\{x\}} = \downarrow x \subseteq \overline{A}$, so $\cup\{\downarrow x : x \in A\} \subseteq \overline{A}$. On the other hand, if $x \in A$ then $x \in \downarrow x \subseteq \cup\{\downarrow x : x \in A\}$, so $A \subseteq \cup\{\downarrow x : x \in A\}$, which is a closed set. Therefore $\overline{A} \subseteq \cup\{\downarrow x : x \in A\}$.

(4) If $x \in A^{'}$ then $x \in \overline{A}$ and $\uparrow x \cap A\backslash\{x\} \neq \emptyset$, so x is not maximal in A, and hence $A^{'} \subseteq \overline{A}\backslash\{x : x \text{ is maximal in } A\}$. For the other inclusion, if $y \in \overline{A}$, and y is not maximal in A, we have that $\uparrow y \cap A \neq \emptyset$. If $\uparrow y \cap A = \{y\}$ then y is a maximal in A, and this is not true so we have that $y \in A^{'}$ □

The boundary of a given subset A is the set $bd(A) = \overline{A}\backslash A^{o} = \overline{A} \cap \overline{A^{c}}$, so it is a closed set, $bd(A) = \cup\{\downarrow x : x \in A\}\backslash\{x : \uparrow x \subseteq A\}$, and if $A^{o} = \emptyset$, then $bd(A) = \overline{A}$.

Theorem 3.2. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then*

(1) $A^{o} = \emptyset \Leftrightarrow A \cap M = \emptyset$.
(2) $\overline{A} = \cup\{\downarrow x : x \in M(A)\} = \downarrow M(A)$.
(3) $A^{'} = \cup\{(\downarrow x)\backslash\{x\} : x \in M(A)\} = (\downarrow M(A))\backslash M(A)$.
(4) *the subset A is dense if and only if $M \subseteq A$.*
(5) *the subset A is nowhere dense if and only if $M \cap A = \emptyset$.*
(6) *if $|M| = 1$, then any subset is either dense or nowhere dense.*
(7) *open sets and closed sets are convex (but not conversely, i.e. convex sets need not be open sets or closed sets).*

Proof. (1) $(\Rightarrow) A^{o} = \emptyset$ and $x \in A \cap M$ so x is maximal of X in A, and hence $\uparrow x = \{x\} \subseteq A$ contradicting $A^{o} = \emptyset$.
$(\Leftarrow)$Suppose that $A \cap M = \emptyset$, and $y \in A^{o}$, so $\uparrow y \subseteq A$. Since X satisfies the ACC, we get a maximal element z in X such that $y \leq z$ and so $z \in \uparrow y \subseteq A$. But this implies that $z \in A \cap M$ which is a contradiction.

(2) If $x \in A$, then there exists a maximal element y in A such that $x \leq y$, so $\downarrow x \subseteq \downarrow y$, and this implies that $\cup\{\downarrow x : x \in A\} \subseteq \cup\{\downarrow x : x \text{ is maximal in } A\}$. The other inclusion is obvious.

(3) Since X satisfies the ACC, it follows that

$$\begin{aligned} A^{'} &= \overline{A}\backslash\{x : x \text{ is maximal in } A\} \\ &= \cup\{\downarrow x : x \in M(A)\}\backslash M(A) \\ &= \cup\{(\downarrow x)\backslash\{x\} : x \in M(A)\}. \end{aligned}$$

(4) ($\Rightarrow$) Suppose that A is dense, and let $x \in M$. Then
$\uparrow x \cap A \neq \emptyset$. But $\uparrow x = \{x\}$ so $x \in A$.
($\Leftarrow$) Suppose that $M \subseteq A$, so $M(A) = M$. By part 2,

$$\overline{A} = \cup\{\downarrow x : x \in M(A)\} = \cup\{\downarrow x : x \in M\} = X.$$

(5) ($\Rightarrow$) $\overline{A}^{o} = \emptyset$, so by Theorem 3.2(1), $M \cap \overline{A} = \emptyset$, and hence $M \cap A = \emptyset$.
($\Leftarrow$)Suppose that $M \cap A = \emptyset$, so no maximal element of X in A. By Theorem 3.1 part 3 no maximal element of X in $\overline{A}$, and hence $\overline{A}^{o} = \emptyset$.

(6) Let $M = \{\top\}$, and let A be a subset of X, then either $\top \in A$ or $\top \notin A$, and by parts (a) and (b) either A is dense or nowhere dense.

