بسم الله الرحمن الرحيم

Islamic University –Gaza
Deanery of Graduate Science
Faculty of Science
Physics Department

TM Nonlinear Electromagnetic Waves in Semiconductor Superlattices Waveguiding Systems

By
Abdallah Abo-Shabab
B.Sc. In physics (1989)
An-Najah University
Nablus
Palestine

Supervised By

Prof. M.M.Shabat
Professor of Theoretical Physics

Dr. S. S. Yassin
Associate Prof. of Nuclear Physics

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بسم الله الرحمن الرحيم

الله لا اله إلا هو الحي القيوم

لا تأخذه سنه ولا نوم له ما في السماوات وما في الأرض

من ذا الذي يشفع عنده إلا بإذنه يعلم ما بين أيديهم وما خلفهم

ولا يحيطون بشيء من علمه إلا بما شاء وسع كرسيه السماوات

والأرض ولا يئوده حفظهما وهو العلي العظيم

صدق الله العظيم

Dedication

TO

Every one who helps me to achieve this research

& my wife, sons and my parents

Abdallah Abo-Shabab

ACKNOWLEDGEMENTS

It gives me pleasure to thank my supervisors, Professor M. M. Shabat and Dr. Samir S. Yassin for continuous guidance, useful discussions and kind help. I would also like to thank my wife and my friends for their patience and continuous encouragement that kept me stable and productive throughout this work. Thanks due to the staff members of physics department at the Islamic University. Finally, I thank the Islamic University for this chance in science.

Abstract

Considerable attention has been devoted to nonlinear waves propagation in various waveguide structure due to their applications in photonic-microwave devices. In the present work, we have investigated theoretically, by using the transfer matrix technique, the dispersion characteristics of transverse magnetic polarized TM waves propagating in a multilayer semiconductor superlattices waveguides which is surrounded in one side by nonlinear magnetic cover. The two sublattice uniaxial antiferromagnetic crystal is considered as a nonlinear magnetic medium where the permeability is treated as a function of the magnetic field. Numerical results are demonstrated for a waveguiding system containing some number of layers of superlattices. Also we shall show that the effect of quasiperiodic layering structure is led to increase the number of bulk bands and surface modes. In addition, the new surface modes are shown nonreciprocal with respect to propagation direction in the presence of an applied magnetic field. An application of optical devices such as switching, optical thresholding and optical bistability can be achieved throughout this work.

الموجات الكهرومغناطيسية الغير خطيه المنتشرة في أنصاف الموصلات

وجه كثير من الاهتمام نحو الأمواج الكهرومغناطيسية الغير خطية التي تنتشر خلال طبقات الألياف الضوئية، وذلك لتطبيقاتها الكثيرة في الأجهزة التي تعمل بالأمواج القصيرة جداً.

في هذا العمل استخدمنا مصفوفة الانتقال لحساب معادلة التشتت لتلك الأمواج التي تنتشر خلال أنصاف الموصلات المكونة من عدة طبقات ذات كثافة كهربيه ودرجة توصيل مختلفة ، وهذه الطبقات مجتمعة مغطاة بغطاء من طبقة مغناطيسية غير خطية تعتمد فيها النفاذية المغناطيسية على قيمة المجال المغناطيسي المؤثر، وعليه تكون النفاذيه المغناطيسية داله في قيمه المجال المغناطيسي .

وقمنا بحساب القيم العددية لعدد معين من الطبقات والتي تشكل قاعدة البلور، ووجدنا أن زيادة عدد الطبقات يحدث زيادة مقابلة في كفاءة الأمواج السطحية والحجمية أيضاً، وهذه الأمواج لا تعتمد على الاتجاه فهي تنتشر بنفس القوة في جميع الاتجاهات بين الأسطح المغناطيسية.

ونأمل بأننا قدمنا بعض التطبيقات التي تساعد على تصميم العديد من الأجهزة الضوئية ذات المزايا والأغراض المختلفة.

contents

Preface

Optical properties of linear and nonlinear multilayered media have been the subject of considerable theoretical and applied interest. We study the surface polariton modes in linear superlattice microstructures and the analysis of the optical response of nonlinear multiplayer systems such as bilayers and superlattices. These structures play an important role in many applications such as multilayer high reflectance optical coatings, multilayer thin films for the magneto-optical read-out of magnetically stored information and semiconductor superlattice media for optoelectronic devices and optical processing. The plan of this thesis as follows ;

Chapter one a general theory about the present work . Chapter two describes the theory of nonlinear TM waves propagating along the interface between nonlinear semi-infinite magnetic media and linear dielectric media, where we study the solution of Maxwell's equations to find the dispersion equation of the surface . we shall Also investigate the frequency characteristics of magnetic spatial solitons on the surface of two-sublattice uniaxial antiferromagnetic crystal. The study presented in this work suggests that the magnetic surface spatial soliton has frequency passband(s) and stopband(s), which are switchable by the power because the nonlinear permeability for both power and frequency are magnetic field dependent.

The power level required for the spatial soliton excitation is quite high by calculation. If ω is close to resonance frequency ω_C, χ_{NL} becomes frequency dependent and the damping has to be taken into account, then the nonlinear permeabilty becomes complex, thus the different soliton passband(s) or stopband(s) may occur, since the curve of $\mathrm{Re}[\chi_{NL}]$ has positive slope in off- resonance region and has negative slope in resonance region.

Since χ_{NL} is insensitive to frequencies compared to the linear permeability $\mu_L(\omega)$ in the off-resonance range, we have regarded it as a constant throughout this work. Considering that there has been recent progress on artificial enhanced nonlinear nonmagnetic dielectric medium at microwave frequencies, there is every reason to expect the magnetic systems, also with a desired enhanced nonlinearity. The calculation in this work represents a starting point for a new area of work in magnetodynamic wave propagation.

Chapter three will illustrate an exact theory for the TM polarized nonlinear guided waves propagating in a finite periodic multilayered dielectric structure in contact with nonlinear dielectric cover and linear dielectric base. By using the transfer matrix, we shall investigate the stationary field distribution , the nonlinear dispersion curve and the power of the system.

Chapter four describes the dispersion relations and the power of magnetoplasmons propagating in a semi-infinite quasiperiodic superlattice. We will examine a superlattice whose unit cells are composed of two different thicknesses of bilayers which are arranged in a Fibonacci sequence, specifically we have looked at a unit cell composed of three bilayers- two of one thickness separated by a third of a different thickness. We have shown that there exist new additional bulk and surface plasmons in the quasiperiodic structure which do not exist in the periodic case, and that these surface modes show nonreciprocity in frequency with respect to direction of propagation.

In chapter five, we have discussed the nonlinear waves, propagating in semiconductor superlattices covered by a nonlinear cladding. A transfer matrix is used to simplify the algebraic equations. We then derive the dispersion equation of the surface and the bulk modes. The power of the system in a special case is also calculated. Numerical results have shown the effect of the quasiperiodic superlattices is to increase the number of bulk bands and surface modes. In addition dispersion curves for surface and bulk modes are displayed.

Chapter 1

Nonlinear waves in solid state physics

In the magnetostatic limit, a lot of work has been done on magnetic/non-magnetic structures. In Ref. [1,2], Eshbach and Damon obtained the bulk and surface modes in a ferromagnetic/non-magnetic interface and a ferromagnetic slab. After that, there were many theoretical studies on the spin waves in magnetic/non-magnetic superlattices, using different techniques, including transfer matrix formalism and Green function method [3,4]. Some interesting nonlinear properties have been discovered and predicted such as the modulation instability of spin waves [5], the formation of magnetic solitons, three and four magnon decays [7 – 9], and the bistability and multistability of magnetostatic waves in periodical structures [9],…..etc.

Although the spectra of linear dipole spin waves (magnetostatic waves) in two sublattice uniaxial antiferromagnetic slabs have been calculated and predicted for many years, nonlinear wave behavior in antiferromagnetic materials has only recently been studied [10]. The nonlinear susceptibility χ_{NL} was derived for the first time in the study of the nonlinear infrared responses of antiferromagnets [10]. χ_{NL} was used to explore the power dependent transmission of electromagnetic radiation through thin antiferromagnetic films.

In Refs. [8,9], Boardman proved theoretically the existence of temporal envelope solitons in an antiferromagnetic film when the external magnetic field was applied parallel to the film surface , and the calculation indicated the power threshold was about several milliwatts. Wang indicated [6] that in some conditions, in either stable or unstable case, a linearly polarized magnetic beam propagating in a bulk antiferromagnet took the form of spatial soliton, and the necessary condition for the steady propagation of a nonlinear magnetic plane wave is $\chi_{NL} < 0$.

The nonlinear phenomena of electromagnetic waves in antiferromagnets are not only interesting in itself but also important in connection with the behavior of antiferromagnetic devices at infrared frequencies. The antiferromagnetic resonance frequencies and the infrared part of electromagnetic wave spectra. These facts make the use of antiferromagnetic media in different applications very attractive.

Spatial solitons are beams of electromagnetic energy that rely on balancing diffraction and nonlinearity to retain their shapes[7]. Although there are at least hundreds of papers dealing with the optical spatial solitons in dielectric waveguides, there is a little work on spatial solitons in antiferromagnetic materials. In our work we give a report on the frequency characteristics of magnetic spatial solitons on the plane surface of an antiferromagnetic crystal . Physically, any possible spatial soliton behavior of an electromagnetic wave in an antiferromagnet ought to be ,in principle, similar to the optical spatial soliton case . A major difference, of course, is the use of nonlinear permeability, rather than the nonlinear permittivity on optical case, and therein lies a major of difficulty and interest in this new area .The distinguishing feature of the magnetic surface spatial solitons reported in this work is the existence of the frequency passband(s) and stopband(s) that can be switched into each other by varying the power[8] .

The frequency band switching effect of the solitons is actually caused by the fact that the nonlinear permeability is not only power dependent but also frequency dependent in infrared frequency region. In the case of the optical soliton the nonlinear permittivity is always treated as frequency independent since where light frequencies usually have large departures from the resonance frequencies of the dielectric materials. It is noted that for frequencies near the resonance the dispersive and nonlinear response of the material may become substantial, and the damping of the system has to be taken into account. In order to obtain an approximate solution without bringing in many complicated expressions, we neglect the damping of the material. It certainly leads to some approximations but makes the main key point simpler and clearer.

An exact theory for the TM polarized nonlinear waves propagating in a finite periodic multilayered dielectric structure in contact with nonlinear magnetic cladding is presented in Refs.[9 – 13]. By using the transfer matrix technique the stationary field distribution and the nonlinear dispersion curve are obtained exactly in the following chapters.

Chapter 2

TM waves between nonlinear medium and dielectric medium

2.1 **General theory**

The surface waveguide structure to be considered is shown in Fig. 2.1. The nonlinear medium, with the relative dielectric constant ε_1 and nonlinear permeability μ_{NL}, occupies the semi- infinite region $z < 0$ and its plane surface extends to infinite in the zoy plane .The linear medium, with the relative dielectric constant ε_2 and nonlinear permeability μ_2, is the substrate which occupying $z > 0$ region. We begin with the assumption that the crystal is at low temperature and the magnetization of each sublattice can be regarded as saturated. We define the $\pm x$- directions to be the directions of the spontaneous magnetizations of the two sublattices respectively and parallel to the surface crystal. The net magnetizations of the two sublattices are equal to each other.

The nonlinear medium assumed in this chapter is linearly isotropic and the waves propagate along the x- axis with a wave number k and angular frequency ω with field vectors (electric and magnetic fields) :

$$\vec{E} = \left[E_x(z), 0, E_z(z)\right] e^{i(kx-\omega t)} \tag{2.1}$$

and

$$\vec{H} = \{ \text{ o}, H_Y(z), \text{o } \} e^{i(kx-\omega t)} \tag{2.2}$$

The components $E_x(z)$ and $E_z(z)$ are $\frac{\pi}{2}$ out of phase so, the transformation

$$H_y \;\rightarrow\; H_y,$$

$$E_x \;\rightarrow\; iE_x,$$

and

$$E_z \;\rightarrow\; E_z$$

can be made and we reduce Maxwell's equations to :

$$\frac{\partial}{\partial z}E_x - k_x E_x = \omega\mu_0 H_y\,, \tag{2.3}$$

$$\frac{\partial}{\partial z}H_y = -\omega\varepsilon_0\varepsilon_{xx}E_x\,, \tag{2.4}$$

and

$$k_x H_y = -\omega\varepsilon_0\varepsilon_{zz}E_z \tag{2.5}$$

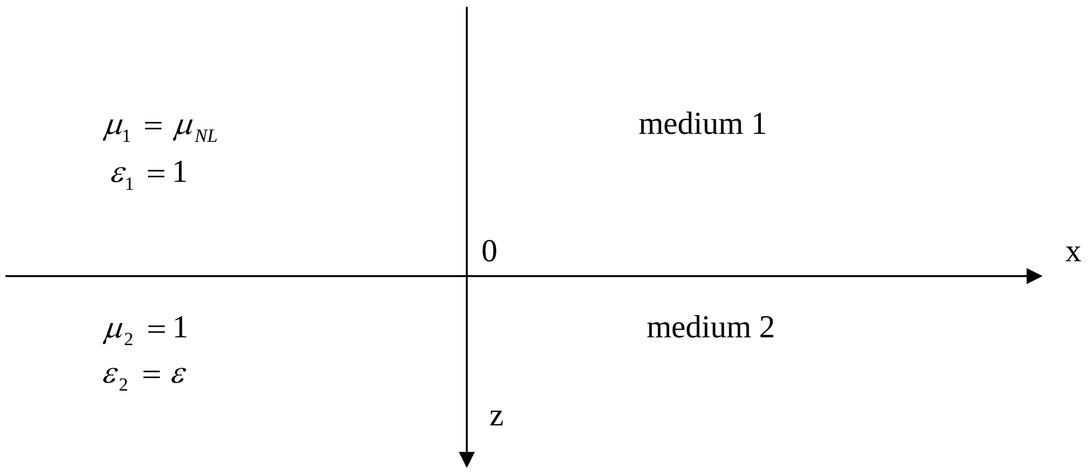

Fig. 2.1. Nonlinear cladding in contact with dielectric substrate

In the case of nonlinear medium the permeability can be written as Ref. [13],

$$\mu_{NL} = \mu_L + \alpha H_y^2 \tag{2.6}$$

where α is the nonlinear factor, and μ_L is the linear part of the permeability.

Accordingly, Maxwell's equations takes the form:

$$\nabla\times\vec{E} = i\mu_0\omega\mu_{NL}\vec{H} \tag{2.7}$$

and

$$\nabla\times\vec{H} = -i\omega\mu\varepsilon_0\varepsilon_1\vec{E} \tag{2.8}$$

By using equations: (2.1) and (2.2) in equation (2.7) and (2.8) we have:

$$\nabla \times \vec{E} = (\frac{\partial}{\partial y} E_z(z) i - (\frac{\partial}{\partial x} E_z(z) - \frac{\partial}{\partial z} E_x(z)) j - \frac{\partial}{\partial y} E_x(z) k \,) e^{i(kx-\omega t)}$$

$$= \left[-ikE_z(z) + \frac{\partial}{\partial z} E_x(z) \right] \; e^{i(kx-\omega t)}$$

$$= i\omega\mu_0\mu_{NL}\vec{H}$$

where

$$\vec{H} = H_y(z) \, e^{i(kx-\omega t)}$$

Then we get:

$$ikE_z(z) - \frac{\partial}{\partial z} E_x(z) = -i\omega\mu_0\mu_{NL} H_y(z) \tag{2.9}$$

By the same way $\nabla \times \vec{H}$ give:

$$\frac{\partial}{\partial z} H_y(z) = i\omega\varepsilon_0\varepsilon_1 E_x(z) \tag{2.10}$$

and

$$kH_y(z) = -\omega\varepsilon_0\varepsilon_1 E_z(z) \tag{1.11}$$

Eliminating E_x and E_z from the equations (2.9), (2.10) and (2.11) we have :

$$ik \frac{k}{-\omega\varepsilon_0\varepsilon_1} H_y(z) - \frac{1}{i\omega\varepsilon_0\varepsilon_1} \frac{\partial^2}{\partial z^2} H_y(z) = -i\omega\mu_0\mu_{NL} H_y(z)$$

The above equation can be rearranged to give :

$$\frac{\partial^2}{\partial z^2} H(z) - k^2 H_y(z) + \omega^2 \mu_0\varepsilon_0\varepsilon_1\mu_{NL} H_y(z) = 0 \tag{2.12}$$

which can be written as

$$\frac{\partial^2}{\partial z^2} H_y(z) - k^2 H_y(z) + k_0^2 \varepsilon_1 \mu_{NL} H_y(z) = 0 \tag{2.13}$$

Substituting the equation (2.6) into the equation (2.13), yields :

$$\frac{\partial^2}{\partial z^2} H_y(z) - k^2 H_y(z) + k_0^2 \varepsilon_1 (\mu_L + \alpha H_y^2(z)) H_y(z) = 0 \tag{2.14}$$

where

$$k_0^2 = \frac{\omega^2}{c^2} \tag{2.15a}$$

and

$$k_1^2 = k^2 - k_0^2 \varepsilon_1 \mu_L \tag{2.15b}$$

The equation (2.14) can be rewritten as:

$$\frac{\partial^2}{\partial z^2} H_y(z) - (k_1^2 - k_0^2 \varepsilon_1 \alpha H_y^2(z)) H_y(z) = 0 \tag{2.16}$$

Multiplying the equation (2.16) by $2\frac{\partial}{\partial z} H_y(z)$ and then integrating over z, we have :

$$2\frac{\partial}{\partial z} H_y(z). \frac{\partial^2}{\partial z^2} H_y(z) - 2\frac{\partial}{\partial z} H_y(z) k_1^2 H_y(z) + 2\frac{\partial}{\partial z} H_y(z) k_0^2 \varepsilon_1 \alpha H_y^3(z) = 0$$

the first integration of this equation is :

$$\left[\frac{\partial}{\partial z} H_y(z)\right]^2 - (k_1^2 - \frac{1}{2} k_0^2 \varepsilon_1 \alpha H_y^2(z)) H_y^2(z) = \ const. \tag{2.17}$$

which can be written as:

$$\frac{\partial}{\partial z} H_y(z) = \sqrt{const. + (k_1^2 - \frac{1}{2} k_0^2 \varepsilon_1 \alpha H_y^2(z)) H_y^2(z)} \tag{2.18}$$

So the integration of this equation is :

$$H_y(z) = \frac{1}{k_0}.\sqrt{\frac{2}{\varepsilon_1 \alpha}}.\frac{k_1}{\cosh(k_1(z - z_0))} \tag{2.19}$$

To find the electric field components we use the equations (2.4) and (2.5) giving :

$$E_{1x}(z) = \frac{k_1^2}{\varepsilon_1 k_0 \omega \varepsilon_0} \sqrt{\frac{2}{\alpha \varepsilon_1}} \frac{\sinh(k_1(z - z_0))}{\cosh^2(k_1(z - z_0))} \tag{2.20}$$

and

$$E_{1z}(z) = -\frac{k_1^2}{\varepsilon_1 k_0 \omega \varepsilon_0} \sqrt{\frac{2}{\alpha \varepsilon_1}} \frac{1}{\cosh(k_1(z - z_0))} \tag{2.21}$$

and from the equations (2.20) and (2.21) we have:

$$E_{1z}(z) = \frac{1}{\tanh(k_1(z - z_0))} E_{1x}(z) \tag{2.22}$$

In the dielectric medium we suggest that the permeability $\mu_2 = 1$ and the relative dielectric function ε_2 is given in the tensor form as:

$$\varepsilon_2 = \begin{bmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_{yy} & 0 \\ -\varepsilon_{xz} & 0 & \varepsilon_{zz} \end{bmatrix} \tag{2.23}$$

where

$$\varepsilon_{xx} = \varepsilon_{zz} = \varepsilon_\infty \left[1 - \frac{\omega_C \omega_P^2}{\omega(\omega^2 - \omega_C^2)} \right], \tag{2.24}$$

$$\varepsilon_{xz} = -\varepsilon_{zx} = i\varepsilon_\infty \frac{\omega_C \omega_P^2}{\omega(\omega^2 - \omega_C^2)}, \tag{2.25}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{zy} = \varepsilon_{yz} = 0, \tag{2.26}$$

$$\varepsilon_{yy} = \varepsilon_\infty \left[1 - \frac{\omega_P^2}{\omega^2} \right] \tag{2.27}$$

and

$$\omega_p^2 = \frac{n^2 e}{\varepsilon_\infty m}$$

where ω_p is the plasma frequency.

