Linear Power Field

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Abstract

The power field is defined. It is found that the motion of the power field oscillates linearly on the second harmonics, 2œω, where ω is the fundamental harmonic.

1 Introduction

The power field plays as important parameter in some physical sciences [1]. For example, if an electrical current is passing through a conductor then the electric power is going to be dissipated into the conductor causing it to melt down. Such phenomenon is not understood yet. Furthermore in radio frequency gas discharge a generation of harmonics were observed in research laboratories [2]. Also, no explanation for such kind of phenomena is reported in the literature.

In this paper we define the time-independent power field operator and try to understand the motion of the expected value of this operator. It is found that if the field oscillates on the fundamental harmonic, ω, then the power field oscillates on the second harmonic, 2ω. This means that if a second harmonic is observed in the field the cause of these harmonics is the power dissipated into the field. The generation of harmonics is very important in gas discharge, since it has numerous applications [3]. One of this applications is the dry etching and the manufacture of microelectronic devices [4].

2 Definition of the Power Field

The dynamical equation of a time dependent operator $\hat{A}$ is given by

$$\frac{d\langle \hat{A} \rangle}{dt} = \frac{i}{\hbar}\langle [\hat{\mathcal{H}}, \hat{A}] \rangle + \langle \frac{\partial \hat{A}}{\partial t} \rangle. \tag{1}$$

The Hamiltonian of the quantum mechanical harmonic oscillator is given by

$$\hat{\mathcal{H}} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right), \tag{2}$$

and

$$\hat{a}\hat{a}^\dagger - \hat{a}^\dagger \hat{a} = 1, \tag{3}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the quantum mechanical harmonic oscillator [5, 6].

If we define the current and the voltage fields [7, 8, 9] by the Hermitian operators

$$\hat{I} = \sqrt{\frac{\hbar\omega}{2L}} \left(\hat{a}^\dagger + \hat{a} \right), \tag{4}$$

$$\hat{V} = i\sqrt{\frac{\hbar\omega}{2C}} \left(\hat{a}^\dagger - \hat{a} \right), \tag{5}$$

then the time-independent operator which is defined by

$$\hat{I}\hat{V} = i\frac{\hbar\omega^2}{2} \left(\hat{a}^\dagger \hat{a}^\dagger - \hat{a}\hat{a} + 1 \right) \tag{6}$$

is not Hermitian operator, because its complex conjugate, adjoint, is given by

$$\left(\hat{I}\hat{V} \right)^\dagger = \hat{V}^\dagger \hat{I}^\dagger = \hat{V}\hat{I} = i\frac{\hbar\omega^2}{2} \left(\hat{a}^\dagger \hat{a}^\dagger - \hat{a}\hat{a} - 1 \right). \tag{7}$$

Subtract Eq.(7) from Eq.(6), we get

$$\hat{I}\hat{V} - \hat{V}\hat{I} = i\hbar\omega^2. \tag{8}$$

This means that the $\hat{I}\hat{V}$ operator and its complex conjugate are not commute.

Now let us define the time-independent power field operator by the following Hermitian operator

$$\hat{P} = \frac{1}{2}\left(\hat{I}\hat{V} + \hat{V}\hat{I}\right). \qquad (9)$$

Substitute from Eq.(7) into Eq.(9), we get

$$\hat{P} = \hat{I}\hat{V} = i\frac{\hbar\omega^2}{2}. \qquad (10)$$

Substitute from Eq.(6) into the above equation, to obtain

$$\hat{P} = i\frac{\hbar\omega^2}{2}\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right). \qquad (11)$$

3 The Number State Fluctuation in the Power Field

Let us assume that the number state of the harmonic oscillator, $|n\rangle$ which is an energy eigenstate [9], be an eigenstate of the power field $\hat{P}$ then the expected value of the power field is given by

$$\langle\hat{P}\rangle = \langle n|\, i\frac{\hbar\omega^2}{2}\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right)|n\rangle = 0 \qquad (12)$$