(7) Obvious. $\square$

If A is a subset of an Artinian T_o- Alexandroff space X, we have that $bd(A) \cap M = \emptyset$, and if $x \in A$ such that $\hat{x} \nsubseteq A$, then $x \notin A^{\circ}$.

If the T_o- Alexandroff space satisfies the ACC then as part (iii) of the above theorem illustrates, the set of all isolated points of a subset A is the set $M(A)$ which is a subset of A, and hence no subset of X is dense-in-itself. Moreover, we know that M is the set of all isolated points of X. So we get the following theorem.

Theorem 3.3. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then*

(1) *X is scattered.*
(2) *X is $\alpha-$scattered.*

Example 3.4.

(1) Consider the poset X with the partial order in Figure 3

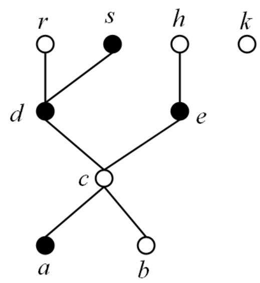

FIGURE 3

So X satisfies the ACC and $M = \{r, s, h, k\}$.

If $A = \{a, d, e, s\}$, then

(i) $A^{o} = \uparrow s = \{s\}$
(ii) $\overline{A} = \downarrow s \cup \downarrow e = \{a, b, c, d, e, s\}$
(iii) $bd(A) = \overline{A} \backslash A^{o} = \{a, b, c, d, e\}$ which is a closed set that contains no maximal element of X.
(iv) $A' = (\downarrow s) \backslash \{s\} \cup (\downarrow e) \backslash \{e\} = \{a, b, c, d\}$

If $B = \{a, d, e\}$

(i) $B^{o} = \emptyset$
(ii) $\overline{B} = \downarrow d \cup \downarrow e = \{a, b, c, d, e\}$

(iii) $bd(B) = \overline{B} = \{a, b, c, d, e\}$
(iv) $B' = (\downarrow d)\backslash\{d\} \cup (\downarrow e)\backslash\{e\} = \{a, b, c\}$
Note that $M \cap B = \emptyset$ so by Theorem 3.2(5), B is nowhere dense and hence $\overline{B}^o = \emptyset$.

If $C = \{r, s, h, k, a, b\}$ then $M \subseteq C$ and by Theorem 3.2(4), C is a dense subset.

(2) Consider the poset X with the partial order in Figure 4

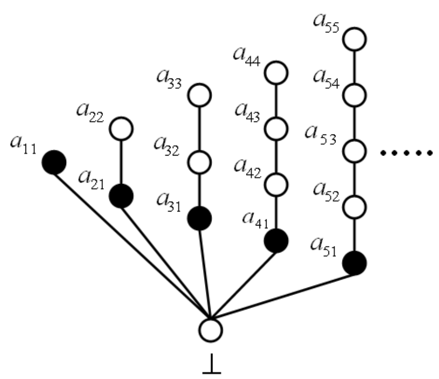

FIGURE 4

So X satisfies the ACC where the set $M = \{a_{11}, a_{22}, a_{33}, a_{44}, ...\}$ and any subset contains M is dense.
Let $A = \{a_{11}, a_{21}, a_{31}, a_{41}, ...\}$. Then
(i) $A^o = \{a_{11}\}$
(ii) $\overline{A} = A \cup \{\perp\}$
(iii) $bd(A) = \{\perp, a_{21}, a_{31}, a_{41}, ...\}$
(iv) $A' = \{\perp\}$.

Note that the only set that contains $\perp$ in its interior is X itself.

4. Identification of Some Generalized Open Sets

In this section, we will focus our study on the class of Artinian T_o-Alexandroff spaces.

Recall that a topological space (X, τ) is called *locally finite* if each element x of X is contained in a finite open set and a finite closed set. If $(X, \tau(\leq))$ is a locally finite T_o- Alexandroff space, then $(X, \leq)$ satisfies both ACC and DCC. The converse is not true. To see this, consider the T_o- Alexandroff space $\{\perp\} \cup \mathbb{N}$ with anti-chain order on $\mathbb{N}$. It satisfies the ACC which is not locally finite. So, in a T_o- Alexandroff space the ACC is weaker condition than the locally finite condition and the class of Artinian T_o- Alexandroff spaces is larger than the class of locally finite T_o- Alexandroff spaces.