After we eliminate the magnetic field from Maxwell's curl equations we get:

$$\nabla^2 \vec{E} - \nabla(\nabla \cdot \vec{E}) = \varepsilon_0 \mu_0 \varepsilon \frac{\partial^2}{\partial t^2} \vec{E} \tag{2.28}$$

The divergence equation

$$\nabla \cdot \vec{D} = 0 \tag{2.29}$$

where

$$\vec{D} = \varepsilon_0 \varepsilon . \vec{E} \tag{2.30}$$

is the displacement vector, does not mean that $\nabla \cdot \vec{E} = 0$ hence the second term on the left hand side of equation (2.28) must be remained.

For TM waves propagating in the $x-$direction with wave number k and frequency ω the solution of the equation (2.28) is :

$$\vec{E} = (E_{2x}, 0, E_{2z})\, e^{i(kx - \omega t) + \alpha_2 z} \tag{2.31}$$

where α_2 is the decay constant and E_{2x}, E_{2z} are the components of the electric field

Substituting the equation (2.31) into the equation (2.28) we get :

$$\nabla^2 \vec{E} = ((ik)^2 E_{2x} i + 0\, j + (\alpha_2)^2 E_{2z} k)\ e^{i(kx-\omega t)+\alpha_2 z}, \tag{2.32}$$

$$\nabla \cdot \vec{E} = \left[ikE_{2x} + \alpha_2 E_{2z}\right]\ e^{i(kx-\omega t)+\alpha_2 z} \tag{2.33}$$

and

$$\nabla(\nabla \cdot \vec{E}) = \left\{\left[(ik)^2 E_{2x} + (ik)\alpha_2 E_{2z}\right] i + 0\, j + \left[(ik)\alpha_2 E_{2x} + \alpha_2^2 E_{2z}\right] k\right\} e^{i(kx-\omega t)+\alpha_2 z} \tag{2.34}$$

in the right hand side of the equation (2.28) we have:

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_{yy} & 0 \\ -\varepsilon_{xz} & 0 & \varepsilon_{zz} \end{bmatrix} \tag{2.35}$$

and

$$\frac{\partial^2}{\partial t^2} \vec{E} = (-i\omega)^2 \left[E_{2x}, 0, E_{2z}\right]\ e^{i(kx-\omega t)+\alpha_2 z} \tag{2.36}$$

from the equations (2.28), (2.32), (2.34) and (2.36) we have :

$$(\alpha_2^2 + k_0^2 \varepsilon_{xx}) E_{2x} - (ik\alpha_2 - k_0^2 \varepsilon_{xz}) E_{2z} = 0 \tag{2.37}$$

and

$$(ik\alpha_2 - k_0^2 \varepsilon_{xz}) E_{2x} + (k^2 - k_0^2 \varepsilon_{xx}) E_{2z} = 0 \tag{2.38}$$

if we eliminate E_{2x} and E_{2z} from the equations (2.37) and (2.38) we have :

$$E_{2z} = \frac{\alpha_2^2 + k_0^2 \varepsilon_{xx}}{ik\alpha_2 - k_0^2 \varepsilon_{xz}} E_{2x} \tag{2.39}$$

and

$$E_{2z} = -\frac{ik\alpha_2 - k_0^2 \varepsilon_{zx}}{k^2 + k_0^2 \varepsilon_{xx}} E_{2x} \tag{2.40}$$

which means that:

$$\frac{\alpha_2^2 + k_0^2 \varepsilon_{xx}}{ik\alpha_2 - k_0^2 \varepsilon_{xz}} = \frac{k_0^2 \varepsilon_{zx} - ik\alpha_2}{k^2 + k_0^2 \varepsilon_{xx}} \tag{2.41}$$

from the equation (2.41) we have:

$$\alpha_2^2 = k^2 - k_0^2 \varepsilon_v \tag{2.42}$$

and

$$\varepsilon_v = \varepsilon_{xx} + \frac{\varepsilon_{xz}}{\varepsilon_{xx}} \tag{2.43}$$

2.2 **The dispersion equation**

To find the dispersion equation we use the continuity of a tangent componental $\vec{E}$ and a normal componental $\vec{D}$ as the boundary conditions between the two mediums as the following:

$$E_{1x}(z) = E_{2x}(z) \tag{2.44}$$

and

$$D_{1z}(z) = D_{2z}(z) \tag{2.45}$$

From the equation (2.22) at $z = 0$ we have:

$$E_{1z}(z) = \frac{-1}{\tanh(k_1(z_0))} E_{1x}(z)\Big|_{z=0} \tag{2.46}$$

and $D_{1Z}(z)$ can be found as the following:

$$\vec{D}_1 = \varepsilon_0 \varepsilon_1 \vec{E}_1$$

in medium 1 the equation (2.21) gives:

$$\begin{aligned} D_{1z}(z) &= \varepsilon_0 E_{1z}(z) \\ &= -\frac{k_1^2}{\varepsilon_1 \omega k_0} \sqrt{\frac{2}{\varepsilon_1 \alpha}} \frac{1}{\cosh(k_1(z - z_0))} \end{aligned} \tag{2.47}$$

D_{2z} can be found as the following:

$$D_{2Z} = \varepsilon_0 \varepsilon_2 E_{2Z}(z) \tag{2.48}$$

$$\varepsilon_2 = \begin{bmatrix} \varepsilon_1 & 0 & i\varepsilon_2 \\ 0 & \varepsilon_3 & 0 \\ -i\varepsilon_2 & 0 & \varepsilon_1 \end{bmatrix} \tag{2.49}$$

where:

$$\varepsilon_1 = \varepsilon_\infty \left[1 + \frac{\omega_P^2}{\omega_C^2 - \omega^2} \right], \tag{2.50}$$

$$\varepsilon_2 = \varepsilon_\infty \omega_C \frac{\omega_P^2}{\omega(\omega_C^2 - \omega^2)}, \tag{2.51}$$

and

$$\varepsilon_3 = \varepsilon_\infty \left[1 - \frac{\omega_P^2}{\omega^2}\right] \tag{2.52}$$

for simplicity :

$$\vec{E}_2(z) = (E_{2x}(z), 0, E_{2z}(z)) \tag{2.53}$$

and

$$D_{2z}(z) = \varepsilon_0 \varepsilon_1 E_2(z) = \varepsilon_0 \begin{bmatrix} \varepsilon_{xx} & 0 & \varepsilon_{xz} \\ 0 & \varepsilon_{yy} & 0 \\ -\varepsilon_{xz} & 0 & \varepsilon_{xx} \end{bmatrix} \cdot \begin{bmatrix} E_{2x}(z) \\ 0 \\ E_{2z}(z) \end{bmatrix} \tag{2.54}$$

then

$$D_{2z}(z) = \{-\varepsilon_{xz} E_{2x}(z) + \varepsilon_{xx} E_{2z}(z)\} \tag{2.55}$$

at the point $z = 0$ we have:

$$E_{1x}(z)\big|_{Z=0} = E_{2x}(z)\big|_{Z=0} \tag{2.56}$$

from the equations (2.20) and (2.53) at the point $z = 0$ we have :

$$E_{2x}(z) = \frac{-k_1^2}{\varepsilon_1 k_0 \varepsilon_0 \omega} \sqrt{\frac{2}{\alpha \varepsilon_1}} \frac{1}{\cosh(k_1(z - z_0))} \cdot e^{\alpha_2 z} \tag{2.57}$$

and from the equations (2.39) , (2.40) , (2.47) and (2.55) we have:

$$\varepsilon_0(-\varepsilon_{xz} E_{2x} + \varepsilon_{zz} E_{2z}) = E_{1z} \tag{2.58}$$

which gives the dispersion relation :

$$-\varepsilon_{xz} + \varepsilon_{xx} \left[\frac{\alpha_2^2 + k_0^2 \varepsilon_{xx}}{ik\alpha_2 - k_0^2 \varepsilon_{xz}}\right] = \frac{-\varepsilon_1}{\tanh(k_1 z_0)} \tag{2.59}$$

2.3 The power flow

The total power is the sum of the power in two media

***** In the nonlinear medium :

$$P = \frac{1}{2} \int (\vec{E} \times \vec{H}^*)_x \, dz \tag{2.60}$$

where

$$\vec{E} = (E_x(z), 0, E_z(z)) \; e^{i(kx - \omega t)}$$

and

$$\vec{H} = (\text{o}, H_y(\text{z}), \text{o}) \; e^{i(kx - \omega t)}$$

where

$$H_y(z) = \frac{1}{k_0}\sqrt{\frac{2}{\alpha\varepsilon_1}}\frac{k_1}{\cosh(k_1(z-z_0))} \tag{2.61}$$

From the solution of Maxwell's equations we can find E_x, E_z as a function of H_y

The right hand side of the equation (2.60) can be rewritten as:

$$\frac{1}{2}\int(\vec{E}\times\vec{H}^*)_x dz = -\frac{1}{2}\int E_z H_y^* dz \tag{2.62}$$

we want to find just E_z from equation (2.11) as the following:

$$E_z = \frac{-k}{\omega\varepsilon_1\varepsilon_0}H_y \tag{2.63}$$

So that the power becomes:

$$P_1 = \frac{k}{2\omega\varepsilon_1\varepsilon_0}\int H_y^2 dz \tag{2.64}$$

Using Eq. (2.61) we get:

$$P_1 = \frac{kk_1^2}{\varepsilon_0\varepsilon_1^2\alpha k_0^2\omega}\int_0^\infty \frac{1}{\cosh^2(k_1(z-z_0))}dz \tag{2.65}$$

if we let $z = z' + z_0$ then the power becomes:

$$P_1 = \frac{kk_1^2}{\varepsilon_0\varepsilon_1^2\alpha k_0^2\omega}\int_{-Z_0}^{\infty}\frac{1}{\cosh^2(k_1 z')}dz' \tag{2.66}$$

where the quantity z_0 is the position of maximum power density and when it moves to infinity , the power flow reaches the following equation:

$$P_1 = \frac{kk_1^2}{\varepsilon_0\varepsilon_1^2\alpha k_0^2\omega}\int_0^{\infty}\frac{1}{\cosh^2(k_1 z')} = \frac{kk_1}{\varepsilon_0\varepsilon_1^2\alpha k_0^2\omega} \tag{2.67}$$

****** In the dielectric medium:

$$p = \frac{1}{2}\int(\vec{E}\times\vec{H}^*)_x dz \quad = -\frac{1}{2}\int E_{2z}H_{2y}^* dz \tag{2.68}$$

From the equation (2.57) we have :

$$E_{2x} = \frac{k_1^2}{\varepsilon_1\ \varepsilon_0 k_0\omega}\sqrt{\frac{2}{\varepsilon_1\alpha}}\frac{1}{\cosh(k_1 z_0)} \tag{2.69}$$

Then, we want to find E_{2z}, H_{2z} from the solution of Maxwell's equations in the dielectric medium ε_2 as the following :

$$\frac{\partial}{\partial z}E_{2x} - k_x E_{2z} = \omega\mu_0 H_{2y}, \tag{2.70}$$

$$\frac{\partial}{\partial z}H_{2y} = -\omega\varepsilon_0\varepsilon_{xx}E_{2x}, \tag{2.71}$$

and

$$k_x H_{2y} = -\omega\varepsilon_0\varepsilon_{zz}E_{2z} \tag{2.72a}$$

From the equation (2.71) we have:

$$H_{2y} = -\omega\varepsilon_0\varepsilon_{xx}\int E_{2x}dz \tag{2.72b}$$

where E_{2x} was given in the equation (2.69), then we write:

$$H_{2y} = -\omega\varepsilon_0\varepsilon_{xx}\int \frac{k_1^2}{\varepsilon_1 k_0\omega\varepsilon_0}\sqrt{\frac{2}{\alpha\varepsilon_1}}\frac{1}{\cosh(k_1 z_0)}e^{\alpha_2 z}dz \tag{2.72c}$$

By integration, we have:

$$H_{2y} = -\frac{\varepsilon_{xx}k_1^2}{\varepsilon_1\alpha_2 k_0}\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{1}{\cosh(k_1 z_0)}e^{\alpha_2 z} \tag{2.73}$$

the equation (2.72) gives:

$$E_{2z} = -\frac{k_x}{\omega\varepsilon_0\varepsilon_{xx}}H_{2y} \tag{2.74a}$$

which equivalent to:

$$E_{2z} = \frac{k_X k_1^2}{\varepsilon_1\omega\varepsilon_0\alpha_2 k_0^2\alpha}\sqrt{\frac{2}{\varepsilon_1\alpha}}\frac{1}{\cosh(k_1 z_0)}e^{\alpha_2 z} \tag{2.74b}$$

using the equations (2.73) and (2.74b) in the equation (2.68) we get:

$$p_2 = \frac{1}{2}\int \frac{k_x}{\omega\varepsilon_0\varepsilon_{xx}} H_{2y}^2 dz \qquad (2.75)$$

which becomes:

$$P_2 = \frac{1}{2}\cdot\frac{k_x}{\omega\varepsilon_0\varepsilon_{xx}}\int\left[\frac{k_1^2\sqrt{\frac{2}{\alpha}\varepsilon_{xx}}}{\alpha_2 k_0^2\cosh(k_1 z_0)}\right]^2 e^{2\alpha_2 z}dz$$

this integral gives:

$$P_2 = \frac{1}{2}\cdot\frac{k_x\varepsilon_{xx}k_1^4\left[\frac{2}{\alpha\varepsilon_1}\right]}{\varepsilon_1^2\omega\varepsilon_0\alpha_2^3 k_0^2\cosh^2(k_1 z_0)}\cdot\frac{1}{2}\, e^{2\alpha_2 z}\Bigg|_{0}^{-\infty} \qquad (2.76)$$

which equivalent to:

$$P_2 = \frac{1}{2}\cdot\frac{k_x\varepsilon_{xx}k_1^4\left[\frac{2}{\alpha\varepsilon_1}\right]}{\varepsilon_1^2\omega\varepsilon_0\alpha_2^3 k_0^4\cosh^2(k_1 z_0)} \qquad (2.77)$$

The total power now is :

$$P_{total} = P_1 + P_2 \qquad (2.78)$$

The numerical results of the equation (2.78) are ulstrated in the Fig. 2.2. where $\mu_{NL} = 1.29, \varepsilon_1 = 1, \mu_L = 1,\ \varepsilon_\infty = 13.13, \omega_P = 0.04075eV$, $n = 10^{18}cm^{-3}$, $\omega_C = 0.004075eV$ and $\alpha = 8.869\times10^8 m^2A^{-2}$ are taken into account. Note that we have three curves corresponding to three different values of α such that the decrease value of α the upper curve become.

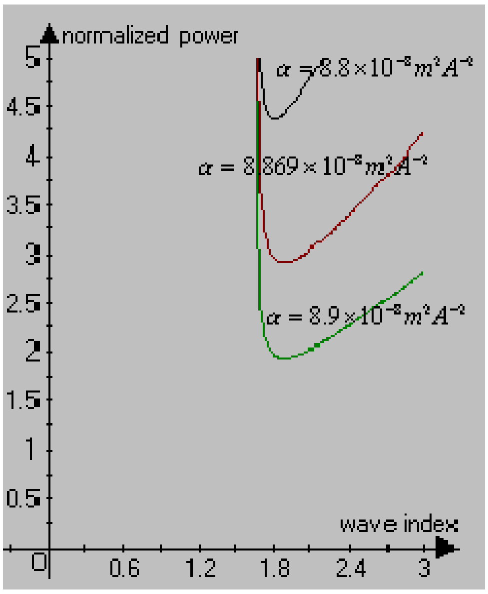

Fig. 2.2 Power flow versus wave index for TM surface/guided waves at a single interface between a nonlinear medium and a dielectric medium

2.4- **Frequency characteristics of the magnetic spatial solitons on the surface of an antiferromagnet**

In the absence of an applied Zeeman field, the permeability tensor describing the nonlinear response of the crystal to the intense field is a diagonal one. For a TM wave special soliton investigated in this work, the permeability is given in a tensor form [6] as:

$$\mu_{YY} = \mu_{NL}(\omega) = \mu_L(\omega) + \chi_{NL}(\omega)\left|\vec{H}\right|^2 \tag{2.79}$$

where :

$$\mu_L = 1 + \chi(\omega) = 1 + \frac{2\omega_\mu \omega_A}{\omega_C^2 - \omega^2} \tag{2.80}$$

is linear permeability, $\omega_M = \gamma\mu_0 M_0$, $\omega_A = \gamma\mu_0 H_0$, $\omega_E = \gamma\omega\mu_0 H_E$ and $\omega_C^2 = \omega_A^2 + 2\omega_A\omega_E$ which is the resonance frequency of the system, and $M_0, \vec{H}_A, and, \vec{H}_E$ are saturation magnetization , anisotropy and exchange fields of the crystal respectively.

The dispersion relation can be written in general form as [10]

$$\tanh(k_1 z_0) = \frac{k_2\varepsilon_1}{k_1\varepsilon_2} \tag{2.81}$$

where

$$k_1 = \sqrt{k^2 - k_0^2\varepsilon_1\mu_L}\ , \tag{2.82}$$

$$k_2 = \sqrt{k^2 - k_0^2\varepsilon_v}\ , \tag{2.83}$$

and

$$\varepsilon_v = \varepsilon_{xx} + \frac{\varepsilon_{xz}^2}{\varepsilon_{xx}} \tag{2.84}$$

where $k_0 = \frac{\omega}{c}$, k is the pointing vector , and $\varepsilon_1, \varepsilon_v$ represents the dielectric constants of the two mediums.

From equation (2.81) we get:

$$0 \le \frac{k_2\varepsilon_1}{k_1\varepsilon_2} \le 1 \tag{2.85}$$

where

$$0 \le \tanh(x) \le 1 \tag{2.86}$$

squaring each side of this inequality using the equations (2.82),(2.83) and (2.85) we get:

$$k_1^2 = k^2 - k_0^2\varepsilon_1\mu_L\ , \tag{2.87}$$

$$k_2^2 = k^2 - k_0^2\varepsilon_v\ , \tag{2.88}$$

$$\frac{k_2^2\varepsilon_1^2}{k_1^2\varepsilon_2^2} < 1 \; , \tag{2.89}$$

$$\frac{\varepsilon_1^2(k^2 - k_0^2\varepsilon_v)}{\varepsilon_2^2(k^2 - k_0^2\varepsilon_1\mu_L)} < 1 \, , \tag{2.90}$$

and

$$k^2 < \frac{k_0^2\varepsilon_V\varepsilon_1^2 - k_0^2\varepsilon_2^2\mu_L}{\varepsilon_1^2 - \varepsilon_2^2} \tag{2.91}$$

which means that $k \to \infty$ when $\varepsilon_1 \to \varepsilon_2$ actually as $z_0 \to \infty$ when $\tanh(k_1 z_0) \to 1$ so that the cut-off corresponding to a self-focused peak in the field moving out to infinity.

The requirement that $k_1^2 > 0$ and $k_2^2 > 0$ leads to:

$$k^2 > k_0^2\varepsilon_v \tag{2.92}$$

which with Eq.(2.91) gives the range of frequency.