This does not mean, of course, that the field is zero as the expected value of the power field squared is given by

$$\begin{aligned}
\langle\hat{P}^2\rangle &= \left\langle\frac{(i\hbar\omega^2)^2}{4}\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right)\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right)\right\rangle \\
&= \left\langle\frac{(i\hbar\omega^2)^2}{4}\left(\hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger - \hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} - \hat{a}\hat{a}\hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}\hat{a}\hat{a}\right)\right\rangle \\
&= \left\langle\frac{(i\hbar\omega^2)^2}{4}\left[-a^\dagger\left(\hat{a}^\dagger\hat{a}\right)a - a\left(aa^\dagger\right)a^\dagger\right]\right\rangle \\
&= \left\langle\frac{(i\hbar\omega^2)^2}{4}\left[-a^\dagger\left(aa^\dagger - 1\right)a - a\left(\hat{a}^\dagger\hat{a} + 1\right)a^\dagger\right]\right\rangle \\
&= \left\langle\frac{(i\hbar\omega^2)^2}{4}\left[-\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a} + \hat{a}^\dagger\hat{a} - a\hat{a}^\dagger\hat{a}a^\dagger - aa^\dagger\right]\right\rangle
\end{aligned}$$

$$= \left\langle \frac{(i\hbar\omega^2)^2}{4}\left[-\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a} + \hat{a}^\dagger\hat{a} - \left(\hat{a}^\dagger\hat{a}+1\right)\left(\hat{a}^\dagger\hat{a}+1\right) - \left(\hat{a}^\dagger\hat{a}+1\right)\right]\right\rangle$$

$$= \left\langle \frac{(i\hbar\omega^2)^2}{4}\left[-\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a} + \hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a} - 2\hat{a}^\dagger\hat{a} - 1 - \hat{a}^\dagger\hat{a} - 1\right]\right\rangle$$

$$= \left\langle \frac{(i\hbar\omega^2)^2}{4}\left[-2\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a} - 2\hat{a}^\dagger\hat{a} - 2\right]\right\rangle$$

$$= -\left\langle \frac{(i\hbar\omega^2)^2}{2}\left[n^2 + n + 1\right]\right\rangle. \qquad (13)$$

For a single mode the power field described by a number state $|n\rangle$, the root mean square deviation in the power field strength is

$$\Delta\left(\hat{P}\right) = \sqrt{\langle\hat{P}^2\rangle - \langle\hat{P}\rangle^2}$$

$$= \frac{\hbar\omega^2}{\sqrt{2}}\sqrt{(n^2+n+1)} \qquad (14)$$

If the single mode power field is unoccupied, $n = 0$, then

$$\Delta\left(\hat{P}\left[vacuum\right]\right) = \frac{\hbar\omega^2}{\sqrt{2}} \qquad (15)$$

4 The Power Field Equation

Since the number state $|n\rangle$ are not an eigenstate of the power field operator $\hat{P}$ let us assume that the state $\psi(r,t)$ be an eigenstate of the power field operator. The expected value of the power field $\hat{P}$ is given by

$$\langle\hat{P}\rangle = \left\langle i\frac{\hbar\omega^2}{2}\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right)\right\rangle \qquad (16)$$

Substitute for $\hat{P}$, which is time-independent, into Eq.(1), we get

$$\frac{d\langle\hat{P}\rangle}{dt} = \frac{i}{\hbar}\langle[\hat{\mathcal{H}},\hat{P}]\rangle. \qquad (17)$$

Let us calculate the commutator by substituting from Eq.(2) and Eq.(16) into Eq.(17), we get

$$\left[\hat{\mathcal{H}},\hat{P}\right] = \hat{\mathcal{H}}\hat{P} - \hat{P}\hat{\mathcal{H}}$$

$$
\begin{aligned}
&= \hbar\omega\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right) i\frac{\hbar\omega^2}{2}\left(\hat{a}^\dagger\hat{a}^\dagger-\hat{a}\hat{a}\right) \\
&\quad - i\frac{\hbar\omega^2}{2}\left(\hat{a}^\dagger\hat{a}^\dagger-\hat{a}\hat{a}\right)\hbar\omega\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right) \\
&= i\frac{\hbar^2\omega^3}{2}\Big\{\Big[\hat{a}^\dagger\hat{a}\hat{a}^\dagger a^\dagger - \hat{a}^\dagger\hat{a}\hat{a}\hat{a}+\frac{1}{2}\left(\hat{a}^\dagger\hat{a}^\dagger-\hat{a}\hat{a}\right) \\
&\quad - \hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger\hat{a} - \hat{a}\hat{a}\hat{a}^\dagger\hat{a}+\frac{1}{2}\left(\hat{a}^\dagger\hat{a}^\dagger-\hat{a}\hat{a}\right)\Big]\Big\} \\
&= i\frac{\hbar^2\omega^3}{2}\Big\{\Big[a^\dagger\left(\hat{a}^\dagger\hat{a}+1\right)\hat{a}^\dagger - \hat{a}^\dagger\hat{a}^\dagger\left(\hat{a}\hat{a}^\dagger-1\right)\Big] \\
&\quad - \Big[\left(\hat{a}\hat{a}^\dagger-1\right)\hat{a}\hat{a} - a\left(\hat{a}^\dagger\hat{a}+1\right)\hat{a}\Big]\Big\} \\
&= i\hbar^2\omega^3\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right).
\end{aligned}
\tag{18}
$$