Preopen, semi-open, α-open and some other concepts have been considered. The name preopen was used for the first time by Mashhour, Abd El-Monsef and El-Deeb [4]. The definition of a preopen set was introduced by Corson and Michael [14] under the name of "locally dense". The other types of generalization of open sets such as α-open, semi-open β-open, b-open were studied by Njåstad, Levine, and others. They introduced the concepts of preclosed, semi-closed, α-closed, then they derived the semi-interior, semi-closure, pre-interior, pre-closure, α-interior and α-closure (for more information, see [14, 26, 27])

Definitions 4.1. A subset A of a topological space (X, τ) is

(1) *a semi-open set* [26] if $A \subseteq \overline{A^o}$, and *a semi-closed set* [33] if A^c is semi-open. Thus A is semi-closed if and only if $\overline{A}^o \subseteq A$. If A is both semi-open and semi-closed then A is called *semi-regular* [7].

(2) *a preopen set* [4] if $A \subseteq \overline{A}^o$, and *a preclosed set* [24] if A^c is preopen. Thus A is preclosed if and only if $\overline{A^o} \subseteq A$.

(3) *an α-open set* [27] if $A \subseteq \overline{A^o}^o$, and *an α-closed set* [18] if A^c is α-open. Thus A is α-closed if and only if $\overline{\overline{A}^o} \subseteq A$.

The family of all semi-open (resp. preopen, α-open) is denoted by $SO(X)$ (resp. $PO(X)$, τ_α). Njåstad [27] proved that τ_α is a topology on X. In general, $SO(X)$ and $PO(X)$ need not be topologies on X. A set A is preopen [23] if and only if $A = U \cap D$ where U is an open set and D is a dense set. In [18], it has been shown that a set is α-open if and only if it is semi-open and preopen. If $A \subseteq X$, then $pInt(A)$ (resp. $sInt(A)$) is the largest preopen set (resp. semi-open set) inside A. $pCl(A)$ (resp. $sCl(A)$) is the smallest preclosed set (resp. semi-closed set) contains A.

Recall that a space (X, τ) is called *resolvable* [8] if and only if $X = D \cup D^c$ where both D and D^c are dense. A subset A is *resolvable* if the subspace $(A, \tau|_A)$ is resolvable. A space (X, τ) is *irresolvable* if it is not resolvable. It is *strongly irresolvable* [21] if no nonempty open set is resolvable, and it is *hereditarily irresolvable* [8] if no nonempty subset is resolvable. A space (X, τ) is *nodec* [22] if all nowhere dense sets are closed and *hyperconnected* if every open subset of X is dense. If X is not hyperconnected then it is *hyperdisconnected*. Finally a space (X, τ) is *submaximal* [15] if each dense subset is open. The following implications hold

$$submaximal \Rightarrow hereditarily\ irresolvable \Rightarrow$$

$$strongly\ irresolvable \Rightarrow irresolvable.$$

Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. The set M belongs to all dense subsets of X, so no disjoint dense subsets exist in X and hence X is surely irresolvable. Since a subspace of X is also an Artinian T_o- Alexandroff space, we get that X is hereditarily irresolvable, and hence it is strongly irresolvable. We will prove this using the following two theorems and after characterizing preopen, semi-open, and $\alpha-$open sets.

Theorem 4.2. [2] *For a space (X, τ), the following are equivalent:*

(i) (X, τ) *contains an open, dense and hereditarily irresolvable subspace.*

(ii) *Every open ultrafilter on X is a base for an ultrafilter on X.*

(iii) *X is strongly irresolvable.*

(iv) *For each dense subset D of X, D^o is dense.*

(v) *For $A \subseteq X$ where $A^o = \emptyset$, A is nowhere dense.*

Theorem 4.3. [23] *For a space (X, τ), the following are equivalent:*

(i) *(X, τ) contains an open, dense, and hereditarily irresolvable subspace.*

(ii) *$PO(X) \subseteq SO(X)$.*

(iii) *$PO(X) = \tau_\alpha$.*

(iv) *The space (X, τ_α) is submaximal.*

Proposition 4.4. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. If A is a preopen set then each maximal element in A belongs to M, i.e. $M(A) \subseteq M$.*

Proof. A is preopen so $A \subseteq \overline{A}^o$. Let x be maximal in A. Since X satisfies the ACC, a maximal element $y \in X$ exists such that $x \leq y$. If $x \neq y$ then $y \notin A$ and by Theorem 3.2(2), $y \notin \overline{A}$ and hence $y \notin \overline{A}^o$. But $x \in A \subseteq \overline{A}^o$ so $y \in \uparrow x \subseteq \overline{A}^o$ which is a contradiction and so $x = y$.
Alternative Proof: $A = U \cap D$ where U is open and D is dense. Let $x \in M(A)$ so $x \in U$ and hence $\hat{x} \subseteq U$. Since X satisfies the ACC, there exist $y \in M \subseteq D$ such that $y \in \hat{x}$, so we get $y \in U \cap D = A$, but $x \in M(A)$, so $x = y \in M$ and therefore $M(A) \subseteq M$. □

The following example shows that the converse of the above theorem is not true, i.e. if each maximal element in A is maximal in X then A need not be preopen.