Another inequality can be found by eliminating z_0 from the dispersion equation and the boundary conditions which can be written as:

$$\varepsilon_2^2 k_1^2 - \varepsilon_1^2 k_2^2 = \frac{1}{2}\varepsilon_2^2 k_0^2 \chi_{NL}\varepsilon_1 H_{2y}^2 \tag{2.93}$$

If we eliminate k_1^2 and k_2^2 from the equation (2.93) using the equations (2.87) and (2.88), and solving for k^2 we get:

$$k^2 = \frac{k_0^2\varepsilon_2\varepsilon_1}{\varepsilon_1^2 - \varepsilon_2^2}\left\{\frac{\varepsilon_1}{\varepsilon_2}\varepsilon_v - \varepsilon_2\mu_L - \frac{1}{2}\varepsilon_2\chi_{NL}H_{2y}^2\right\} \tag{2.94}$$

which is the dispersion equation of the system

Since $\frac{k^2}{k_0^2} > 0$, then equation (2.94) in the case of $\varepsilon_2 < \varepsilon_1$ gives:

$$\varepsilon_1\mu_2 - \varepsilon_2\mu_L - \frac{1}{2}\varepsilon_2\chi_{NL}H_2^2 < 0 \tag{2.95a}$$

or

$$\varepsilon_2\mu_2 - \varepsilon_2\mu_L - \frac{1}{2}\varepsilon_2\chi_{NL}H_2^2 < 0 \tag{2.95b}$$

in the case of $\varepsilon_2 > \varepsilon_1$

Since $k_1^2 > 0$ and $k_2 > 0$ for a surface wave we have :

$$k^2 > k_0^2 \varepsilon_1 \mu_L \tag{2.96a}$$
$$k^2 > k_0^2 \varepsilon_2 \mu_L \tag{2.96b}$$

2.5- **Results Analysis**

The frequency characteristics have two cases

*** The case of** $\varepsilon_1 < \varepsilon_2$**:**

The equations (2.94) and (2.96a,b) give:

$$\mu_L > \frac{\varepsilon_2}{\varepsilon_1}\mu_2 - \frac{1}{2}\chi_{NL} H_2^2 \tag{2.97}$$

the variation range of μ_L is shown in figure (2.3), in this case μ_L can be either positive or negative, and the wave frequency can be either smaller or larger than the resonance frequency ω_c. From the equation (2.97) we have two forms:

I- If we assume that $\mu_2 = 1$ and $\frac{1}{2}\chi_{NL} H_2 < \frac{\varepsilon_2}{\varepsilon_1} - 1$, Then the equation (2.97) gives:

$$\omega' = \sqrt{\frac{\omega_C - 2\omega_A \omega_M}{\frac{\varepsilon_2}{\varepsilon_1} - 1 - \frac{1}{2}\chi_{NL} H_2^2}}\, \omega < \omega_C \tag{2.98}$$

Then we have surface special soliton has a frequency passband and the bandwidth of ω^2 is:

$$(\Delta\omega^2)_P = \frac{2\omega_M \omega_A}{\frac{\varepsilon_2}{\varepsilon_1} - 1 - \frac{1}{2}\chi_{NL} H_2^2} \tag{2.99}$$

which comes directly from the equation (2.98).

The variation of ω' with $\frac{1}{2}\chi_{NL} H_2^2$ are shown in figure (2.4). The increase of $\frac{1}{2}\chi_{NL} H_2^2$ reduces the lower frequency limit ω' and widens the passband.The maximum passband width is ω_C as $\frac{1}{2}\chi_{NL} H_2^2$ approaches $\left[\frac{\varepsilon_1}{\varepsilon_2} - 1 - \frac{2\omega_M \omega_A}{\omega_C^2}\right]$, at which the lower stopband vanishes .

In the limit of $\frac{1}{2}\chi_{NL}H_2^2 \to 0$, $(\Delta\omega^2)_P = 2\omega_M\omega_A \frac{\varepsilon_1}{\varepsilon_2 - 1}$ which is the narrowest passband related to $\frac{\varepsilon_2}{\varepsilon_1}$. Asmaller difference between ε_1 and ε_2 leads to a wider passband. Increasing $\frac{\varepsilon_2}{\varepsilon_1}$ will widen the variation range of $\chi_{NL}H_2^2$. Curve 1 varies with $\chi_{NL}H_2^2$ more sharply than curve 2 and 3 do. This indicates that the spatial soliton propagating in the structure with a smaller value of $\frac{\varepsilon_2}{\varepsilon_1}$ depends more strongly on the power. When $\omega_M = 10^{10} rad/s, \omega_A = 3.5\times10^{12} rad/s$. $\omega_C = 8.895\times10^{12} rad/s$, $\frac{1}{2}\chi_{NL}H_2^2 = 0.5$ and $\frac{\varepsilon_2}{\varepsilon_1} = 2$, then the minimum passband is $\sqrt{(\Delta\omega^2)_P} = 3.7\times10^{11} rad/s$.

II- when $\frac{1}{2}\chi_{NL}H_2^2 > \frac{\varepsilon_2}{\varepsilon_1} - 1$ then the equation (2.97) gives:

$$\omega < \omega_C \tag{2.100}$$

or:

$$\omega > \sqrt{\frac{\omega_C^2 + 2\omega_M\omega_A}{\frac{1}{2}\chi_{NL}H_2^2 - \frac{\varepsilon_2}{\varepsilon_1} + 1}} = \omega'' \tag{2.101}$$

So the surface spatial soliton has a low frequency passband and a high-frequency passband . The low limit ω'' of the high-frequency passband varies with the power and the parameters of the materials, while the low-frequency passband is power independent. Between the two passbands there is a stopband with a width of ω^2 being:

$$(\Delta\omega^2)_S = \frac{2\omega_M\omega_A}{\frac{1}{2}\chi_{NL}H_2^2 - \frac{\varepsilon_2}{\varepsilon_1} + 1} \tag{2.102}$$

If $\frac{1}{2}\chi_{NL}H_2^2 = 0.6$, $\frac{\varepsilon_2}{\varepsilon_1} = 1.5$ we have $\sqrt{(\Delta\omega^2)_S} = 8.36\times10^{11} rad/s$, the increase of $\chi_{NL}H_2^2$ leads to a decrease of ω'' and a narrowing of the stopband .As $\chi_{NL}H_2^2$ approaches infinite the stopband vanishes. On the contrary, as $\chi_{NL}H_2^2$ becomes smaller then ω'' becomes larger and finally stops the guided wave with frequency ω which was originally in the high-frequency passband . The variation of ω'' with $\chi_{NL}H_2^2$ is shown in figure (2.5). It is now quite clear that for a fixed wave frequency the propagation and the cut-off states of the soliton can be swiched by varying the power.

Figure (2.6) brings the curves of the two cases together for the ratio $\frac{\varepsilon_2}{\varepsilon_1}=1.5$. As the power increases, $\frac{1}{2}\chi_{NL}H_2^2$ reduces and destroys the lower stopband first, then shrinks the upper stopband until the band vanishes finally. When $\frac{1}{2}\chi_{NL}H_2^2$ value is below $\frac{\varepsilon_2}{\varepsilon_1}-1$, no $\omega>\omega_C$ spatial soliton can exist.

** **The case of** $\varepsilon_1>\varepsilon_2$**:**

From the equations (2.94) and (2.96a,b) we find the necessary condition for the magnetic spatial soliton propagation in the case of $\varepsilon_1>\varepsilon_2>0$:

$$\mu_L<\frac{\varepsilon_2}{\varepsilon_1}\mu_2-\frac{1}{2}\chi_{NL}H_2^2 \tag{2.103}$$

The variation range of μ_L determined by the equation (2.103) is displayed in figure (2.7) . Since $\varepsilon_1>\varepsilon_2$, μ_L is always smaller than one when $\mu_2=1$,so the soliton frequency must be larger than the resonance frequency of the antiferromagnetic medium.

When $\mu_L=1$ the equation (2.103) gives :

$$\omega_C<\omega<\sqrt{\frac{\omega_C^2+2\omega_M\omega_A}{\frac{1}{2}\chi_{NL}H_2^2-\frac{\varepsilon_2}{\varepsilon_1}+1}}=\omega''' \tag{2.104}$$

Obviously there is a passband with the width of ω^2 being:

$$(\Delta\omega^2)_P=\frac{2\omega_M\omega_A}{\frac{1}{2}\chi_{NL}H_2^2-\frac{\varepsilon_2}{\varepsilon_1}+1} \tag{2.105}$$

The variation of the upper frequency limit ω'' with $\chi_{NL}H_2^2$ is shown in Fig. 2.8. It is seen that the increase of $\chi_{NL}H_2^2$ leads to a narrowing of the passband, which vanish as $\chi_{NL}H_2^2$ goes to infinite. Therefore a guided spatial soliton with frequency ω ,which is originally in the passband, will stop as $\chi_{NL}H_2^2$ approaches a critical value $2\times\left[\frac{2\omega_M\omega_A}{\omega^2-\omega_C^2}+\frac{\varepsilon_2}{\varepsilon_1}-1\right]$ and beyond. In the limit of $\chi_{NL}H_2^2 \to 0, (\Delta\omega^2)_P = 2\times\omega_M\omega_A\left[\frac{\varepsilon_1}{\varepsilon_1-\varepsilon_2}\right]$, which is the widest passband related to $\frac{\varepsilon_2}{\varepsilon_1}$. A smaller $\frac{\varepsilon_2}{\varepsilon_1}$ results in a steeper variation of the passband at small $\chi_{NL}H_2^2$, $if\ \frac{1}{2}\chi_{NL}H_2^2 = 0.5$, the passbands $\sqrt{(\Delta\omega^2)_P}$ for $\frac{\varepsilon_2}{\varepsilon_1} = 0.75, 0.5,$ and 0.25 are 3×10^{11} ,2.6×10^{11} ,and 2.36×10^{11} rad/s ,respectively.

The total power flux is $[10]$**:**

$$P = P_L + P_{NL} = \frac{1}{2}\int(E_1\times H_1^*)_z\,dx + \frac{1}{2}\int(E_2\times H_2^*)_z\,dx \tag{2.106}$$

$$P = \frac{kk_1}{\varepsilon_0\varepsilon_1^2\omega\chi_{NL}k_0}\left(1+\frac{k_2\varepsilon_0}{k_1\varepsilon_0}\right) + \frac{kk_1^2\left[1-\sqrt{\frac{k_2\varepsilon_1}{k_1\varepsilon_2}}\right]}{2\varepsilon_0\varepsilon_1\varepsilon_2\omega\chi_{NL}k_0^2k_2^2} \tag{2.107}$$

The power flux in figure (2.9) is normalized with $P_0 = \frac{1}{2}\chi_{NL}\omega\varepsilon_0 \approx 0.43mw/m$. It is quite high but since it is inverse of $\chi_{NL}\omega$ working at higher frequencies requires a smaller χ_{NL} to offset this fact.

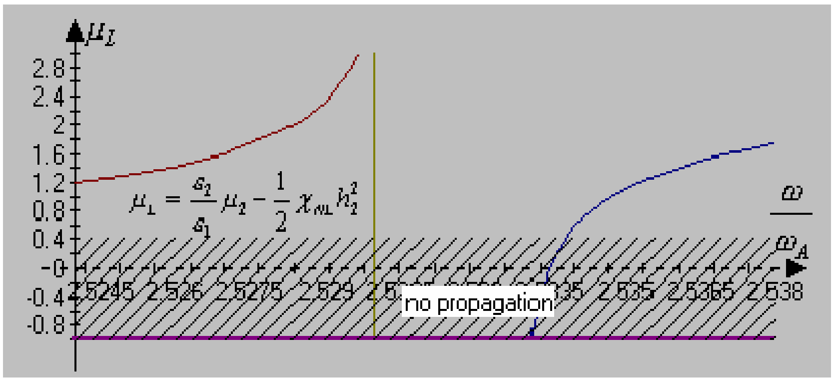

Fig. 2.3: Frequency dependence of the linear permeability μ_L, showing regions of propagation, in the case of $\varepsilon_1 < \varepsilon_2 . \omega_A = 3.5 \times 10^{12}\, rad/s.$

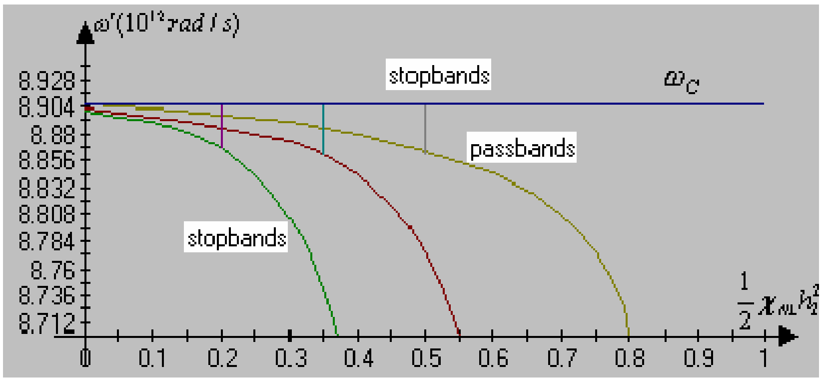

Fig. 2.4: Variation of ω' with $\frac{1}{2}\chi_{NL}h_2^2$, for three different data $\frac{\varepsilon_2}{\varepsilon_1}$: $(1):1.25, (2):1.5, (3):1.75.$

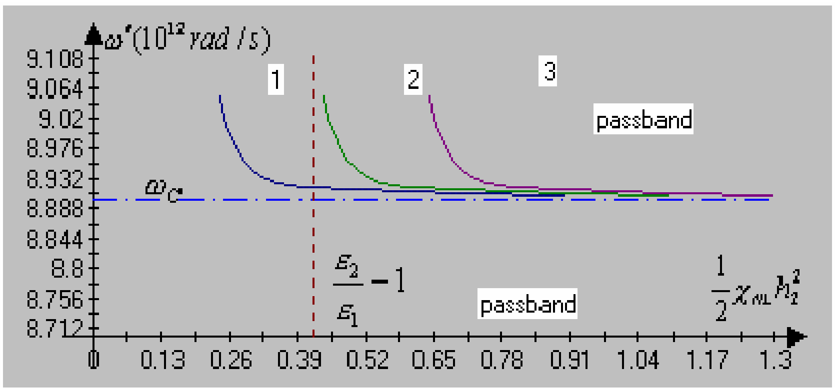

Fig. 2.5: Variation of ω'' with $\frac{1}{2}\chi_{NL}h_2^2$ for the same three data $\frac{\varepsilon_2}{\varepsilon_1}$ as in Fig. 2.4

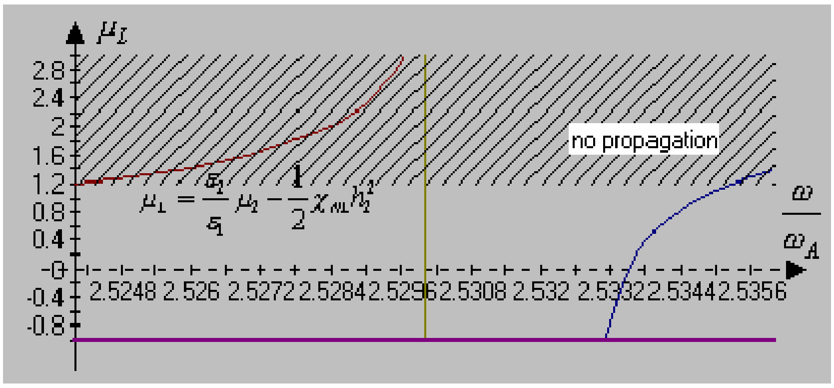

Fig. 2.6: Variation of the passbands with $\frac{1}{2}\chi_{NL}h_2^2$, for $\varepsilon_1 = 4, \varepsilon_2 = 6$.

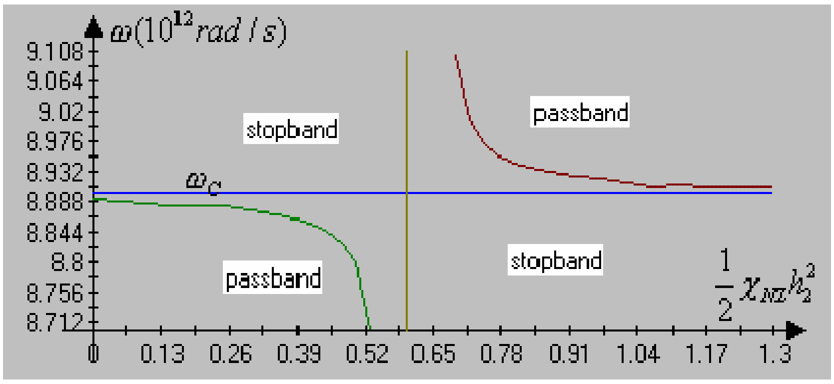

Fig. 2.7: Frequency dependence of the linear permeability, in the case of $\varepsilon_1 > \varepsilon_2, \omega_A = 3.5\times10^{12} rad/s$.

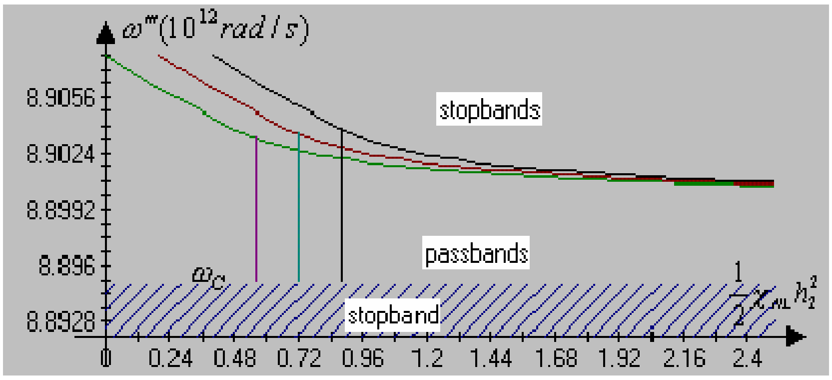

Fig. 2.8: Variation of ω''' with $\frac{1}{2}\chi_{NL}h_2^2$, for three different data $\frac{\varepsilon_2}{\varepsilon_1}$: (1) : 0.75, (2) : 0.5, (3) : 0.25.

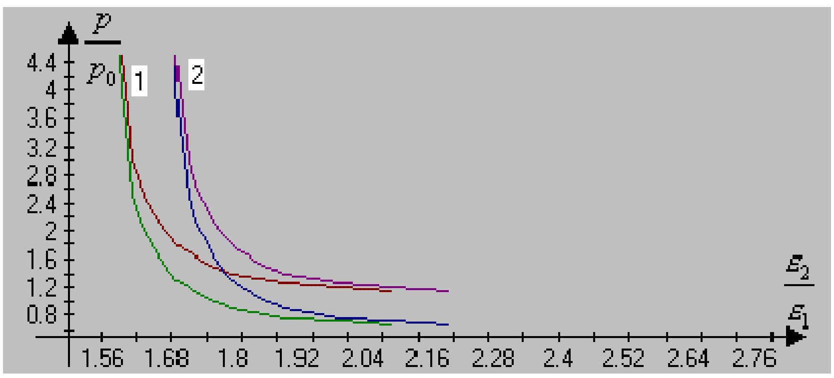

Fig. 2.9: Normalized total power flux $\frac{p}{p_0}$ along the z - direction as a function of $\frac{\varepsilon_2}{\varepsilon_1}$: (1) : 0.625, (2) : 0.75, and for two frequencies, $\omega = 8.898\times10^{12}, 8.899\times10^{12}\, rad/s$.

Chapter 3

TM Nonlinear electromagnetic waves in multilayer dielectric systems

3-1 Introduction

There is at present considerable interest in the study of intrinsically nonlinear effects on the propagation of surface and guided electromagnetic waves along the single and multiple interfaces of optically nonlinear media [5 – 11]. Recently there have also been intensively discussed the properties of linear and nonlinear multilayer systems [13], e.g., the study of surface polariton modes in linear finite and semi-infinite superlattices and the characteristics of the optical response of nonlinear multilayer structures [16].

In this communication we investigate the propagation characteristics of TM-polarized nonlinear guided waves in finite periodic stratified media in contact with nonlinear dielectric cover and linear dielectric base as shown in Fig. 3.1. By using the transfer matrix method, the electromagnetic field distribution and the nonlinear dispersion curve are obtained exactly.

The nonlinear dielectric cover is assumed to be isotropic with a dielectric function given by [15] $\varepsilon = \varepsilon_S + \alpha |E|^2$, α is the nonlinear coefficient in the region $z < 0$. A linear periodic stratified medium with N unit cells consisting of alternating layers of materials 1 and 2 characterized by dielectric functions $\varepsilon_1, \varepsilon_2$ and thicknesses d_1, d_2 in the region $0 < z < N(d_1 + d_2)$ and a linear base with dielectric constant ε_c in the region $z > Nd$ where $d = d_1 + d_2$ is the width of the cell.