Substitute from Eq.(18) into Eq.(17), to get

$$
\frac{d\langle\hat{P}\rangle}{dt} = -\hbar\omega^3\langle\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right)\rangle. \tag{19}
$$

Operate with $\frac{d}{dt}$ from the left on Eq.(19), we get

$$
\frac{d^2\langle\hat{P}\rangle}{dt^2} = -\hbar\omega^3\frac{d}{dt}\langle\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right)\rangle \tag{20}
$$

Now, let us calculate

$$
\frac{d}{dt}\langle\left[\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right]\rangle = \frac{i}{\hbar}\langle\left[\mathcal{H},\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right)\right]\rangle \tag{21}
$$

The commutator is given by

$$
\begin{aligned}
\left[\mathcal{H},\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right)\right] &= \mathcal{H}\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right) - \left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right)\mathcal{H} \\
&= \hbar\omega\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right)\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right) \\
&\quad - \left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right)\hbar\omega\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right) \\
&= \hbar\omega\Big\{\Big[\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}^\dagger+\hat{a}^\dagger\hat{a}\hat{a}\hat{a}+\frac{1}{2}\left(\hat{a}^\dagger\hat{a}^\dagger+\hat{a}\hat{a}\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\left[\hat{a}^\dagger\hat{a}^\dagger\hat{a}^\dagger a + \hat{a}\hat{a}\hat{a}^\dagger\hat{a} + \frac{1}{2}\left(\hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}\right)\right]\} \\
= & \ \hbar\omega\{\left[\hat{a}^\dagger\left(\hat{a}^\dagger\hat{a} + 1\right)\hat{a}^\dagger + \left(\hat{a}\hat{a}^\dagger - 1\right)\hat{a}\hat{a}\right. \\
& \left. -\hat{a}^\dagger\hat{a}^\dagger\left(\hat{a}\hat{a}^\dagger - 1\right) + \hat{a}\left(\hat{a}^\dagger\hat{a} + 1\right)\hat{a}\right]\} \\
= & \ \hbar\omega\{\left[\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}^\dagger\hat{a}\hat{a} - \hat{a}\hat{a}\right. \\
& \left. -\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}^\dagger\hat{a}\hat{a} + \hat{a}\hat{a}\right]\} \\
= & \ 2\hbar\omega\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right)
\end{aligned} \tag{22}
$$

Substitute from Eq.(22) into Eq.(21), we get

$$
\begin{aligned}
\frac{d}{dt}\langle\left[\hat{a}^\dagger\hat{a}^\dagger + \hat{a}\hat{a}\right]\rangle &= \langle i2\omega\left(\hat{a}^\dagger\hat{a}^\dagger - \hat{a}\hat{a}\right)\rangle \\
&= \frac{4}{\hbar\omega}\langle\hat{P}\rangle
\end{aligned} \tag{23}
$$

Substitute from Eq.(23) into Eq.(20), we get

$$
\frac{d^2\langle\hat{P}\rangle}{dt^2} = -(2\omega)^2\langle\hat{P}\rangle \tag{24}
$$

which can be written as

$$
\frac{d^2\langle\hat{P}\rangle}{dt^2} + (2\omega)^2\langle\hat{P}\rangle = 0 \tag{25}
$$

which means that the power field oscillates linearly on the second harmonics, 2ω.

5 Conclusion

The quantum mechanical harmonic oscillator field oscillates on the fundamental harmonics, ω. However, according to the above definition of the time-independent power field, it is found that the power field oscillates linearly on the second harmonics, 2ω.

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