Example 4.5. Let $X = \{a, b, c, d, r, e\}$ be a poset with the partial order in Figure 5

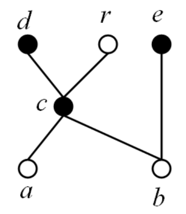

FIGURE 5

and let $A = \{c, d, e\}$. Then $M = \{d, r, e\}$ and so the maximal elements of A are in M. Note that $\overline{A} = \downarrow d \cup \downarrow e = \{a, b, c, d, e\}$, $\overline{A}^o = \{d, e\}$ and hence $A \nsubseteq \overline{A}^o$ so A is not a preopen.

The set A is preopen if and only if $A = U \cap D$ where U is open and D is dense. So the set A is preclosed if and only if $A = F \cup D^c$ where F is closed and D is dense. This implies the following theorem

Theorem 4.6. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then the set A is preclosed if and only if $\downarrow x \subseteq A$ for all $x \in A \cap M$.*

Proof. $(\Rightarrow)$ A is preclosed then $A = F \cup D^c$ where F is closed and D is dense it follows that $M \subseteq D$. Let $x \in A \cap M$ so $x \in F \cup D^c$ and $x \in M$ and this implies that $x \in F$ so $\downarrow x \subseteq F \subseteq A$.

$(\Leftarrow)$ Suppose that for all $x \in A \cap M$, $\downarrow x \subseteq A$.
Set $F = \cup\{\downarrow x : x \in A \cap M\}$ and $D^c = A \backslash F$, so $F \subseteq A$ which is closed and $M \cap D^c = \emptyset$ and hence D is dense. Moreover $A = F \cup D^c$, and therefore A is preclosed. □

Corollary 4.7. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then*

(a) *the set A is preopen if and only if $\downarrow x \cap A = \emptyset$ for all $x \in A^c \cap M$. Equivalently, A is preopen if and only if $\hat{x} \subseteq A$ for all $x \in A$.*

(b) *if X contains a top element $\top$, then a nonempty subset A is preopen if and only if $\top \in A$ if and only if A is dense.*

Proof. (a) ($\Rightarrow$) If $x \in A$ and $\hat{x} \cap A^c \neq \emptyset$, and if $y \in \hat{x} \cap A^c$, then $y \in M \cap A^c$ and $x \in \downarrow y \cap A$ and hence A^c is not preclosed.

($\Leftarrow$) Let $y \in A^c \cap M$. If $\downarrow y \cap A \neq \emptyset$, then there exists $x \in A$ such that $y \in \hat{x}$ and hence $\hat{x} \nsubseteq A$.

(b) ($\Rightarrow$) Suppose that A is preopen and let $x \in A$. Then by part (a) $\hat{x} = \{\top\} \subseteq A$.

($\Leftarrow$) Obvious. □

If A contains no elements of M, then $A^o = \emptyset$ and hence A is not semi-open. The following theorem characterizes semi-open sets.

Theorem 4.8. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then a set A is semi-open if and only if $M(A) \subseteq M$.*

Proof. ($\Rightarrow$) Suppose that $A \subseteq \overline{A^o}$ and $x \in A$ is a maximal element of A, then $x \in \overline{A^o}$. By Theorem 3.2(2), there exists z which is maximal in A^o and $x \in \downarrow z$. Since x is maximal in A, we get that $x = z$, and hence $x \in A^o$ so $\hat{x} \subseteq A$. If $y \in \hat{x}$, then $y \in M$ and $x \leq y$ in A, and since x is maximal in A, we have $x = y$ in M.

($\Leftarrow$) Let $x \in A$ and choose y as maximal in A such that $x \leq y$, so $y \in M$ and $\uparrow y = \{y\} \in A$, which implies that $y \in A^o$ and $x \in \downarrow y \subseteq \overline{A^o}$, and therefore $A \subseteq \overline{A^o}$ □

Corollary 4.9. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then $PO(X) \subseteq SO(X)$, that is if A is a preopen then it is semi-open.*

Proof. Follows from Proposition 4.4. □

As a consequence of this corollary, we have that if A is preclosed then A is semi-closed. To see this, suppose A is preclosed. Then A^c is preopen and by Corollary 4.9, A^c is semi-open. Therefore A is semi-closed.