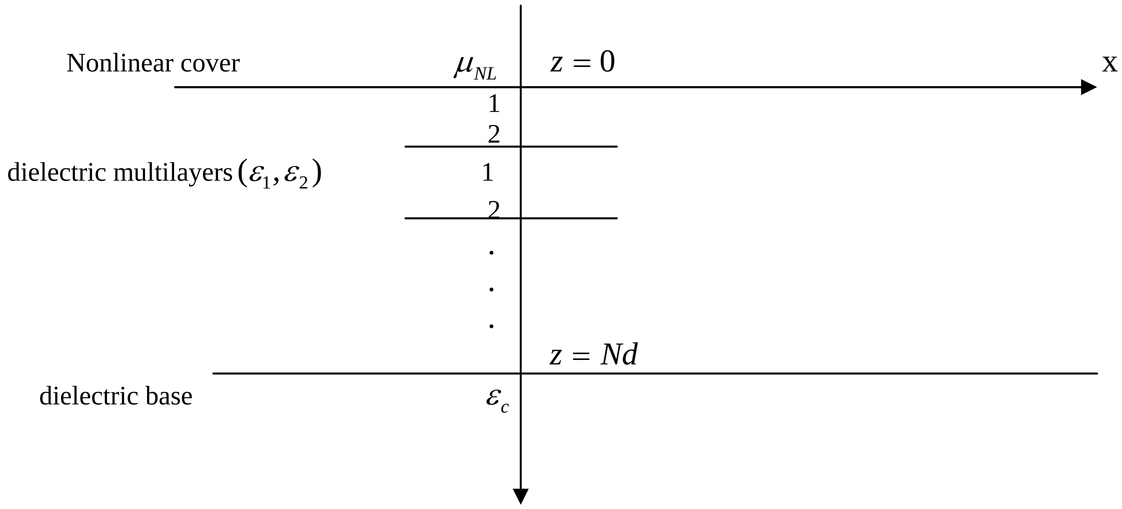

Fig. 3.1: Nonlinear magnetic cladding lies in the region $z < 0$,dielectric multilayers lies in the region $0 < z < Nd$ where $d = d_1 + d_2$ and the dielectric cladding lies in the region $z > Nd$

3.2- Theoretical model

In this chapter we introduce our system as shown in Fig. 3.1. The TM wave is assumed to propagate along the x-axis, i.e., the magnetic field may be written as:

In the nonlinear medium the magnetic field is [16] :

$$H_y = \frac{q_S}{k_0} \cdot \sqrt{\frac{2}{\alpha \varepsilon_S}} \cdot \frac{1}{\cosh[q_S(z - z_0)]} \tag{3.1}$$

where

$$q_S = \sqrt{k^2 - \varepsilon_S k_0^2}$$

In the multilayer dielectric system, the magnetic field is

$$\vec{H} = \frac{1}{2} H_Y e^{i(\beta \cdot k_0 x - \omega t)}$$

where $k_0 = \frac{\omega}{c}$ and β is the propagation constant

The general solution of Maxwell's equations in the nonlinear periodic stratified structure which satisfy the boundary conditions at $z = md$ where $d = d_1 + d_2$, as shown in Ref. [24], has the form:

$$H_y = A_m^c = \cosh[k_0 q_1 (z - md - d_1)] + a_{ms} \cdot \frac{\varepsilon_1}{q_1} \cdot \sinh[k_0 q_1 (z - md)], \quad md \le z \le m + d_1 \tag{3.2}$$

$$H_y = A_{m+1}^c \cosh[k_0 q_2 (z - md - d_1)] + A_{m+1}^s \cdot \frac{\varepsilon_2}{q_2} \sinh[k_0 q_2 (z - md - d_1)], \quad md + d_1 \le z \le (m+1)d \tag{3.3}$$

where

$$md + d_1 \le z \le (m+1)d, m = 0,1,2,...., N-1, q_{1,2} = \sqrt{\beta^2 - \varepsilon_{1,2}}, \varepsilon_{1,2} = n_{1,2}^2, A_m^c, A_m^s$$

are constants .

In the linear dielectric medium, the magnetic field is

$$H_y = H_N e^{-k_0 q_C (z - Nd)} \tag{3.4}$$

where $q_c = \sqrt{\beta^2 - \varepsilon_{1,2}^2}$, H_N is constant which comes directly from the boundary conditions.

Using the boundary conditions between equation (3.2) and equation (3.3) at two points $z = md + d_1$ and $z = (m+1)d$, with the aid we yield the following :

$$\begin{bmatrix} A_m^c \\ \\ A_m^s \end{bmatrix} = T \begin{bmatrix} A_{m+1}^c \\ \\ A_{m+1}^s \end{bmatrix} \tag{3.5}$$

where the transfer matrix T as shown in Ref. [15] is given by:

$$T = \begin{bmatrix} \cosh\gamma_1 \cosh\gamma_2 + \frac{q_2 \varepsilon_1}{q_1 \varepsilon_2} \sinh\gamma_1 \sinh\gamma_2 & -\frac{\varepsilon_2}{q_2} \sinh\gamma_2 \cosh\gamma_1 - \frac{\varepsilon_1}{q_1} \sinh\gamma_1 \cosh\gamma_2 \\ \\ -\frac{q_2}{\varepsilon_2} \sinh\gamma_2 \cosh\gamma_1 - \frac{q_1}{\varepsilon_1} \sinh\gamma_1 \cosh\gamma_2 & \cosh\gamma_1 \cosh\gamma_2 + \frac{q_1 \varepsilon_2}{q_2 \varepsilon_1} \sinh\gamma_1 \sinh\gamma_2 \end{bmatrix} \tag{3.6}$$

where we used the following :

1. $\gamma_{1,2} = k_0 q_{1,2} d_{1,2}$
2. $E_{x1} = E_{x2} \quad at \quad z = md + d_1$ (3.7)
3. $E_{x2} = E_{x1} \quad at \quad z = (m+1)d$ (3.8)

4. $D_{z1} = D_{z2} \quad at \quad z = md + d_1$ (3.9)

5. $D_{z2} = D_{z1} \quad at \quad z = (m+1)d$ (3.10)

6. $m = 0,1,2,3,........, N-1$

Using the boundary conditions at $z = 0$ between the equations (3.1) and (3.2) and note that $H_y, \frac{d}{dz}H_y$ are continuos along z axis we have :

$$A_0^c = H_0 = \sqrt{\frac{2}{\alpha\varepsilon_S}} \cdot \frac{\sqrt{\beta^2 - \varepsilon_S}}{\cosh(z_0 \cdot \sqrt{\beta^2 - \varepsilon_S})} \Bigg|_{Z=0} \tag{3.11}$$

and

$$A_0^s = \frac{1}{k_0} \cdot \frac{\partial}{\partial z} H_y \Bigg|_{Z=0} \tag{3.12}$$

if we let

$$U = \frac{1}{k_0 q_s H_0} \cdot \frac{\partial}{\partial z} H_y \Bigg|_{Z=0} \tag{3.13}$$

we have

$$\begin{bmatrix} A_0^c \\ \\ A_0^s \end{bmatrix} = H_0 \begin{bmatrix} 1 \\ \\ uq_S / \varepsilon_S \end{bmatrix} \tag{3.14}$$

Using the same way at the point $z = Nd$ we have the following :

$$\begin{bmatrix} A_N^c \\ \\ A_N^s \end{bmatrix} = H_N \begin{bmatrix} 1 \\ \\ -q_c / \varepsilon_c \end{bmatrix} \tag{3.15}$$

The eignvalues and the eigenvectors of T can be found from the equation

$$TV = \lambda V \tag{3.16}$$

where V is the column eigenvectors and λ is the eigenvalues of T.

The equation (3.16) becomes :

$$[T - \lambda I] \cdot V = 0 \tag{3.17}$$

which has a nontrivial solution if

$$\left|T - \lambda I\right| = 0 \tag{3.18}$$

The solution of this equation is given in Ref. [12] and the result is

$$\lambda_{\pm} = \frac{2f(\beta) \pm \sqrt{\left[-2f(\beta)\right]^2 - 4}}{2} = se^{\pm tk_0 d} \tag{3.19}$$

where:

$$f(\beta) = \cosh\gamma_1 \cosh\gamma_2 + \frac{1}{2}\cdot\left[\frac{\varepsilon_2 q_1}{\varepsilon_1 q_2} + \frac{\varepsilon_1 q_2}{\varepsilon_2 q_1}\right]\cdot\sinh\gamma_1 \sinh\gamma_2 \tag{3.20a}$$

and

$$s = \sin\left[f(\beta)\right] = \begin{cases} 1, f(\beta) \geq 0 \\ -1, f(\beta) < 0 \end{cases} \tag{3.20b}$$

To find the eigenvalues we write:

$$V = \begin{bmatrix} V_1 \\ \\ V_2 \end{bmatrix} \tag{3.21}$$

the equation (3.16) becomes:

$$T\begin{bmatrix} V_1 \\ \\ V_2 \end{bmatrix} = \lambda\begin{bmatrix} V_1 \\ \\ V_2 \end{bmatrix} \tag{3.22}$$

the solution of this equation is :

$$V_{\pm} = \begin{bmatrix} 1 \\ \\ a_{\mp} \end{bmatrix} \tag{3.23}$$

where

$$a\pm = \frac{1}{2}\cdot\frac{\left[\frac{\varepsilon_1 q_2}{\varepsilon_2 q_1} - \frac{\varepsilon_2 q_1}{\varepsilon_1 q_2}\right]\cdot\sinh\gamma_1 \sinh\gamma_2 \pm s\sqrt{\left[f(\beta)\right]^2 - 1}}{\frac{\varepsilon_1}{q_1}\sinh\gamma_1 \cosh\gamma_2 + \frac{\varepsilon_2}{q_2}\sinh\gamma_2 \cosh\gamma_1} \tag{3.24}$$

We can introduce a column vector as the linearly independent eigenvectors $V\pm$ as the following :

$$\begin{bmatrix} A_N^c \\ \\ A_M^s \end{bmatrix} = A^+V_+ - A^-V_- \tag{3.25}$$

where:

$$A^{\pm} = H_N \frac{a_{\pm} + q_c / \varepsilon_c}{a_+ - a_-} \tag{3.26}$$

then we can prove that :

$$\begin{bmatrix} A_N^c \\ \\ A_N^s \end{bmatrix} = H_N \begin{bmatrix} 1 \\ \\ -q_c / \varepsilon_c \end{bmatrix} = A^+V_+ - A^-V_- \tag{3.27}$$

By using the equation (3.25) in the equation (3.5), we find the following :

$$\begin{bmatrix} A_m^c \\ \\ A_m^s \end{bmatrix} = s^{N-m} \left\{ A^+V_+ e^{(N-m)tk_0 d} - A^-V_- e^{-(N-m)tk_0 d} \right\} \tag{3.28}$$

when $m \to N - m$ then equation (3.28) becomes :

$$\begin{bmatrix} A_{N-m}^c \\ \\ A_{N-m}^s \end{bmatrix} = s^m \left\{ A^+V_+ e^{mtk_0 d} - A^-V_- e^{-mtk_0 d} \right\} \tag{3.29}$$

when $m = N$ then the equation (3.29) becomes :

$$\begin{bmatrix} A_0^c \\ \\ A_0^s \end{bmatrix} = s^N \left\{ A^+V_+ e^{Ntk_0 d} - A^-V_- e^{-Ntk_0 d} \right\} \tag{3.30}$$

from the equations (3.30) and (3.14) we have :

$$H_0 \begin{bmatrix} 1 \\ \\ Uq_s / \varepsilon_s \end{bmatrix} = s^N \left\{ A^+V_+ e^{Ntk_0 d} - A^-V_- e^{-Ntk_0 d} \right\} \tag{3.31}$$

when we use the values of $A^{\pm}, V_{\pm}$ we find that :

$$H_N = \frac{s^N (a_+ - a_-)H_0}{\left[\frac{q_c}{\varepsilon_c} + a_+\right]e^{Ntk_0 d} - \left[\frac{q_c}{\varepsilon_c} + a_-\right]e^{-Ntk_0 d}} \tag{3.32}$$

$$U = \frac{1}{q_s} \cdot \frac{\left[\frac{q_c}{\varepsilon_c} + a_+\right]a_- e^{Ntk_0 d} - \left[\frac{q_c}{\varepsilon_c} + a_-\right]a_+ e^{-Ntk_0 d}}{\left[\frac{q_c}{\varepsilon_c} + a_+\right]e^{Ntk_0 d} - \left[\frac{q_c}{\varepsilon_c} + a_-\right]e^{-Ntk_0 d}} \tag{3.33}$$

3.3- The power of the system

The total power is in three systems the first one is nonlinear cover the second is dielectric multilayers and the third one is the linear cladding

In the nonlinear cover:

The power can be written as

$$p_{NL} = \frac{k}{2\omega\varepsilon_S\varepsilon_0}\int_{-\infty}^{0} H_y^2 dz \tag{3.34a}$$

and this integral gives

$$p_{NL} = \frac{kq_S}{\alpha\omega k_0^2\varepsilon_0\varepsilon_S^2} \tag{3.34b}$$

In the multilayer dielectric medium:

In each layer 1 or 2 the power equation is:

$$p_{f1,2} = \frac{k}{2\omega\varepsilon_0\varepsilon_{1,2}}\int H_{y1,2}^2 dz \tag{3.35}$$

where $H_{y1,2}$ can be found from the equations (3.2) and (3.3). The constants in these equations comes from the equation (3.28) as the following:

$$A_m^c = s^{N-m}\left\{A^+ e^{(N-m)tk_0 d} - A^- e^{-(N-m)tk_0 d}\right\} \tag{3.36}$$

$$A_m^s = s^{N-m}\left\{A^+ a_- \cdot e^{(N-m)tk_0 d} - A^- a_+ \cdot e^{-(N-m)tk_0 d}\right\} \tag{3.37}$$

From the values of A^{+}, A^{-} and H_N we find that:

$$A_m^c = s^{N-m} H_N \left\{ \frac{a_+ + q_c/\varepsilon_c}{a_+ - a_-} \right\} e^{(N-m)tk_0 d} - s^{N-m} H_N \left\{ \frac{a_- + q_c/\varepsilon_c}{a_+ - a_-} \right\} e^{-(N-m)tk_0 d} \tag{3.38}$$

$$A_m^s = s^{N-m} a_- H_N \left\{ \frac{a_+ + q_c/\varepsilon_c}{a_+ - a_{+-}} \right\} e^{(N-m)tk_0 d} - s^{N-m} a_+ H_N \left\{ \frac{a_- + q_c/\varepsilon_c}{a_+ - a_-} \right\} e^{-(N-m)tk_0 d} \tag{3.39}$$

We can find the power in the multilayers as the following:

$$p_f = \sum_{m=0}^{m=N-1} p_{f1,2} \tag{3.40}$$

For simplicity we can use the following:

$$F_i^{\pm} = \frac{\cosh\gamma_i \sinh\gamma_i + \gamma_i}{\varepsilon_i q_i} + (-1)^{i+1} [2a_{\pm}] \cdot \left\{ \frac{\sinh\gamma_i}{q_i} \right\}^2 + \frac{\varepsilon_i a_{\pm}}{q_i^3} \cdot \{\cosh\gamma_i \sinh\gamma_i - \gamma_i\} \tag{3.41}$$

$$F_i = \frac{\cosh\gamma_i \sinh\gamma_i + \gamma_i}{\varepsilon_i q_i} + (-1)^{i+1} (a_+ + a_-) \left\{ \frac{\sinh\gamma_i}{q_i} \right\}^2 + \varepsilon_i a_+ a_- \cdot \left\{ \frac{\cosh\gamma_i \sinh\gamma_i - \gamma_i}{q_i^3} \right\} \tag{3.42}$$

where i =1 or 2 , now the power in the multilayer system is:

$$p_f = \frac{\beta H_0}{4\varepsilon_0 \omega} \cdot \left\{ \left(a_+ + \frac{q_c}{\varepsilon_c} \right) e^{Ntk_0 d} - \left(a_- + \frac{q_c}{\varepsilon_c} \right) e^{-Ntk_0 d} \right\}^{-2} \times$$
$$\left(\left[a_+ + \frac{q_c}{\varepsilon_c} \right]^2 \cdot \left(F_1^- + F_2^- e^{-2tk_0 d} \right) \cdot \frac{e^{2Ntk_0 d} - 1}{1 - e^{2tk_0 d}} + \left[a_- + \frac{q_c}{\varepsilon_c} \right]^2 \cdot \left(F_1^+ + F_2^+ e^{2tk_0 d} \right) \cdot \frac{1 - e^{-2Ntk_0 d}}{e^{2tk_0 d} - 1} \right.$$
$$\left. - 2N (F_1 + F_2) \cdot \left[a_- + \frac{q_c}{\varepsilon_c} \right] \cdot \left[a_+ + \frac{q_c}{\varepsilon_c} \right] \right) \tag{3.43}$$

In the dielectric cladding the power is:

$$p_c = \frac{k}{2\omega \varepsilon_0 \varepsilon_C} \int_{Z=Nd}^{\infty} H_{yN}^2 dz \tag{3.44}$$

where H_{yN} was given in equation (3.4) the result is :

$$p_c = \frac{\beta H_0^2}{4\omega\varepsilon_0\varepsilon_c}\left[\frac{a_+ - a_-}{(a_+ + q_c/\varepsilon_c)e^{Ntk_0d} - (a_- + q_c/\varepsilon_c)e^{-Ntk_0d}}\right]^2 \tag{3.45}$$

The total power becomes:

$$p_{total} = p_{NL} + p_f + p_c \tag{3.46}$$

3.4- **Numerical results**

We present our numerical results for the following parameter values $n_s = 3.502,\ n_c = 1,\ d_1 = d_2 = 0.075\lambda,\ N = 10,\ n_1 = 3.590,\ n_2 = 3.513.$

Fig. 3.2 shows the dependence of the propagation constant β on the normalized power flow p/p_0. The solid lines shows the case (i) in which the material with $n = 3.59$ in contact with the nonlinear medium and the dotted lines shows the case (ii) in which the medium with $n = 3.513$ in contact with nonlinear medium.

The unique feature of the zero'th order nonlinear guided wave solutions are the existence of wave propagation for $\beta >$ max (n_1, n_2) and the local maximum in guided wave power, see branches(a) in Fig. 3.2. The self-focusing action of the nonlinear substrate leads to a field maximum in that medium and the nonlinear guided wave branches (a) degenerate at high powers into single interface surface wave. Self-focusing in the nonlinear substrate also occurs for higher order nonlinear guided solutions (branches (b) in Fig. 3.2). Note also that the values of the local maximum of the power flow corresponding to the branches (a) and the values of the absolute maximum of the power flow corresponding to the branches (b) in Fig. 3.2 are clearly different in two cases (i) and (ii).

Fig. 3.3 shows the dependence of $\alpha(\mu_0/\varepsilon_0)(H^2/\varepsilon)$ on the dimensionless coordinate k_0z for three values of the propagation constant β corresponding to the same value of the dimensionless power flow $p/p_0 \cong 12$ and for $n_1 = 3.590, n_2 = 3.513$. For $\beta = 3.5414$ the electromagnetic field is concentrated in the finite superlattice medium and there exist several local maximum localized in the nonlinear self-focusing substrate occurs.

In Fig. 3.4 we illustrate the dependence of $\alpha(\mu_0/\varepsilon_0)H^2$ on the dimensionless coordinate k_0z for three values of the propagation constant β corresponding to the same value of $p/p_0 \cong 7$ and for $n_1 = 3.590, n_2 = 3.513$. In this case there are several local field maximum in the superlattice medium and the field minimum closest to the nonlinear substrate moves with increasing β into that medium.

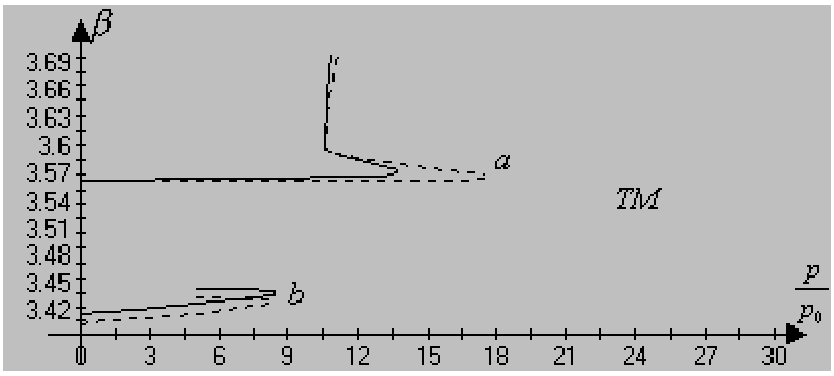

Fig. 3.2: β vs, $\frac{p}{p_0}$ for n_s =3.502, n_c = 1, $d_1 = d_2$ =0.075 λ, N=10, for two cases

* case I : n_1 =3.95, n_2 = 3.513 * case II : n_1 =3.513, n_2 = 3.

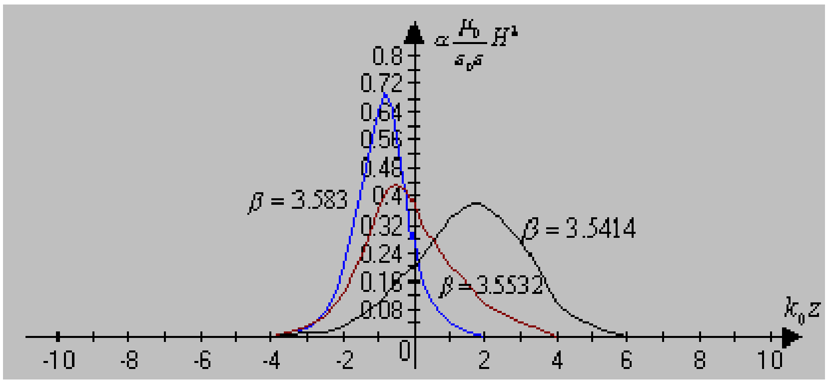

Fig. 3.3: $\alpha \frac{\mu_0}{\varepsilon_0 \varepsilon} H^2$ vs. $k_0 z$ for three values of β

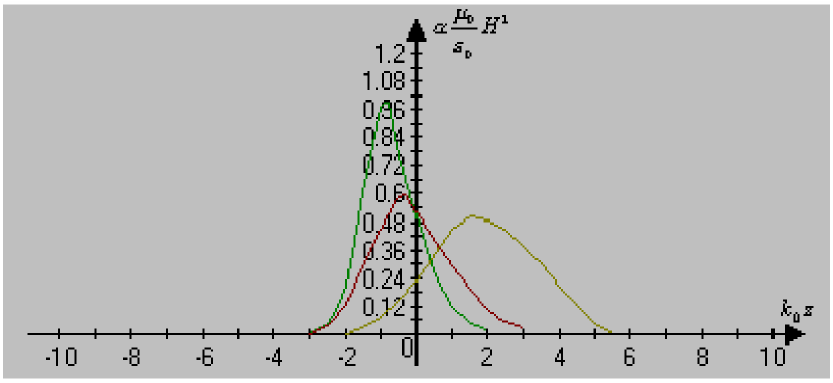

Fig. 3.4: $\alpha \frac{\mu_0}{\varepsilon_0} H^2$ vs. $k_0 z$ for the same three values of β as in Fig. 3.3.

Chapter 4

Nonlinear TM waves between nonlinear medium and superlattices in the long wavelength

4.1- Introduction

There has been considerable interest recently in the properties of quasiperiodic structures. Theoretical investigations have focused on one dimensional Schrodinger equations with two values of potentials arranged in a quasiperiodic sequence $[17]$, and superlattices with two thicknesses of films arranged in a quasiperiodic sequence $[18]$. The quasiperiodicity in the potentials or the superlattice layering has been imposed analytically by requiring that these parameters follow a Fibonacci sequence $[19]$, i.e., if a system is constructed of building blocks α and β,then the system will be a sequence of blocks which obeys the recursion relation $F_m = F_{m-1} + F_{m-2}$, for integer $m \geq 3$, with $F_1 = \{\alpha\}$ and $F_2 = \{\alpha\beta\}$.To illustrate the procedure , then , the next iteration produces $F_3 = \{\alpha\beta\alpha\}$. Therefore, we see that the extended sequence will be $\{\alpha\beta\alpha\alpha\beta\alpha\alpha\beta\alpha\cdots\}$.

We investigate the dispersion relations in semi-infinite superlattices in contact with nonlinear magnetic cladding $[20-26]$. The unit cells in the superlattices, are composed of two different thicknesses of bilayer minicells arranged in a Fibonacci sequence. We find that there exists another spectrum of both bulk and surface modes in the quasiperiodic structure, which is not present in the periodic structure. These surface modes become nonreciprocal with respect to the direction of propagation in an applied magnetic field. Since the number and frequency of these modes depend upon the layering of the unit cell, and since the surface modes are nonreciprocal, our results could be important to surface-wave-device applications.

4.2- The dispersion equation of the periodic superlattices

We seek a solution to Maxwell's equations corresponding to a TM electromagnetic wave propagating along the x-axis in the xz-plane with wave number k and angular frequency ω. The electric and magnetic vectors of the electromagnetic field then take the form.

$$\vec{E} = [E_x(z), 0, E_z(z)]e^{i(kx-\omega t)}$$

$$\vec{H} = [0, H_y(z), 0]e^{i(kx-\omega t)}$$

The nonlinear magnetic cladding is assumed to be isotropic with a permeability $\mu_{NL} = \mu_L + \alpha|H|^2 = \mu_L + \alpha H_Y^2$ where μ_L is the linear part of the permeability and α is the nonlinear coefficient. This expression arises from the expansion of the permeability about an applied static field H_0, and terms that could lead to harmonic generation are neglected. Hence, $\vec{H}$ is an ac magnetic field carried by the TM wave . H_y is real because only stationary, non-radiating waves will be considered .

The magnetic field in the nonlinear cladding is given by [16]:

$$H_y = \frac{1}{k_0}\sqrt{\frac{2}{\alpha\varepsilon_1}}\frac{k_1}{\cosh[k_1(z-z_0)]} \tag{4.1}$$

where k is the pointing vector in the $x-$direction which is the direction between superlattices and nonlinear cladding .

In Fig. 4.1, we have the superlattices $\varepsilon_A, \varepsilon_B, \varepsilon_A, \varepsilon_B \cdots$ as unit cells , each of them has a depth L_n where $L_n = d_{nA} + d_{nB}$. Now we introduce the solution to multi- layer $\varepsilon_A, \varepsilon_B$ in one unit cell.

In the long wave length, static approximation, we can write:

$$\nabla \times \vec{E} = 0 \qquad (4.2)$$

If we make the assumption that only material A contains free charges and material B is an insulator, then we may write the dielectric functions of the materials as the following:

$$\varepsilon_A = \begin{bmatrix} \varepsilon_1 & 0 & i\varepsilon_2 \\ 0 & \varepsilon_3 & 0 \\ -i\varepsilon_2 & 0 & \varepsilon_1 \end{bmatrix} \qquad (4.3a)$$

$$\varepsilon_B = \begin{bmatrix} \varepsilon_B & 0 & 0 \\ 0 & \varepsilon_B & 0 \\ 0 & 0 & \varepsilon_B \end{bmatrix} = \varepsilon_{\infty B} I \qquad (4.3b)$$

where :

$$\varepsilon_1 = \varepsilon_{\infty A}\left\{1 + \frac{\omega_P^2}{\omega_C^2 - \omega^2}\right\}, \ \ \varepsilon_2 = \varepsilon_{\infty A}\left\{\frac{\omega_C \omega_P^2}{\omega(\omega_C^2 - \omega^2)}\right\}, \ \ and \ \ \varepsilon_3 = \varepsilon_{\infty A}\left\{1 - \frac{\omega_P^2}{\omega^2}\right\}$$

In this equation ω_p is the plasma frequency, ω_c is the cyclotron frequency $\frac{eB}{m^* c}$, and the subscript ∞ refers to the background dielectric constant of the given material and m^* is the effective mass of the charge carriers .

The equation (4.2) allows us to introduce a scalar potential Φ given by $\vec{E} = -\vec{\nabla}\Phi$. Using this and the relationship, $\vec{D} = \varepsilon \vec{E}$, we find the following:

$$\nabla \cdot \vec{D} = 0$$

$$\nabla \cdot \vec{D} = \varepsilon_1 \left[\frac{\partial^2}{\partial x^2}\Phi + \frac{\partial^2}{\partial y^2}\Phi \right] + \varepsilon_3 \frac{\partial^2}{\partial z^2}\Phi \qquad (4.4)$$

The potential does not depend on z, then the equation (4.4) becomes :

$$\frac{\partial^2}{\partial x^2}\Phi + \frac{\partial^2}{\partial y^2}\Phi = 0 \qquad (4.5)$$

and the solution of this equation is :

$$\Phi_{An} = (A_{n1}e^{k\delta z} + A_{n2}e^{-k\delta z})\ e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.6}$$

$$\Phi_{Bn} = (B_{n1}e^{k\delta z} + B_{n2}e^{-k\delta z})\ e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.7}$$

where δz is the depth along $z-$axis and Q_n is the block wave vector.

To find the electric field, we introduce the following equations:

$$\vec{E} = -\nabla\Phi \tag{4.8}$$
$$\vec{D} = \varepsilon\vec{E} \tag{4.9}$$

$$E_{Anz} = -k(A_{n1}e^{k\delta z} - A_{n2}e^{-k\delta z})\ e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.10}$$

$$E_{Anx} = -ik(A_{n1}e^{k\delta z} + A_{n2}e^{-k\delta z})\ e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.11}$$

$$D_{Anz} = (-\{\varepsilon_1 + \varepsilon_2\}A_{n1}e^{k\delta z} + \{\varepsilon_1 - \varepsilon_2\}A_{n2}e^{-k\delta z})\ e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.12}$$

$$D_{A(n+1)z} = \left\{-(\varepsilon_1 + \varepsilon_2)A_{(n+1)1}e^{k\delta z} + (\varepsilon_1 - \varepsilon_2)A_{(n+1)2}e^{-k\delta z}\right\} e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.13}$$

$$E_{A(n+1)x} = -ik\left[A_{(n+1)1}e^{k\delta z} + A_{(n+1)2}e^{-k\delta z}\right] e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.14}$$

By the same way we have :

$$D_{Bnz} = -k\varepsilon_B\left[B_{n1}e^{k\delta z} - B_{n2}e^{-k\delta z}\right] e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.15}$$

$$D_{B(n+1)z} = -k\varepsilon_B\left[B_{(n+1)1}e^{k\delta z} - B_{(n+1)2}e^{-k\delta z}\right] e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.16}$$

$$E_{Bnx} = -ik\left[B_{n1}e^{k\delta z} - B_{n2}e^{-k\delta z}\right] e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.17}$$

$$E_{B(n+1)x} = -ik\left[B_{(n+1)1}e^{k\delta z} - B_{(n+1)2}e^{-k\delta z}\right] e^{i(kx-\omega t)} \cdot e^{iQ_n L} \tag{4.18}$$

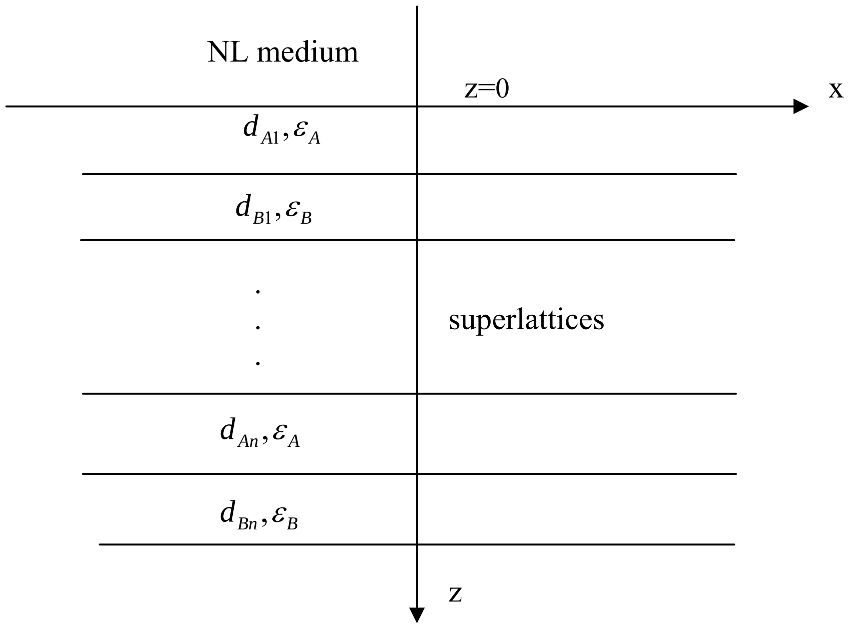

Fig. 4.1: The geometry of the system

In figure (4.1) we have: $L = d_{An} + d_{Bn}$, or $L = L_{n\alpha} + L_{n\beta} + L_{n\alpha}$, where L is the depth of one unit cell.

The potential Φ and the displacement vector D_z must be continuous at the boundary, now we want to use the boundary conditions at two steps :

The first step: between the last depth of A_n and the beginning depth of B_n.
The second step: between the last depth of B_n and the beginning depth of $A_{(n+1)}$.
Then we get the following equations:

$$A_{n1}e^{kdAn} + A_{n2}e^{-kdAn} = B_{n1} + B_{n2} \tag{4.19}$$

$$(\varepsilon_1 + \varepsilon_2)A_{n1}e^{kdAn} + (\varepsilon_2 - \varepsilon_1)A_{n2}e^{-kdAn} = \varepsilon_B(B_{n1} - B_{n2}) \tag{4.20}$$

$$B_{n1}e^{kdBn} + B_{n2}e^{-kdBn} = A_{(n+1)1} + A_{(n+1)2} \tag{4.21}$$

$$(\varepsilon_1+\varepsilon_2)A_{(n+1)1}+(\varepsilon_2-\varepsilon_1)A_{(n+1)2}=\varepsilon_B(B_{n1}e^{kdBn}-B_{n2}e^{-kdBn}) \tag{4.22}$$

Eliminating B_{n1} and B_{n2} from the equations (4.19-4.22), as shown in (Appendix A), we can write the following result:

$$\begin{bmatrix} A_{(n+1)1} \\ \\ A_{(n+1)2} \end{bmatrix} = T_1 \cdot \begin{bmatrix} A_{n1} \\ \\ A_{n2} \end{bmatrix} \tag{4.23}$$

where

$$T_1=\frac{1}{-4\varepsilon_1\varepsilon_B}\begin{bmatrix} xy\cdot e^{A1}\cdot e^{B2}-wz\cdot e^{A1}\cdot e^{B1} & wy\cdot e^{A2}\cdot e^{B2}-wz\cdot e^{A2}\cdot e^{B1} \\ \\ xz\cdot e^{A1}\cdot e^{B1}-xz\cdot e^{A1}\cdot e^{B2} & xy\cdot e^{A2}\cdot e^{B1}-wz\cdot e^{A2}\cdot e^{B2} \end{bmatrix}$$

is the transfer matrix that gives the constants A_{n+1} in terms of A_n constants

and

$$\left.\begin{aligned} w &= \varepsilon_B-\varepsilon_2+\varepsilon_1 \\ x &= \varepsilon_B-\varepsilon_1-\varepsilon_2 \\ y &= \varepsilon_B-\varepsilon_1+\varepsilon_2 \\ z &= \varepsilon_B+\varepsilon_2+\varepsilon_1 \\ e^{B1} &= e^{kdBn} \\ e^{B2} &= e^{-kdBn} \\ e^{A1} &= e^{kdAn} \\ e^{A1} &= e^{-kdAn} \end{aligned}\right\} \tag{4.24}$$

In a similar way we can find another transfer matrix T_2, which related the constants B_{n+1} to the constants B_n, as shown in (Appendix B) as the following:

$$\begin{bmatrix} B_{(n+1)1} \\ \\ B_{(n+1)2} \end{bmatrix} = T_2 \cdot \begin{bmatrix} B_{n1} \\ \\ B_{n2} \end{bmatrix} \tag{4.25}$$

where

$$T_2 = R\begin{bmatrix} x^2(\varepsilon_1-\varepsilon_2)e^{A2+B1} - zw(\varepsilon_1+\varepsilon_2)e^{A1+B1} & zy(\varepsilon_1+\varepsilon_2)e^{B2+A1} - xz(\varepsilon_1-\varepsilon_2)e^{A2+B2)} \\ \\ xz(\varepsilon_1-\varepsilon_2)e^{B1+A2} - xw(\varepsilon_1+\varepsilon_2)e^{A1+B1} & xy(\varepsilon_1+\varepsilon_2)e^{B2+A1} - z^2(\varepsilon_1-\varepsilon_2)e^{A2+B2} \end{bmatrix} \tag{4.26}$$

and

$$R = \frac{1}{(\varepsilon_1-\varepsilon_2)(x^2-z^2)}$$

Now we want to find T the transfer matrix between the cells ($n,\ n+1$) such that we have just two layers $\varepsilon_A, \varepsilon_B$ in each cell :

$$T = T_2 T_1 \tag{4.27}$$

the constants between the cells ($n,\ n+1$) are :

$$\begin{bmatrix} C_{(n+1)1} \\ \\ C_{(n+1)2} \end{bmatrix} = T \cdot \begin{bmatrix} C_{n1} \\ \\ C_{n2} \end{bmatrix} \tag{4.28}$$

where the transfer matrix T gives the dispersion relation :

$$\cos[Q_n(d_A+d_B)] = \frac{1}{2}tr(T) \tag{4.29}$$

4.3- The dispersion equation of the surface

To find the dispersion for surface waves between nonlinear cladding and semi-infinite superlattice, we match the boundary conditions at the surface $z=0$, then we use the continuity of tangent $\vec{E}$ and normal $\vec{D}$ as the following.

In the nonlinear medium the fields are :

$$E_x = -\frac{1}{\omega\varepsilon_0}\cdot\frac{k_1^2}{k_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{\sinh[k_1(z-z_0)]}{\cosh^2[k_1(z-z_0)]}\cdot e^{i(kx-\omega t)} \tag{4.30}$$

$$D_{1z} = \varepsilon_1 E_z \tag{4.31}$$

$$D_{1z} = \frac{-k}{\omega\varepsilon_0}\cdot\frac{k_1}{k_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{1}{\cosh[k_1(z-z_0)]}e^{i(kx-\omega t)} \tag{4.32}$$

In the first layer in medium 2 we have:

$$\vec{E}_A = -\nabla\Phi_A \tag{4.33}$$

$$\Phi_{An} = \{A_{n1}e^{k\delta z} + A_{n2}e^{-k\delta z}\}e^{i(kx-\omega t)+iQnL} \tag{4.34}$$

$$\vec{E} = -\left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\cdot\Phi_{An} \tag{4.35}$$

$$E_{xA} = -(ik)[A_{n1}e^{k\delta z} + A_{n2}e^{-k\delta z}]e^{i(kx-\omega t+QnL)} \tag{4.36}$$

$$E_{zA} = -k[A_{n1}e^{k\delta z} - A_{n2}e^{-k\delta z}]e^{i(kx-\omega t+QnL)} \tag{4.37}$$

At the point $z = 0$ the boundary conditions give:

$$E_{xnL} = E_{xA} \tag{4.38}$$

$$D_{znL} = D_{zA} \tag{4.39}$$

From the equations (4.38) and (4.39) we can write:

$$\frac{1}{\omega\varepsilon_0}\cdot\frac{k_1^2}{k_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\frac{\sinh[k_1 z_0]}{\cosh^2[k_1 z_0]} = k(A_{01} + A_{02})e^{iQ_0 L} \tag{4.40}$$

$$\frac{1}{\omega\varepsilon_0}\cdot\frac{k_1}{k_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{1}{\cosh[k_1 z_0]} = (A_{01} - A_{02})e^{iQ_0 L} \tag{4.41}$$

$$A_{01}e^{iQ_0 L} + A_{02}e^{iQ_0 L} = \frac{k_1^2}{\omega\varepsilon_0}\cdot\frac{1}{kk_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{\sinh[k_1 z_0]}{\cosh^2[k_1 z_0]} \tag{4.42}$$

$$A_{01}e^{iQ_0L} + A_{02}e^{iQ_0L} = \frac{k_1}{\omega\varepsilon_0}\cdot\frac{1}{k_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{1}{\cosh[k_1z_0]} \tag{4.43}$$

$$A_{01}e^{iQ_0L} = \frac{1}{2}\cdot\frac{1}{\omega\varepsilon_0}\cdot\frac{k_1}{k_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{k_1}{k}\frac{\tanh[k_1z_0]}{\cosh[k_1z_0]} \tag{4.44}$$

$$A_{02}e^{iQ_0L} = \frac{1}{2}\cdot\frac{1}{\omega\varepsilon_0}\cdot\frac{k_1}{k_0}\cdot\sqrt{\frac{2}{\varepsilon_1\alpha}}\cdot\frac{1}{\cosh[k_1z_0]}[-1] \tag{4.45}$$