M. Ganster in [23] showed that the collection $PO(X)$ is a topology if and only if the intersection of any two dense sets is preopen. In an Artinian T_o- Alexandroff space, the intersection of any two dense sets is dense and hence preopen. So $PO(X)$ is a topology on X. If U is preopen then by Corollary 4.9, it is semi-open and hence it is $\alpha-$open, and we have the following corollary.

Corollary 4.10. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then $PO(X) = \tau_\alpha$, that is a set A is preopen if and only if it is $\alpha-$open.*

Proof. Obvious. □

As a consequence of Corollary 4.10, A set is $\alpha-$closed if and only if it is preclosed if and only if $\downarrow x \subseteq A$ for all $x \in A \cap M$.

Corollary 4.11. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then*

(i) *X contains an open, dense and hereditarily irresolvable subspace.*
(ii) *every open ultrafilter on X is a base for an ultrafilter on X.*
(iii) *X is strongly irresolvable.*
(iv) *for each dense subset D of X, D^o is dense.*
(v) *(X, τ_α) is submaximal.*
(vi) *for $A \subseteq X$ where $A^o = \emptyset$, A is nowhere dense.*

Proof. Direct consequence from Theorem 4.2, Theorem 4.3, and Corollary 4.9. □

Part (i) tells us that there exists an open, dense, and hereditarily irresolvable subspace. It is not difficult to see that one open, dense, and hereditarily irresolvable subspace is the subspace M. In fact X may contain more than one and we can take it as any up set that contains M. For part (iii) X is strongly irresolvable. In fact X is hereditarily irresolvable. In [8], E. Hewitt proved that every topological space can be represented uniquely as a disjoint union $X = F \cup G$, where F is closed and resolvable, and G is open and hereditarily irresolvable. Moreover, X is resolvable iff $G = \emptyset$ and X is hereditarily irresolvable iff $F = \emptyset$. So if X is an Artinian T_o- Alexandroff space, then $F = \emptyset$ and $G = X$ in the Hewitt representation. Part (iv) can easily be proved directly, since if D is dense then $M \subseteq D$ and hence $M \subseteq D^{^o}$, so $D^{^o}$ is dense. Part (vi) of the above corollary is studied in description of the closure of a set (see Theorem 3.2(5)).

M. Ganster in [23] proved that if X has the Hewitt representation $X = F \cup G$ and if x is nonisolated point of G then $\{x\} \notin \tau^*$ where τ^* is the topology on X having $PO(X)$ as a subbase. The point is isolated in a T_o- Alexandroff space if it is maximal, and if X satisfies the ACC, each point not in M is nonisolated and so it is not preopen as a set. In fact it is nowhere dense by Theorem 3.2(5).

Corollary 4.12. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space. Then a subset A is semi-closed if and only if $\forall x \notin A$, there exists $y \in M \backslash A$ such that $x \leq y$. Equivalently, A is semi-closed if and only if $\forall x \notin A$, $\hat{x} \nsubseteq A$.*

Proof. ($\Rightarrow$) Let A be semi-closed, and let $x \in A^{^c}$. If x is maximal in $A^{^c}$ then $x \in M$. So take $y = x$. In the other case, if x is not maximal in $A^{^c}$, we get a maximal element y in $A^{^c}$ such that $x \leq y$ and by the above theorem, $y \in M \cap A^{^c}$.

($\Leftarrow$) Suppose that $\forall x \in A^{^c}$, there exists $y \in M \cap A^{^c}$ such that $x \leq y$. Let z be a maximal element in $A^{^c}$. Then there exists $y \in M \cap A^{^c}$ such that

$z \leq y$. But $y \in A^c$ and z is maximal in A^c. Hence we get that $z = y$, and hence $z \in M$, so A^c is semi-open. $\square$

Example 4.13. Let $X = \{a, b, c, d, r, e\}$ be a poset with the partial order in Figure 6 below

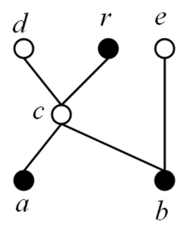

FIGURE 6

Let $A = \{a, b, r\}$. Then A is semi-closed since A^c is semi-open. Corollary 4.12 is applies to on A. Note that A is semi-regular and since $\downarrow r \nsubseteq A$, it is not preclosed.

Proposition 4.14. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space, and let A be a subset of X. Then*

(i) $sCl(A) \subseteq pCl(A)$.
(ii) $pint(A) \subseteq sint(A)$.