We can use the relation:

$$T\left|A_n\right\rangle = e^{iQnL}\left|A_n\right\rangle \tag{4.46}$$

to write the following equations:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\cdot\begin{bmatrix} A_{01} \\ A_{02} \end{bmatrix} = e^{iQnL}\begin{bmatrix} A_{01} \\ A_{02} \end{bmatrix} \tag{4.47}$$

$$A_{01}T_{11} + A_{02}T_{12} = e^{iQ_{0L}}A_{01} \tag{4.48}$$

$$A_{01}T_{21} + A_{02}T_{22} = e^{iQ_{0L}}A_{02} \tag{4.49}$$

From equations (4.48) and (4.94) we find

$$T_{11} + \frac{A_{02}}{A_{01}}T_{12} = \frac{A_{01}}{A_{02}}T_{21} + T_{22} \tag{4.50}$$

If we let

$$\frac{A_{02}}{A_{01}} = \lambda \tag{4.51}$$

then the equation (4.50) gives:

$$T_{11} + \lambda T_{12} = \frac{1}{\lambda}T_{21} + T_{22} \tag{4.52}$$

which leads to

$$\lambda\{T_{11} + \lambda T_{12}\} = T_{21} + \lambda T_{22} \tag{4.53}$$

The equation (4.53) is the dispersion equation of the surface where

$$\lambda = -\frac{k}{k_1 \tanh[k_1 z_0]} \tag{4.54}$$

4.4- The power for superlattices and nonlinear medium

The power of the system, where, $L = L_{\alpha n} = L_{\beta n}$, is the sum of the total power of all powers in each layer as the following equations:

$$P = \frac{1}{2} \cdot \int (\vec{E} \times \vec{H}^*)_x \, dz \tag{4.55}$$

$$P = -\frac{1}{2}\int E_z H_y^* dz \tag{4.56}$$

$$P_{nA,B} = \frac{k_x}{2\omega\varepsilon_0\varepsilon_{A,B}} \int H_{yn,A,B}^2 dz \tag{4.57}$$

$$P_{NA} = \frac{k_x}{2\omega\varepsilon_0\varepsilon_A} \sum_{n=0}^{N} \int H_{ynA}^2 dz \tag{4.58a}$$

$$P_{NB} = \frac{k_x}{2\omega\varepsilon_0\varepsilon_B} \sum_{n=0}^{N} \int H_{ynB}^2 dz \tag{4.58b}$$

$$E_{xnA} = -ik\left(A_{n1}e^{k\delta z} + A_{n2}e^{-k\delta z}\right) \cdot e^{i(kx - \omega t + QnL)} \tag{4.59}$$

$$E_{xnB} = -ik(B_{n1}e^{k\delta z} + B_{n2}e^{-k\delta z})e^{i(kx - \omega t + QnL)} \tag{4.60}$$

$$H_{ynA,B} = \int i\omega\varepsilon_0\varepsilon_{A,B} E_{xnA,B} dz \tag{4.61}$$

$$H_{ynA} = \omega\varepsilon_0\varepsilon_A (A_{nl}e^{k\delta Z} - A_{n2}e^{-k\delta z}) \tag{4.62}$$

$$H_{ynB} = \omega\varepsilon_0\varepsilon_B (B_{n1}e^{k\delta z} - B_{n2}e^{-k\delta z}) \tag{4.63}$$

$$P_{nA,B} = \int \frac{k_x}{2\omega\varepsilon_0\varepsilon_{A,B}} H^2_{ynA,B} dz \tag{4.64}$$

To find the constants A, B we write,

$$T = T_1 T_2 \tag{4.65}$$

and the vectors comes from the following equation;

$$\begin{bmatrix} A_{n1} \\ \\ A_{n2} \end{bmatrix} = T^n \begin{bmatrix} A_{01} \\ \\ A_{02} \end{bmatrix} = e^{itnk_0L} \begin{bmatrix} A_{01} \\ \\ A_{02} \end{bmatrix}; \tag{4.66}$$

where the $A_{n1,n2,01,02}$ are constants to be calculated from the boundary conditions as the following:

$$A_{01} = \frac{1}{2} \cdot \frac{1}{\omega\varepsilon_0} \cdot \frac{k_1}{k_0} \cdot \sqrt{\frac{2}{\varepsilon_1 \alpha}} \cdot \frac{1}{\cosh[k_1 z_0]} \cdot \frac{k_1}{k} \cdot \tanh[k_1 z_0] \tag{4.67}$$

$$A_{02} = -\frac{1}{2} \cdot \frac{1}{\omega\varepsilon_0} \cdot \frac{k_1}{k_0} \cdot \sqrt{\frac{2}{\varepsilon_1 \alpha}} \cdot \frac{1}{\cosh[k_1 z_0]} \tag{4.68}$$

$$A_{n1} = A_{01} e^{-itnk_0L} \tag{4.69}$$

$$A_{n2} = A_{02} e^{-itnk_0L} \tag{4.70}$$

The $B's$ constants are coming directly from the $A's$ constants as:

$$\begin{bmatrix} B_{n1} \\ \\ B_{n2} \end{bmatrix} = T_2 \begin{bmatrix} A_{n1} \\ \\ A_{n2} \end{bmatrix} \tag{4.71}$$

$$\begin{bmatrix} B_{01} \\ \\ B_{02} \end{bmatrix} = T_2 \begin{bmatrix} A_{01} \\ \\ A_{02} \end{bmatrix} \tag{4.72}$$

$$\begin{bmatrix} B_{n1} \\ \\ B_{n2} \end{bmatrix} = T^n \begin{bmatrix} B_{01} \\ \\ B_{02} \end{bmatrix} \tag{4.73}$$

From the equations (4.66-4.73) we have :

$$B_{n1} = A_{01}\left\{x^2(\varepsilon_1 - \varepsilon_2)e^{A2+B1} - wz(\varepsilon_1 + \varepsilon_2)e^{A1+B1}\right\} \cdot e^{-itnk_0L} + A_{02}\left\{yz(\varepsilon_1 + \varepsilon_2)e^{A1+B1} - xz(\varepsilon_1 - \varepsilon_2)e^{A2+B2}\right\} \cdot e^{-itnk_0L} \tag{4.74}$$

$$B_{n2} = A_{01}\left\{xz(\varepsilon_1 - \varepsilon_2)e^{A2+B1} - wx(\varepsilon_1 + \varepsilon_2)e^{A1+B1}\right\} \cdot e^{-itnk_0L} + A_{02}\left\{xy(\varepsilon_1 + \varepsilon_2)e^{A1+B2} - z^2(\varepsilon_1 - \varepsilon_2)e^{A2+B2}\right\} \cdot e^{-itnk_0L} \tag{4.75}$$

$$P_{NA} = \frac{k_x}{2\omega\varepsilon_0\varepsilon_A} \sum_{n=0}^{N} \int \omega^2 \varepsilon_0^2 \varepsilon_A^2 \left\{A_{n1}e^{k\delta z} - A_{n2}e^{-k\delta z}\right\}^2 dz \tag{4.76}$$

$$P_{NA} = \frac{k_x \omega \varepsilon_0 \varepsilon_A}{2} \left\{ \sum_{n=0}^{N} \int A_{n1}^2 \cdot e^{2k\delta z} dz + \sum_{n-0}^{N} \int A_{n2}^2 \cdot e^{-2k\delta z} dz - 2\sum_{n=0}^{N} \int A_{n1}A_{n2} dz \right\} \tag{4.77}$$

$$P_{NB} = \frac{k_x \omega \varepsilon_0 \varepsilon_B}{2} \left\{ \sum_{n=0}^{N} \int B_{n1}^2 \cdot e^{2k\delta z} + \sum_{n=0}^{N} \int B_{n2}^2 \cdot e^{-2k\delta z} dz - 2\sum_{n=0}^{N} \int B_{n1}B_{n2} dz \right\} \tag{4.78}$$

To find the power, we can use the following integrals:

$$1: \int_0^A e^{2k\delta z} dz = \frac{1}{2k}\left[e^{2k\delta z}\right]_0^A = \frac{1}{2k}\left[e^{2kA} - 1\right] \tag{4.79}$$

$$2: \int_0^B e^{2k\delta z} dz = \frac{1}{2k}\left[e^{2k\delta z}\right]_0^B = \frac{1}{2k}\left[e^{2kB} - 1\right] \tag{4.80}$$

$$3: \int_0^A e^{-2k\delta z} dz = \frac{1}{2k}\left[1 - e^{-2kA}\right] \tag{4.81}$$

$$4: \int_0^B e^{-2k\delta z} dz = \frac{1}{2k}\left[1 - e^{-2kB}\right] \tag{4.82}$$

so that the total power can be written as:

$$P_{total} = P_{nonlinear} + P_{NA} + P_{NB} \tag{4.83}$$

4.5- Results and discussions

In our numerical results, we assume that material A is doped semiconductor GaAs, while material B is undoped simiconductor GaAs. For these materials, we use $\varepsilon_\infty = 13.13,$ and the doping concentration is $n = 10^{18} cm^{-3}$. The plasma frequency is $\omega_p = 0.04075 eV$, and $\mu_L = 1.29$.

If we label the two different bilayers α and β, then a unit cell composed of three bilayers will be look like $\alpha\beta\alpha$. The total thickness of each bilayer will be denoted d_α and d_β respectively, and we label the thicknesses of the individual films $d_{\alpha A}, d_{\alpha B}, d_{\beta A}$ and $d_{\beta B}$. In all of what follows, we take $d_\alpha / d_\beta = 1.618$, $d_{\alpha A} = 2d_{\alpha B}$, and $d_{\beta A} = 2d_{\beta B}$.

For the purpose of comparison, we begin by showing the dispersion curves for the periodic superlattice $(F_1 = 1)$. In Fig. 4.2 the bulk bands between $Q_n L = 0$ and $Q_n L = \pi$ in this case the applied field is zero, and the equation is $\cos(Q_n L) = \frac{1}{2} tr(T)$.

In Fig. 4.3 we show the dispersion curves for bulk plasmons in the case $F_3 = 3$. Here we take the applied magnetic field to be zero and the equation is $\cos(Q_n L) = \frac{1}{2} tr(T)$. The uppermost and lowermost bands are extremely narrow, note that the boundaries of the bulk bands are given by $QL = 0$, and $QL = \pi$.

Fig. 4.4 depicts the surface modes for the same superlattice, again without an applied field. Both $\pm k$ are shown, and we note that all modes are nonreciprocal with respect to propagation direction. The surface dispersion equation is given by equation (4.53).

Fig. 4.5 shows the dispersion for the surface modes again, but this time with an applied field given by $\omega_C = 0.004075\ eV$.

Both $\pm k$ are shown, and there are several points of interest.

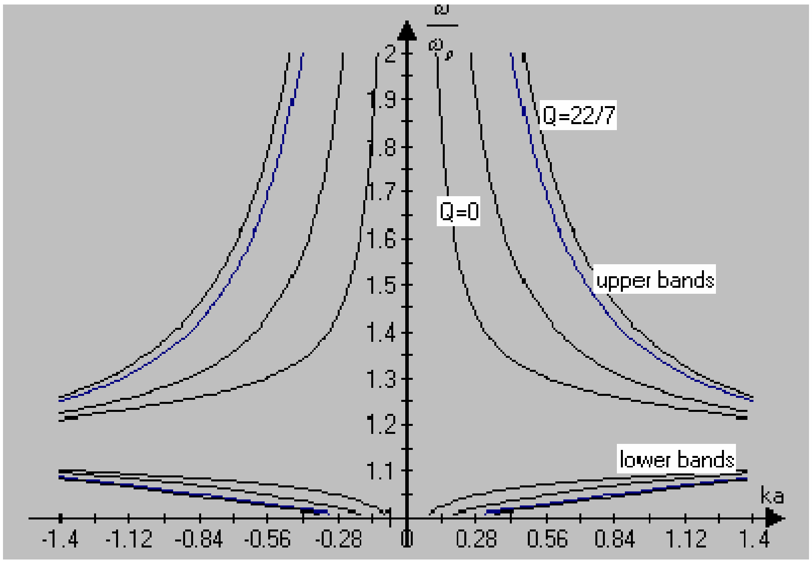

Fig. 4.2: The Computed dispersion curves for bulk modes (F_1=1)

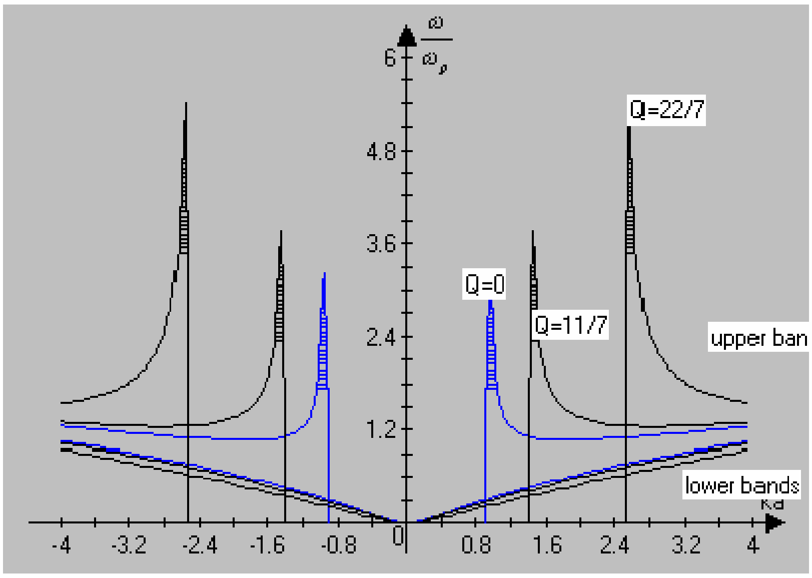

Fig. 4.3: The Computed dispersion curves for bulk modes (F_3=3)

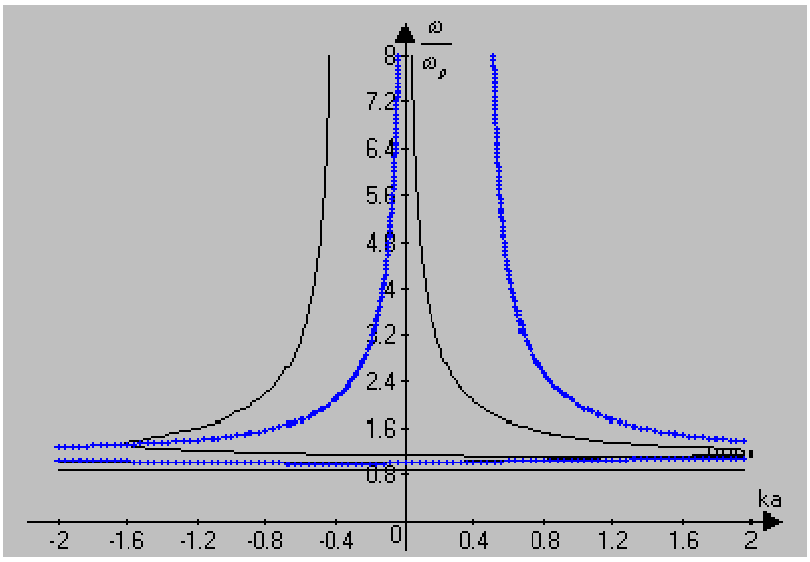

Fig. 4.4: The Computed dispersion curves for surface modes the field is zero

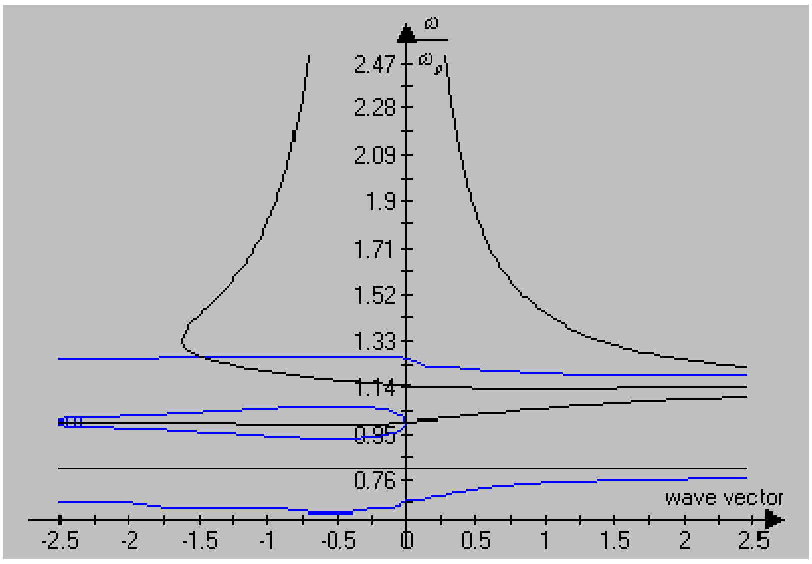

Fig. 4.5: The Computed dispersion curves for surface modes $\omega_C = 0.004075\ eV$

Chapter 5

Polaritons in an n-i-p-i semiconductor superlattice covered by a nonlinear magnetic cladding

5.1- Introduction

Recently a number of papers has appeared dealing with the propagation of bulk and surface plasmons in semiconductor superlattices of various types. Binary superlattices consisting of alternating layers of materials a and b with or without a two-dimensional electron (hole) gas at the interfaces where studied by many authors $[17-21]$. A particular superlattice structure, the so-called n, i, p, i superlattices has also been investigated and its particular features discussed $[26]$.

In this chapter, we present a full theory of the bulk and surface-plasmon excitation spectrum of a finite superlattice covered by a nonlinear magnetic cladding. We have included the effects of both retardation and an external magnetic field and we have obtained the dispersion relation for surface magnetoplasmon polaritons in this structure.

Our model is based on a transfer-matrix treatment, already presented earlier, to simplify the algebra which is otherwise quite involved. Since the quantization of the electronic states into subbands is quite negligible due to our basic assumption that the layer thicknesses are sufficiently large, we can describe the properties of the layers by macroscopic dielectric functions. Thus the electromagnetic fields in each layer are described by solving Maxwell's equations subject to the appropriate boundary conditions.

5.2 Theoretical model

The guiding structure to be considered consists of a nonlinear magnetic cladding in contact with superlattices everywhere on the $z = 0$ plane, where the z - axis points into the structure, the applied magnetic field is along y - axis and the propagation is along x - axis. The nonlinear magnetic cladding is assumed to be isotropic with a permeability given by $[13]$.

$$\mu_{NL} = \mu_L + \alpha |H|^2 = \mu_L + \alpha H_Y^2 \qquad (5.1)$$

where μ_L is the linear part of the permeability and α is the nonlinear coefficient. This expression arises from an expansion of the permeability about an applied static field H_0, and terms that could lead to harmonic generation are neglected. Hence, H is the ac magnetic field carried by the TM wave. H_y is real because only stationary, non-radiating waves will be considered.

We have a solution to the Maxwell's equations corresponding to a TM electromagnetic wave propagating in the nonlinear cladding [13]

$$H_y(z) = \frac{1}{k_0}\sqrt{\frac{2}{\alpha\varepsilon_s}} \cdot \frac{\alpha_s}{\cosh[k_s(z - z_0)]} \qquad (5.2)$$

where z_0 is a constant of integration that defines the position of a self-focused peak in H_Y and $\alpha_s = \sqrt{k_x^2 - k_0^2\varepsilon_s\mu_L}$, ε_s is the dielectric constant of the nonlinear medium .

The semiconductor superlattice consists of multilayer materials in cells along the z - direction. Materials a and c are n type and p type, with dielectric constants $\varepsilon_a(\omega)$ and $\varepsilon_c(\omega)$, and with thickness a and c respectively. Materials b and d are intrinsic semiconductors with frequency independent tensors ε_b and ε_d and thickness b and d respectively. The unit cell has length $L = a + b + c + d$ and is designated by the index n.

In the $n'th$ unit cell, at the interfaces $z = nL$ and $z = nL + a$ there is a two-dimensional electron gas, while at $z = nL + a + b$ and $z = nL + a + b + c$ there is a two dimensional hole gas. We assume that a uniform external magnetic field is imposed in the y - direction and that surface magnetoplasmon polaritons are allowed to propagate in the x - direction with a wave-vector k and frequency ω.

We are going firstly to find the dispersion relation for the system in an infinite superlattice and then for a finite one. In both cases the field amplitudes are assumed to be localized at each interface. In the following we discuss bulk modes and surface modes.