Proof. (i) $pCl(A)$ is preclosed set contains A, so it is semi-closed contains A, and hence $sCl(A) \subseteq pCl(A)$.
(ii) $pint(A)$ is preopen set inside A so it is semi-open in A and hence $pint(A) \subseteq sint(A)$. $\square$

Theorem 4.15. *Let $(X, \tau(\leq))$ be an Artinian T_o- Alexandroff space and A is a subset of X. Then*

(a) $pint(A) = \{x \in A : \hat{x} \subseteq A\}$.

(b) $sint(A) = \{x \in A : \hat{x} \cap A \neq \emptyset\}$.

(c) $pCl(A) = A \cup \{\downarrow x : x \in A \cap M\}$.

(d) $sCl(A) = A \cup \{x : \hat{x} \subseteq A\}$.

Proof. (a) If $y \in A \cap M$ then $\hat{y} = \{y\} \subseteq A$ and so $y \in \{x \in A : \hat{x} \subseteq A\}$. Let $y \in M \backslash \{x \in A : \hat{x} \subseteq A\}$, so $y \notin A$. If $z \in \downarrow y$ then $y \in \hat{z}$ which implies $z \notin \{x \in A : \hat{x} \subseteq A\}$, so $\downarrow y \cap \{x \in A : \hat{x} \subseteq A\} = \emptyset$, and hence by Corollary 4.7(a), $\{x \in A : \hat{x} \subseteq A\}$ is preopen which subset of A. Now if U is preopen in A, and if $z \in U$ such that $z \notin \{x \in A : \hat{x} \subseteq A\}$, then $\hat{z} \cap A^c \neq \emptyset$, say $r \in \hat{z} \cap A^c$, so we get $r \in M \backslash U$, moreover $z \in \downarrow r \cap U$ which contradict by U is preopen. Therefore $z \in \{x \in A : \hat{x} \subseteq A\}$ and hence $pint(A) = \{x \in A : \hat{x} \subseteq A\}$.

(b) Let $z \in M(\{x \in A : \hat{x} \cap A \neq \emptyset\})$ so $\hat{z} \cap A \neq \emptyset$. Let $w \in \hat{z} \cap A$, so $w \in M$ and $\hat{w} = \{w\} \subseteq A$, therefore $w \in \{x \in A : \hat{x} \cap A \neq \emptyset\}$. Since $w \geq z$ and z is maximal in $\{x \in A : \hat{x} \cap A \neq \emptyset\}$, we get that $z = w$, therefore $z \in M$ and $\{x \in A : \hat{x} \cap A \neq \emptyset\}$ is semi-open set contained in A. Now suppose that S is semi-open set contained in A. If $x \in S$ then by ACC there is $y \in M(S)$ such that $x \leq y$, so $y \in M$, hence $\hat{x} \cap A \neq \emptyset$ therefore $S \subseteq \{x \in A : \hat{x} \cap A \neq \emptyset\}$ and $sint(A) = \{x \in A : \hat{x} \cap A \neq \emptyset\}$.

(c) Let $z \in A \cup \{\downarrow x : x \in A \cap M\}$ where $z \in M$. If $z \notin A$ then $z \in \downarrow x$ for some $x \in A \cap M$ which contradict by $z \in M$, so $z \in A \cap M$ and hence $\downarrow z \subseteq \cup\{\downarrow x : x \in A \cap M\}$ which implies that $A \cup \{\downarrow x : x \in A \cap M\}$ is preclosed contains A. Let B be another preclosed set contains A, and let $x \in A \cap M$, so $x \in B$ and hence $\downarrow x \subseteq B$, therefore $A \cup \{\downarrow x : x \in A \cap M\} \subseteq B$ and $pCl(A) = A \cup \{\downarrow x : x \in A \cap M\}$.

(d) If $z \notin A \cup \{x : \hat{x} \subseteq A\}$ then $z \notin A$ and $z \notin \{x : \hat{x} \subseteq A\}$, so $\hat{z} \cap A^c \neq \emptyset$. If $r \in \hat{z} \cap A^c$ then $r \notin A$ and $r \notin \{x : \hat{x} \subseteq A\}$ ($r \in M$

so $\hat{r} = \{r\}$). Therefore $\hat{z} \nsubseteq A \cup \{x : \hat{x} \subseteq A\}$ and by Corollary 4.12 $A \cup \{x : \hat{x} \subseteq A\}$ is semi-closed contains A. If C is another semi-closed set contains A, and if $y \in \{x : \hat{x} \subseteq A\}$ not in C, then $\hat{y} \subseteq A \subseteq C$. By ACC there exist $r \in M(C^c)$ such that $y \leq r$. Since C^c is semi-open, we get $r \in M$ and so $r \in \hat{y} \cap C^c$ which contradict by $\hat{y} \subseteq A \subseteq C$. Therefore $A \cup \{x : \hat{x} \subseteq A\} \subseteq C$ and $sCl(A) = A \cup \{x : \hat{x} \subseteq A\}$. □

Example 4.16. Consider the poset X with the partial order in Figure 7 below

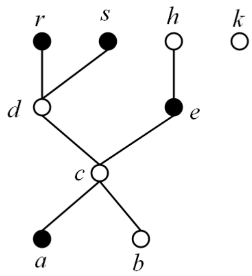

FIGURE 7

X satisfies the ACC and $M = \{r, s, h, k\}$. Let $A = \{a, e, r, s\}$, since $\hat{e} = \{h\} \nsubseteq A$, we get that A is not preopen, and $e \in M(A)$, so $M(A) \nsubseteq M$ and A is not semi-open. Moreover, we have the following:

(a) $pint(A) = \{x \in A : \hat{x} \subseteq A\} = \{r, s\}$.
(b) $sint(A) = \{x \in A : \hat{x} \cap A \neq \emptyset\} = \{a, r, s\}$.
(c) $pCl(A) = A \cup \{\downarrow x : x \in A \cap M\} = \{a, b, c, e, d, r, s\}$.
(d) $sCl(A) = A \cup \{x : \hat{x} \subseteq A\} = \{a, e, d, r, s\}$.

Note that $sCl(A) \subseteq pCl(A)$ and $pint(A) \subseteq sint(A)$.

5. Identification of some topological properties

Theorem 5.1. [15] *Let (X, τ) be a topological space. Then the following are equivalent:*

(1) *X is submaximal.*
(2) *Every preopen set is open.*

The following theorem characterizes the submaximality condition in a T_o- Alexandroff space.

Theorem 5.2. *Let $(X, \tau(\leq))$ be a T_o- Alexandroff space. Then all the following are equivalent:*

(i) *X is submaximal.*
(ii) *Each element of X is either maximal or minimal, i.e. each element of X is pure.*
(iii) *X is nodec.*

Proof. $(i \Rightarrow ii)$ Suppose that X is submaximal, and there exists $x_1 < x_2 < x_3$ in X, then we get a set $X \backslash \{x_2\}$ which is dense subset but not open which is a contradiction.

$(ii \Rightarrow i)$ Suppose that each element of X is either maximal or minimal, so X satisfies both ACC and DCC. If U is a dense subset, then $M \subseteq U$, so U is upset and hence open set,since if $x \in U$, and $y \in X$ such that $x \leq y$, then either $x = y$ or $y \in M \subseteq U$, therefore X is submaximal.

$(ii \Rightarrow iii)$ If each element of X is either maximal or minimal, we get that X is Artinian, and by Theorem 3.2 part 5, a subset A is nowhere sense if and only if $A \cap M = \emptyset$, so $A \subseteq m$ and hence A is closed.

$(iii \Rightarrow ii)$ If there exist $a, b, c \in X$ such that $a < b < c$ then $\{b\}$ is nowhere dense subset which is not closed, so X is not nodec. $\square$

If a T_o- Alexandroff space X is submaximal then X satisfies both ACC and DCC. In fact, the graph of the poset X contains two rows, the set M, and the set m, in this case and by Theorem 5.1 each preopen set is open.

By Corollary 4.9, a preopen set is semi-open. A semi-open set need not be preopen. Example 4.5 gives a semi-open set which is not preopen. But is there a condition on T_o- Alexandroff space that makes $SO(X) = PO(X)$?. The answer of this question is not direct. Njåstad in [27] showed that $SO(X)$ is a topology if and only if (X, τ) is extremally disconnected, where the space is *extremally disconnected* if the closure of every open set is open. In this case, $SO(X) = \tau_\alpha$. In an Artinian T_o- Alexandroff space, $PO(X) = \tau_\alpha$. So we get the following Theorem.