5.2-a **Bulk modes**

To find the dispersion relation for the bulk modes we use the boundary condition to find the field constants in the layers of superlattices as a function of well known constants at the surface. The electric and magnetic fields in superlattices can be written as:

$$\vec{E}^n(x,t) = \left[E_x^n(z|k\omega), 0, E_z^n(z|k\omega)\right]e^{i(kx-\omega t)} \tag{5.3}$$

and

$$\vec{H}^n(x,t) = \left[0, H_y^n(z|k\omega), 0\right]e^{i(kx-\omega t)} \tag{5.4}$$

In each layer these fields satisfy Maxwell's equations:

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E}^n(x,t) = -\varepsilon_0 \varepsilon_j(\omega) \frac{\partial^2}{\partial t^2} \vec{E}^n(x,t), \tag{5.5}$$

and

$$\vec{\nabla} \times \vec{H}^n(x,t) = \varepsilon_0 \varepsilon_j(\omega) \frac{\partial}{\partial t} \vec{E}^n(x,t), \tag{5.6}$$

where ε_0 is the vacuum permittivity, ε_j is the dielectric constant, and $j = a,b,c$ or d.

In the equations (5.3) and (5.6) the x - component of the electric field and the y - component of the magnetic field in each layer of the $n'th$ cell is given by:

$$E_{xj}^n(z|k\omega) = A_{1j}^n e^{-\alpha_j z} + A_{2j}^n e^{\alpha_j z} \tag{5.7}$$

and

$$H_{yj}^n(z|k\omega) = -i\frac{\omega\varepsilon_0\varepsilon_j}{\alpha_j}\left[A_{1i}^n e^{-\alpha_j z} - A_{2j}^n e^{\alpha_j z}\right] \tag{5.8}$$

where

$$\alpha_j = \begin{cases} \left(k_x^2 - \varepsilon_j \omega^2 / c^2\right)^{1/2}, k_x > \varepsilon_j \omega / c \\ i\left(\varepsilon_j \omega^2 / c^2 - k_x^2\right)^{1/2}, k_x < \varepsilon_j \omega / c \end{cases} \tag{5.9}$$

Therefore, using the boundary conditions for the electric and magnetic fields given by the equations (5.7) and (5.8) at the interfaces:

$z = nL + a, nL + a + b, nL + a + b + c \quad and \quad z = (n+1)L,$ we obtain the following equations:

$$A^n_{1a} f_a + A^n_{2a} \bar{f}_a = A^n_{1b} + A^n_{2b}, \tag{5.10}$$

$$\varepsilon'_a\left(A^n_{1a} f_a - A^n_{2a} \bar{f}_a\right) = \left(\varepsilon'_b + \sigma_h\right)A^n_{1b} - \left(\varepsilon'_b + \sigma_h\right)A^n_{2b}, \tag{5.11}$$

$$A^n_{1b} f_b + A^n_{2b} \bar{f}_b = A^n_{1c} + A^n_{2c}, \tag{5.12}$$

$$\varepsilon'_b(A^n_{1b} f_b - A^n_{2b} \bar{f}_b) = (\varepsilon'_c - \sigma_e)A^n_{1c} - (\varepsilon'_c + \sigma_e)A^n_{1c} - (\varepsilon'_c + \sigma_e)A^n_{2c}, \tag{5.13}$$

$$A^n_{1c} f_c + A^n_{2c} \bar{f}_c = A^n_{1d} + A^n_{2d}, \tag{5.14}$$

$$\varepsilon'_c(A^n_{1c} f_c - A^n_{2c} \bar{f}_c) = (\varepsilon'_d - \sigma_e)A^n_{1d} = (\varepsilon'_d + \sigma_e)A^n_{2d}, \tag{5.15}$$

$$A^n_{1d} f_d + A^n_{2d} \bar{f}_d = A^{n+1}_{1a} + A^{n+1}_{2a}, \tag{5.16}$$

$$\varepsilon'_d(A^n_{1d} f_d - A^n_{2d} \bar{f}_d) = (\varepsilon'_a - \sigma_h)A^{n+1}_{1a} - (\varepsilon'_a + \sigma_h)A^{n+1}_{2a} \tag{5.17}$$

In the equations (5.10-5.17) we redefined A^n_{1j} and A^n_{2j} as:

$$A^n_{ma} = A^n_{ma} e^{(-1)^m \alpha_a nL} \tag{5.18}$$

$$A^n_{mb} = A^n_{mb} e^{(-1)^m \alpha_b (nL+a)} \tag{5.19}$$

$$A^n_{mc} = A^n_{mc} e^{(-1)^m \alpha_c (nL+a+b)} \tag{5.20}$$

$$A^n_{md} = A^n_{md} e^{(-1)^m \alpha_d (nL+a+b+c)} \tag{5.21}$$

where $m = 1,2$, $\varepsilon'_j = \dfrac{\varepsilon_j}{\alpha_j}$, $j = a,b,c,d$, $\sigma_p = \dfrac{n_p e^2}{m^*_p \omega^2 \varepsilon_0}$, $p = e,h.$ $f_j = e^{-\alpha_j j}$, $\bar{f}_j = e^{\alpha_j j}$

We define, for each medium, the column vector:

$$\left| A^n_j \right\rangle = \begin{bmatrix} A^n_{1j} \\ A^n_{2j} \end{bmatrix}, \tag{5.22}$$

The equations (5.10-5.17) can be written in a matrix form as:

$$\begin{aligned} M_a \left| A^n_a \right\rangle &= N_b \left| A^n_b \right\rangle, \\ M_b \left| A^n_b \right\rangle &= N_c \left| A^n_c \right\rangle, \\ M_c \left| A^n_c \right\rangle &= N_d \left| A^n_d \right\rangle, \\ M_d \left| A^n_d \right\rangle &= N_a \left| A^{n+1}_a \right\rangle, \end{aligned} \tag{5.23}$$

where we have defined the matrices

$$M_j = \begin{bmatrix} f_j & \bar{f}_j \\ \varepsilon'_j f_j & -\varepsilon'_j \bar{f}_j \end{bmatrix}, \tag{5.24}$$

and

$$N_j = \begin{bmatrix} 1 & 1 \\ \varepsilon'_j - \sigma_p & -\varepsilon'_j - \sigma_p \end{bmatrix}, \tag{5.25}$$

with $p = h$ for $j = a,b$ and $p = e$ for $j = c,d$.

From the equations (5.23-5.25) it is easy to see that:

$$\left| A_j^{n+1} \right\rangle = T \left| A_j^n \right\rangle, \tag{5.26}$$

where the matrix T is given by:

$$T = N_a^{-1} M_d N_d^{-1} M_c N_c^{-1} M_b N_b^{-1} M_a \tag{5.27}$$

The matrix T in the equation (5.27) is a transfer matrix because it relates the coefficients of the electric field in one cell to those in the preceding cell. Taking into account the translational symmetry of the problem, we can use Bloch's theorem [19], that is:

$$\left| A_j^{n+1} \right\rangle = e^{iQL} \left| A_j^n \right\rangle. \tag{5.28}$$

By using the equations (5.26) and (5.27) we have:

$$T \left| A_j^n \right\rangle = e^{iQL} \left| A_j^n \right\rangle, \tag{5.29}$$

$$T^{-1} \left| A_j^n \right\rangle = e^{-iQL} \left| A_j^n \right\rangle, \tag{5.30}$$

and consequently

$$\left[\cos(QL) I - \frac{1}{2}\left(T + T^{-1}\right) \right] \left| A_j^n \right\rangle = 0 \tag{5.31}$$

Since $\left| A_j^n \right\rangle$ is a general vector of the structure considered, the dispersion relation of the bulk polaritons on the superlattice will be given by:

$$\cos(QL)I = \frac{1}{2}(T + T^{-1}) \tag{5.32}$$

From the definition of the transfer matrix in equation (5.27) and using the equations (5.24) and (5.25) we can show that $\det(T) = 1$, therefore

$$T^{-1} = \begin{bmatrix} T_{22} & -T_{12} \\ -T_{21} & T_{11} \end{bmatrix} \tag{5.33}$$

and hence, from the equations (5.32) and (5.33) our dispersion relation for the bulk modes is simply:

$$\cos(QL) = \frac{1}{2}Tr(T) \tag{5.34}$$

5.2b- **Surface modes**

In order to study the surface modes we match the boundary conditions for the electric and magnetic fields at the surface where $z = 0$. Then the periodicity in the z direction is destroyed and we can no longer assume Bloch's theorem. Therefore we have to consider electromagnetic modes that have their excitations localized in the near vicinity of the interface between nonlinear magnetic cladding and superlattices, where we replace Q by $i\beta$, then Eq.5.34 becomes:

$$\cosh(\beta L) = \frac{1}{2}TrT \tag{5.35}$$

We can conclude this prices as:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \cdot \begin{bmatrix} A_{1a}^{0} \\ A_{2a}^{0} \end{bmatrix} = e^{-\beta L} \begin{bmatrix} A_{1a}^{0} \\ A_{2a}^{0} \end{bmatrix} \tag{5.36}$$

$$\begin{aligned} T_{11}A_{1A}^{0} + T_{12}A_{2a}^{0} &= e^{-\beta L}A_{1a}^{0} \\ T_{21}A_{1a}^{0} + T_{22}A_{2a}^{0} &= e^{-\beta L}A_{2a}^{0} \end{aligned} \tag{5.37}$$

if we replace $\frac{A_{1a}^{0}}{A_{2a}^{0}}$ by λ we have the dispersion equation of the surface:

$$\lambda\{T_{11} + \lambda T_{12}\} = T_{21} + \lambda T_{22} \tag{5.38}$$

where

$$\lambda = \frac{\alpha_s \tanh(\alpha_s z_0) - \alpha_a / \varepsilon_a}{\alpha_s \tanh(\alpha_s z_0) + \alpha_a / \varepsilon_a}$$

5.3 Special cases in superlattices

If we consider $a = c$, $b = d$, and $\sigma_h = \sigma_e$. Therefore the periodicity of the superlattice is $L = a + b$ and the transfer matrix will be given by:

$$T_1 = N_a^{-1} M_b N_b^{-1} M_a$$

And the dispersion equation of the surface in this case is:

$$\cos(Q(a+b)) = \frac{1}{2} Trace(T_1) \qquad (5.39)$$

To find the power of the system in this case we have:

$$P_{TOTAL} = P_{\text{NONLINEAR}} + P_{SUPERLATTICES} \qquad (5.40\ \text{I})$$

where

$$P_{\text{NONLINEAR}} = \frac{k_x k_1}{\alpha \omega k_0^2 \varepsilon_0 \varepsilon_1^2} \qquad (5.40\ \text{II})$$

and

$$P_{SUPERLATTICES} = \sum_{n=0}^{N} \left(A_{1a}^n\right)^2 \left[\frac{1}{2\alpha_a}\left(1 - e^{-2\alpha_a a}\right)\right] + \sum_{n=0}^{N} \left(A_{2a}^n\right)^2 \left[\frac{1}{2\alpha_a}\left(e^{2\alpha_a a} - 1\right)\right] \text{-} 2 \sum_{n=0}^{N} A_{1a}^n . A_{2a}^n . a$$

$$+ \sum_{n=0}^{N} \left(A_{1b}^n\right)^2 \left[\frac{1}{2\alpha_b}\left(1 - e^{-2\alpha_b b}\right)\right] + \sum_{n=0}^{N} \left(A_{2b}^n\right)^2 \left[\frac{1}{2\alpha_b}\left(e^{2\alpha_b b} - 1\right)\right] - 2 \sum_{n=0}^{N} A_{1b}^n . A_{2b}^n b \qquad (5.40\ \text{III})$$

where $A^n_{1,2,a,b}$ are constants can be found from the boundary conditions by using the following surface constants at the point $z = 0$

$$E_{xNL} = \frac{1}{i\varepsilon_0 k_0} \sqrt{\frac{2}{\alpha \varepsilon_s}} \frac{\alpha_s \sinh(\alpha_s z_0)}{\cosh^2(\alpha_s z_0)}$$

and

$$H_{yNL} = \frac{1}{k_0} \sqrt{\frac{2}{\alpha \varepsilon_s}} \frac{\alpha_s}{\cosh(\alpha_s z_0)}$$

In the first layer, at the point $z = 0$, we have:

$$E_{xa} = A_{a1}^0 + A_{a2}^0 \qquad (5.41)$$

and

$$H_{ya} = -i\left[\frac{\omega\varepsilon_0\varepsilon_a}{\alpha_a}\right]\cdot(A_{a1}^0 - A_{a2}^0) \tag{5.42}$$

the continuity of the electric and magnetic fields give:

$$E_{xNL} = E_{xa} \tag{4.43}$$

and

$$H_{yNL} = H_{ya} \tag{5.44}$$

then, we have:

$$\frac{1}{i\varepsilon_0 k_0}\sqrt{\frac{2}{\alpha\varepsilon_s}}\frac{\alpha_s}{\cosh(\alpha_s z_0)}\tanh(\alpha_s z_0) = A_{a1}^0 + A_{a2}^0 \tag{5.45}$$

and

$$\frac{1}{k_0}\sqrt{\frac{2}{\alpha\varepsilon_s}}\frac{\alpha_s}{\cosh(\alpha_s z_0)} = -i\left(\frac{\omega\varepsilon_0\varepsilon_a}{\alpha_s}\right)\left\{A_{a1}^0 - A_{a2}^0\right\} \tag{5.46}$$

The solution of the equations (5.45) and (5.46) for the surface constants is:

$$A_{a1}^0 = \frac{i}{2}\cdot\left(\frac{\alpha_s}{\omega k_0\varepsilon_0}\cdot\sqrt{\frac{2}{\alpha\varepsilon_s}}\cdot\frac{1}{\cosh(\alpha_s z_0)}\right)\cdot\left[\alpha_s\tanh(\alpha_s z_0) + \frac{\alpha_a}{\varepsilon_a}\right] \tag{5.47}$$

and

$$A_{a2}^0 = \frac{i}{2}\left(\frac{\alpha_s}{\omega k_0\varepsilon_0}\cdot\sqrt{\frac{2}{\alpha}}\cdot\frac{1}{\cosh(\alpha_s z_0)}\right)\cdot\left[\alpha_s\tanh(\alpha_s z_0) - \frac{\alpha_a}{\varepsilon_a}\right] \tag{5.48}$$

when we find A_{A1}^0, A_{A2}^0, we can find A_j where $j = a \;\; or \;\; b$ as the following:

$$\begin{bmatrix} A_{j1}^{n+1} \\ \\ A_{j2}^{n+1} \end{bmatrix} = T\begin{bmatrix} A_{j1}^{n} \\ \\ A_{j2}^{n} \end{bmatrix} \tag{5.49}$$

Another particular case, if we consider the limit where retardation effects can be neglected, we have that $\alpha_a = \alpha_b$ and equation (5.39) becomes:

$$\left[\frac{\omega}{\Omega}\right]^2 = \frac{K_x\{\cosh[(1+s)K_x)] - \cosh[(1-s)K_x]\}}{(r+1)\sinh[(1+s)K_x] - (r-1)\sinh[(1-s)K_x] \pm \sqrt{F}}, \tag{5.50}$$

with

$$F = 2(r^2+1)\{\cosh[(1+s)K_x]\cosh[(1-s)K_x]-1\} - 2(r^2-1)\sinh[(1+s)K_x]\sinh[(1-s)K_x]$$
$$+4r\cosh[(1+s)Q_Z]\{\cosh[(1+s)K_x]\cosh[(1-s)K_x]\}$$

where we have introduced $K_x = k_x a,\ \ Q_Z = Qa,\ \ r = \varepsilon_a / \varepsilon_b,\ \ s = b/a,$ and $\Omega = \sqrt{\dfrac{n_e e^2}{a m_e^* \varepsilon_0 \varepsilon_b}}$

Another special case of our results is the dispersion relation of bulk plasmons, in this case the superlattice consists of two dielectric materials a and b with dielectric constants $\varepsilon_a(\omega)$ and ε_b frequency independent, respectively. The dispersion relation of bulk plasmons in this case can be obtained from the equation (5.39) by considering $\alpha_a = \alpha_b = k$ and $\sigma_e = \sigma_h = 0$, then we obtain:

$$\cos[Q(a+b)] = \frac{1}{2\varepsilon_a\varepsilon_b}\left[(\varepsilon_a^2+\varepsilon_b^2)\sinh(ka)\sinh(kb) + 2\varepsilon_a\varepsilon_b\cosh(ka)\cosh(kb)\right], \qquad (5.51)$$

which equivalent to:

$$\pm\cos[Q(a+b)] = \frac{1}{2}Tr(T_2) \qquad (5.52)$$

where

$$T = (T_2)^2$$

5.4- Numerical results and conclusion

In this section we present numerical examples of dispersion relations of magnetoplasmons in superlattices. We will show that the effect of quasiperiodic layering is to increase the number of bulk bands and surface modes. We also show that the new surface modes are nonreciprocal with respect to propagation direction in the presence of an applied magnetic field.

In order to obtain numerical results we consider the dielectric materials a and c as Si doped with n and p impurities. Since we do not use highly semiconductors, we assume that $\varepsilon_a(\omega) = \varepsilon_c(\omega)$, and the dielectric constant of the Si can be taken as $\varepsilon(\omega) = \varepsilon_L(1-\omega^2/\omega_p^2)$. Where $\varepsilon_L = 11.7$ is the background dielectric constant of the material, and $\omega_p = 7.65\times10^{13}s^{-1}$ which is the electronic plasma frequency and we consider $\varepsilon(\omega)$ independent of the impurity density. The effective mass of the electrons and holes are related to the electron mass m_0 by $m_e^* = 0.2m_0, m_h^* = 0.4m_0$ respectively. We also assume that the dielectrics b and d consists of SiO_2 with dielectric constant $\varepsilon_b = \varepsilon_d = 3.7.$

Fig.5.1 shows the dispersion for the surface modes with an applied magnetic field field given by $\omega_C = 0.004075eV$. Both $\pm k$ are shown, and there are several points of interest.

Fig. 5.2 displays the frequency (ω / ω_p) of the two lower and upper bands of the bulk polaritons as a function of $k_x a$, for superlattice. we plot the dispersion relation for surface modes and bulk polaritons by considering the semiconductor layers (n and p) with 400 A^0 of thickness and the insulators with 200 of thickness and with: $|\sigma_e| = |\sigma_n| = 2 \times 10^{16}$ $carrier / m^2$, $\mu_L = 1.29, \alpha = 8.869 \times 10^{-8} m^2 A^{-2}$. We observed that the existence of four bands tend to crowed together when k increases.

Fig. 5.3 illustrates the dependence of αH_y^2 on the dimensionless coordinate $k_0 z$ for two values of the propagation constant β we see that for $\beta = 3.90$,the maximum point of the curve becomes greater than the other curve when $\beta = 3.5$.

Fig. 5.4 shows the power flow versus wave index for TM surface guided waves at the interface between a nonlinear magnetic cladding and the first unit cell in superlattices . Note that we have three curves according to three different values of α such that the decrease value of α the upper curve becomes.

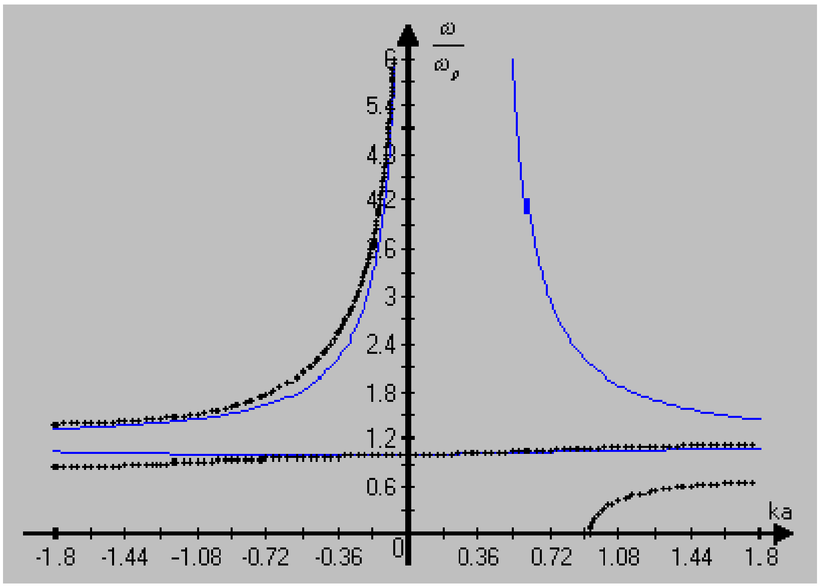

Fig. 5.1 The Computed dispersion curves for surface modes.