Theorem 5.3. *In Artinian T_o- Alexandroff spaces, the following are equivalent:*

(i) (X, τ) *is extremelly disconnected.*
(ii) $PO(X) = SO(X)$.
(iii) *For all $x \in X$, $|\hat{x}| = 1$, i.e. $\forall x \in X$ there exists exactly one element $y \in M$ such that $x \leq y$.*

Proof. (i)⇔(ii) is obvious.
(ii)⇒(iii) If there exists $x_o \in X$ such that $|\hat{x_o}| \geq 2$, then there are two different elements $y, z \in \uparrow x_o \cap M$ and hence $S = \{x_o, y\}$ is semi-open set that is not preopen.
(iii)⇒(ii) Let S be semi-open and x be maximal in S^c. If $\downarrow x \cap S \neq \emptyset$, then there exists $y \in S$ such that $y \leq x$. By ACC of X there exists a maximal element $z \in S$ such that $z \geq y$. Since S is semi-open, $z \in M$ and so $x, z \in \hat{y}$. Therefore $|\hat{y}| \geq 2$. □

Corollary 5.4. *If $(X, \tau(\leq))$ is a submaximal, extremally disconnected T_o- Alexandroff space, then $SO(X)$ is equal to the original topology.*

Proof. Since X is submaximal, X is Artinian and $PO(X) = \tau(\leq)$. Since X is extremally disconnected, $SO(X) = PO(X)$. □

Corollary 5.5. *If X has a top element $\top$, then $PO(X) = SO(X)$, and so X is extremally disconnected.*

The exteremally disconnectedness does not imply disconnectedness. We can see this fact in a T_o- Alexandroff space which has a maximum element. By the above corollary it is exteremally disconnected, and since any open set must contain the top element, no disjoint open sets exist, and therefore it is connected.

Theorem 5.6. [18] *For a topological space (x, τ), the following conditions are equivalent:*

(1) *X is hyperconnected.*
(2) *Every nonempty preopen subset of X is dense.*

Theorem 5.7. *Let $(X, \tau(\leq))$ be a T_o- Alexandroff space.*

(1) *If X is a chain, then X is hyperconnected.*
(2) *If X contains a maximum element $\top$, then X is hyperconnected.*
(3) *If X satisfies the ACC, then X is hyperconnected if and only if X contains a top element $\top$.*
(4) *It may happen that a hyperconnected T_o- Alexandroff space is not a chain and does not contain a maximum element.*

Proof. (1) Let $U \in \tau(\leq)$ be a nonempty open set, so U is an up set. Let $x \in U \subseteq \overline{U}$, and let $y \in X$. Since X is a chain, either $x \leq y$ or $x \geq y$. If $x \leq y$ then $y \in U \subseteq \overline{U}$, since U is an up set, and if $x \geq y$, then $y \in \overline{U}$, since $\overline{U}$ is a down set. Therefore $\overline{U} = X$. Hence U is dense.

(2) Any open set U contains $\top$ and hence $\downarrow \top \subseteq \overline{U}$. Therefore $\overline{U} = X$.

(3) ($\Rightarrow$) If $|M| \geq 2$, so there exist two elements $x_1 \neq x_2$ in M, and hence $\{x_1\}$ is open subset which is not dense.

($\Leftarrow$) If $M = \{\top\}$, then M is a subset of every open set, which implies that every open set is dense, and hence X is hyperconnected.

Alternative Proof: ($\Rightarrow$) Suppose that X is hyperconnected. By Theorem 5.6 , every nonempty preopen subset of X is dense. If there exist $x_1 \neq x_2$ in M, then $(\downarrow x_1)^c$ is preopen subset and $x_1 \notin (\downarrow x_1)^c$, so $(\downarrow x_1)^c$ is not dense and this is a contradiction.

($\Leftarrow$) If $|M| = 1$ then by Corollary 4.7(b), each preopen subset is dense, and hence by Theorem 5.6, X is hyperconnected.

(4) The following example gives a hyperconnected T_o- Alexandroff space which is not a chain and does not contain a maximum element. □

Example 5.8. Let $X = \mathbb{N}$ with the partial order in Figure 8 below

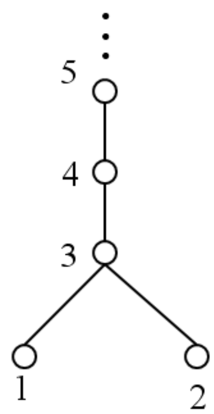

FIGURE 8

If U is open in X then U is an up set. So we can choose $x \in U \subseteq \overline{U}$ such that $x \geq 5$. Let $y \in X$. Then either $y \leq x$ or $y \geq x$. Since $\overline{U}$ is a down set, if $y \leq x$, we get $y \in \overline{U}$. Since U is an up set, if $y \geq x$, we get $y \in U \subseteq \overline{U}$. Therefore $\overline{U} = X$, and hence X is hyperconnected.

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