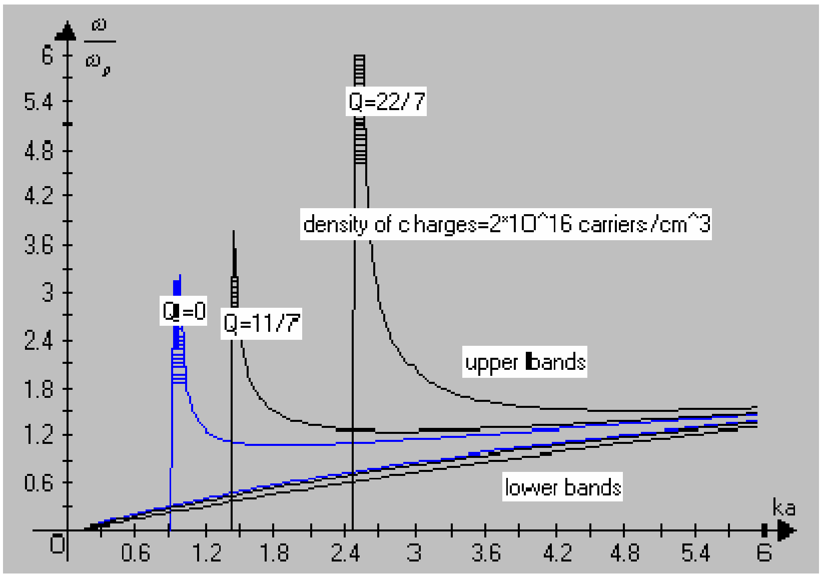

Fig. 5.2 The Computed dispersion curves for bulk modes

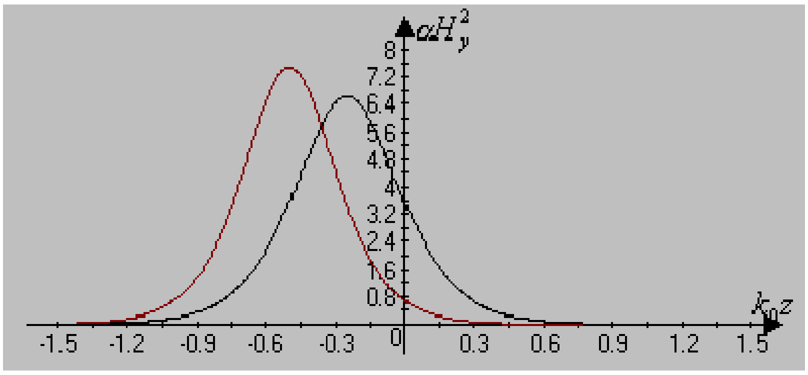

Fig. 5.3 The nonlinearity interface αH_y^2 versus $k_0 z$

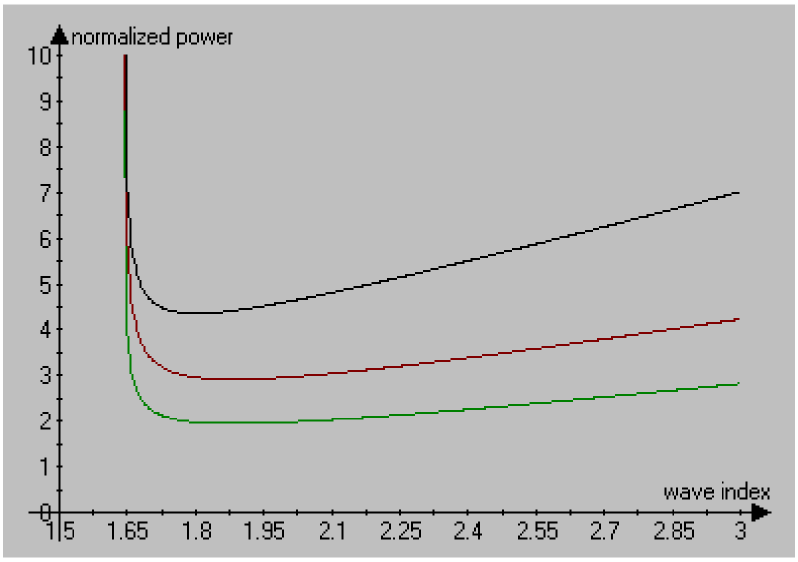

Fig. 5.4 The normalized power flow versus wave index $\beta = k_x / k_0$

Appendices

Appendix A

Here, we want to find the transfer matrix T_1 which gives the field constants A_{n+1} in terms of the constants A_n in superlattices.

The potential Φ and the displacement vector D_z must be continuous at the boundary, now we want to use the boundary conditions at two steps :

The first step: between the last depth of the layer (A_n) and the beginning depth of the layer (B_n).
The second step: between the last depth of the layer (B_n) and the beginning depth of the layer $A_{(n+1)}$.
Then we get the following equations:

$$A_{n1}e^{kdAn} + A_{n2}e^{-kdAn} = B_{n1} + B_{n2}\,, \tag{A.1}$$

$$(\varepsilon_1 + \varepsilon_2)A_{n1}e^{kdAn} + (\varepsilon_2 - \varepsilon_1)A_{n2}e^{-kdAn} = \varepsilon_B(B_{n1} - B_{n2})\,, \tag{A.2}$$

$$B_{n1}e^{kdBn} + B_{n2}e^{-kdBn} = A_{(n+1)1} + A_{(n+1)2}\,, \tag{A.3}$$

and

$$(\varepsilon_1 + \varepsilon_2)A_{(n+1)1} + (\varepsilon_2 - \varepsilon_1)A_{(n+1)2} = \varepsilon_B(B_{n1}e^{kdBn} - B_{n2}e^{-kdBn}) \tag{A.4}$$

From the equation (A.1) we have:

$$B_{n1} = A_{n1}e^{kdAn} - A_{n2}e^{-kdAn} - B_{n2} \tag{A.5}$$

and from the equation (A.2) we have:

$$B_{n1} = \left[\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_B}\right]A_{n1}e^{kdAn} + \left[\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_B}\right]A_{n2}e^{-kdAn} + B_{n2} \tag{A.6}$$

Adding equations (A.5) and (A.6) we have :

$$2B_{n1} = A_{n1}\left[\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_B}+1\right]\cdot e^{kdAn} + A_{n2}\left[\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_B}-1\right]\cdot e^{-kdAn} \qquad \text{(A.7)}$$

In the same manner, from the equation (A.3) we have:

$$B_{n1} = A_{(n+1)1}e^{kdBn} + A_{(n+1)2}e^{-kdBn} - B_{n2}e^{-2kdBn} \qquad \text{(A.8)}$$

and from the equation (A.4) we get:

$$B_{n1} = A_{(n+1)1}\left[\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_B}\right]\cdot e^{kdBn} + A_{(n+1)2}\left[\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_B}\right]\cdot e^{-kdBn} + B_{n2}e^{-2kdBn} \qquad \text{(A.9)}$$

By adding equations (A.8) and (A.9) we have:

$$2B_{n1} = A_{(n+1)1}\left[1+\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_B}\right]\cdot e^{kdBn} + A_{(n+1)2}\left[1+\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_B}\right]\cdot e^{-kdBn} \qquad \text{(A.10)}$$

then from the equations (A.7) and (A.10) we have:

$$A_{n1}(\varepsilon_1+\varepsilon_2+\varepsilon_B)e^{kdBn} + A_{n2}(-\varepsilon_2+\varepsilon_1+\varepsilon_B)e^{-kdBn} = A_{(n+1)1}(\varepsilon_1+\varepsilon_2+\varepsilon_B)e^{+kdBn} + A_{(n+1)2}(\varepsilon_2-\varepsilon_1+\varepsilon_B)e^{-kdBn}$$

(A.11)

Now solving for B_{n2} we get the following relations

from equation (A.1) we have :

$$B_{n2} = A_{n1}e^{kdAn} + A_{n2}e^{-kdAn} - B_{n1} \qquad \text{(A.12)}$$

and from equation (A.2) we find :

$$B_{n2} = B_{n1} - \left[\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_B}\right]A_{n1}e^{kdAn} - A_{n2}\left[\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_B}\right]\cdot e^{-kdAn} \qquad \text{(A.13)}$$

from the equation (A.3) we can write :

$$B_{n2} = \left[A_{(n+1)1} + A_{(n+1)2} - B_{n1}e^{kdBn}\right]\cdot e^{kdBn} \qquad \text{(A.14)}$$

from the equation (A.4) we can write:

$$B_{n2} = \left[B_{n1}e^{kdBn} - \left(\frac{\varepsilon_1+\varepsilon_2}{\varepsilon_B}\right)A_{(n+1)1} - \left(\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_B}\right)A_{(n+1)2}\right]\cdot e^{kdAn} \qquad \text{(A.15)}$$

Adding equations (A.12) and (A.13) we have :

$$2B_{n2} = A_{n1}e^{kdAn} + A_{n2}e^{-kdAn} - \left[\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_B}\right]A_{n1}e^{kdAn} - \left[\frac{\varepsilon_{2-}\varepsilon_1}{\varepsilon_B}\right]A_{n2}e^{-kdAn} \qquad \text{(A.16)}$$

and from the equations (A.14) and (A.15) we have:

$$2B_{n2} = \left[A_{(n+1)1}\left(1 - \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_B}\right) + A_{(n+1)2}\left(1 - \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_B}\right)\right] \cdot e^{kdBn} \qquad \text{(A.17)}$$

Then from equations (A.16) and (A.17) we have:

$$A_{n1}(\varepsilon_B - \varepsilon_1 - \varepsilon_2) \cdot e^{kdAn} + A_{n2}(\varepsilon_B - \varepsilon_2 + \varepsilon_1) \cdot e^{-kdAn} = \left[(\varepsilon_B - \varepsilon_1 - \varepsilon_2)A_{(n+1)1} + (\varepsilon_B - \varepsilon_2 + \varepsilon_1)A_{(n+1)2}\right] \cdot e^{kdBn} \qquad \text{(A.18)}$$

For simplicity, we define the following :

$$\left.\begin{array}{l} w = \varepsilon_B - \varepsilon_2 + \varepsilon_1 \\ x = \varepsilon_B - \varepsilon_1 - \varepsilon_2 \\ y = \varepsilon_B - \varepsilon_1 + \varepsilon_2 \\ z = \varepsilon_B + \varepsilon_2 + \varepsilon_1 \\ e^{B1} = e^{kdBn} \\ e^{B2} = e^{-kdBn} \\ e^{A1} = e^{kdAn} \\ e^{A1} = e^{-kdAn} \end{array}\right\} \qquad \text{(A.19)}$$

Consequently, equations (A.11) and (A.18) can be written as the following:

$$xA_{n1}e^{A1} + wA_{n2}e^{A2} = xA_{(n+1)1}e^{B1} + wA_{(n+1)2}e^{B1} \qquad \text{(A.20)}$$

and

$$zA_{n1}e^{A1} + yA_{n2}e^{A2} = zA_{(n+1)1}e^{B2} + yA_{(n+1)2}e^{B2} \qquad \text{(A.21)}$$

In the matrix form equations (A.20) and (A.21) becomes :

$$\begin{bmatrix} x \cdot e^{B1} & w \cdot e^{B1} \\ z \cdot e^{B2} & y \cdot e^{B2} \end{bmatrix} \cdot \begin{bmatrix} A_{(n+1)1} \\ A_{(n+1)2} \end{bmatrix} = \begin{bmatrix} x \cdot e^{A1} & w \cdot e^{A2} \\ z \cdot e^{A1} & y \cdot e^{A2} \end{bmatrix} \cdot \begin{bmatrix} A_{n1} \\ A_{n2} \end{bmatrix} \qquad \text{(A.22)}$$

which can be written as transfer matrix:

$$\begin{bmatrix} A_{(n+1)1} \\ \\ A_{(n+1)2} \end{bmatrix} = T_1 \cdot \begin{bmatrix} A_{n1} \\ \\ A_{n2} \end{bmatrix} \quad \text{(A.23)}$$

To find T_1, we use the inverse of the matrix

$$\begin{bmatrix} a & b \\ \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ \\ -c & a \end{bmatrix} \quad \text{(A.24)}$$

from equation (A.22) we get :

$$\begin{bmatrix} A_{(n+1)1} \\ \\ A_{(n+1)2} \end{bmatrix} = \frac{1}{xy-wz} \begin{bmatrix} y \cdot e^{B2} & -w \cdot e^{B1} \\ \\ -z \cdot e^{B2} & x \cdot e^{B1} \end{bmatrix} \cdot \begin{bmatrix} x \cdot e^{A1} & w \cdot e^{A2} \\ \\ z \cdot e^{A1} & y \cdot e^{A2} \end{bmatrix} \cdot \begin{bmatrix} A_{n1} \\ \\ A_{n2} \end{bmatrix} \quad \text{(A.25)}$$

Noting that $xy - wz = -4\varepsilon_1\varepsilon_B$ then the equation (A.25) becomes :

$$\begin{bmatrix} A_{(n+1)1} \\ \\ A_{(n+1)2} \end{bmatrix} = \frac{1}{-4\varepsilon_1\varepsilon_B} \begin{bmatrix} xy \cdot e^{A1} \cdot e^{B2} - wz \cdot e^{A1} \cdot e^{B1} & wy \cdot e^{A2} \cdot e^{B2} - wy \cdot e^{A2} \cdot e^{B1} \\ \\ xz \cdot e^{A1} \cdot e^{B1} - xz \cdot e^{A1} \cdot e^{B2} & xy \cdot e^{A2} \cdot e^{B1} - wz \cdot e^{A2} \cdot e^{B2} \end{bmatrix} \cdot \begin{bmatrix} A_{n1} \\ \\ A_{n2} \end{bmatrix} \quad \text{(A.26)}$$

then the matrix T_1 becomes:

$$T_1 = \frac{1}{-4\varepsilon_1\varepsilon_B} \begin{bmatrix} xy \cdot e^{A1} \cdot e^{B2} - wz \cdot e^{A1} \cdot e^{B1} & wy \cdot e^{A2} \cdot e^{B2} - wz \cdot e^{A2} \cdot e^{B1} \\ \\ xz \cdot e^{A1} \cdot e^{B1} - xz \cdot e^{A1} \cdot e^{B2} & xy \cdot e^{A2} \cdot e^{B1} - wz \cdot e^{A2} \cdot e^{B2} \end{bmatrix} \quad \text{(A.27)}$$

Appendix B

In this appendix, we derive another matrix T_2, which gives the field constants B_{n+1} in terms of B_n ones.

In the following equations, we want to eliminate A_{n1} and A_{n2} to find another matrix T_2 such that

$$\begin{bmatrix} B_{(n+1)1} \\ \\ B_{(n+1)2} \end{bmatrix} = T_2 \cdot \begin{bmatrix} B_{n1} \\ \\ B_{n2} \end{bmatrix} \tag{B1}$$

By applying the boundary conditions between the last depth of $A_{(n+1)1}$ and the beginning of $B_{(n+1)1}$, we get:

$$B_{n1}e^{B1} + B_{n2}e^{B2} = A_{(n+1)1} + A_{(n+1)2} \tag{B.2}$$

$$(\varepsilon_1 + \varepsilon_2)A_{(n+1)1} + (\varepsilon_2 - \varepsilon_1)A_{(n+1)2} = \varepsilon_B(B_{n1}e^{B1} - B_{n2}e^{B2}) \tag{B.3}$$

In equations (A.1) and (A.2) when $n \to n+1$, we have :

$$A_{(n+1)1}e^{A1} + A_{(n+1)2}e^{A2} = B_{(n+1)1} + B_{(n+1)2} \tag{B.4}$$

and

$$(\varepsilon_1 + \varepsilon_2)A_{(n+1)1}e^{A1} + (\varepsilon_2 - \varepsilon_1)A_{(n+1)2}e^{A2} = \varepsilon_B(B_{(n+1)1} - B_{(n+1)2}) \tag{B.5}$$

By eliminating A_1 and A_2 from equations (B.2) and (B.3) we find

$$A_{(n+1)1} = B_{n1}e^{B1} + B_{n2}e^{B2} - A_{(n+1)2} \tag{B.6}$$

$$A_{(n+1)1} = \frac{\varepsilon_B}{\varepsilon_1 + \varepsilon_2}\left[B_{n1}e^{B1} - B_{n2}e^{B2}\right] - \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2}A_{(n+1)2} \tag{B.7}$$

So that the equations (B.4) and (B.5) gives:

$$A_{(n+1)1} = (B_{(n+1)1} + B_{(n+1)2}) \cdot e^{A2} - A_{(n+1)2} e^{2A2} \quad \text{(B.8)}$$

$$A_{(n+1)1} = \frac{\varepsilon_B}{\varepsilon_1 + \varepsilon_2} (B_{(n+1)1} - B_{(n+1)2}) \cdot e^{A2} - \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_1 + \varepsilon_2} A_{(n+1)2} e^{2A2} \quad \text{(B.9)}$$

By using the equation (B.7) we can write :

$$\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 - \varepsilon_2} A_{(n+1)1} = \frac{\varepsilon_B}{\varepsilon_1 - \varepsilon_2} (B_{n1} e^{B1} - B_{n2} e^{B2}) + A_{(n+1)2} \quad \text{(B.10)}$$

The equations (B.6) and (B.10) gives the following relation:

$$\left[1 + \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 - \varepsilon_2}\right] A_{(n+1)1} = \left[1 + \frac{\varepsilon_B}{\varepsilon_1 - \varepsilon_2}\right] B_{n1} e^{B1} + \left[1 - \frac{\varepsilon_B}{\varepsilon_1 - \varepsilon_2}\right] B_{n2} e^{B2} \quad \text{(B.11)}$$

From the equation (B.9) we have :

$$\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 - \varepsilon_2} A_{(n+1)1} = \frac{\varepsilon_B}{\varepsilon_1 - \varepsilon_2} B_{(n+1)1} e^{A2} \cdot (B_{(n+1)1} - B_{(n+1)2}) + A_{(n+1)2} e^{2A2} \quad \text{(B.12)}$$

The equations (B.8) and (B.12) gives the following relation:

$$\left[1 + \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 - \varepsilon_2}\right] A_{(n+1)1} = \left[1 + \frac{\varepsilon_B}{\varepsilon_1 + \varepsilon_2}\right] B_{(n+1)1} e^{A2} + \left[1 - \frac{\varepsilon_B}{\varepsilon_1 + \varepsilon_2}\right] B_{(n+1)2} e^{A2} \quad \text{(B.13)}$$

The equations (B.11) and (B.13) gives the following relation:

$$\left[1 + \frac{\varepsilon_B}{\varepsilon_1 - \varepsilon_2}\right] B_{n1} e^{B1} + \left[1 - \frac{\varepsilon_B}{\varepsilon_1 - \varepsilon_2}\right] B_{n2} e^{B2} = \left[1 + \frac{\varepsilon_B}{\varepsilon_1 + \varepsilon_2}\right] B_{(n+1)1} e^{A2} + \left[1 - \frac{\varepsilon_B}{\varepsilon_1 + \varepsilon_2}\right] B_{(n+1)2} e^{A2} \quad \text{(B.14)}$$

By using the equation (A.19), the equation (B.14) can be written as:

$$(\varepsilon_1 + \varepsilon_2)(\varepsilon_1 - \varepsilon_2 + \varepsilon_B) B_{n1} e^{B1} + (\varepsilon_1 + \varepsilon_2)(\varepsilon_1 - \varepsilon_2 - \varepsilon_B) B_{n2} e^{B2} =$$
$$(\varepsilon_1 - \varepsilon_2)(\varepsilon_1 + \varepsilon_2 + \varepsilon_B) B_{(n+1)1} e^{A2} + (\varepsilon_1 - \varepsilon_2)(\varepsilon_1 + \varepsilon_2 - \varepsilon_B) B_{(n+1)2} e^{A2} \quad \text{(B.15)}$$

The equation (B.2) give the following relation:

$$A_{(n+1)2} = B_{n1}e^{B1} + B_{n2}e^{B2} - A_{(n+1)1} \quad \text{(B.16)}$$

By the same way, the equation (B.3) give the following relation:

$$A_{(n+1)2} = \frac{\varepsilon_B}{\varepsilon_2 - \varepsilon_1}\left\{B_{n1}e^{B1} - B_{n2}e^{B2}\right\} - \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_2 - \varepsilon_1}A_{(n+1)1} \quad \text{(B.17)}$$

By using the equation (B.4), we have :

$$A_{(n+1)2} = B_{(n+1)1}e^{A1} + B_{(n+1)2}e^{A1} - A_{(n+1)1}e^{2A1} \quad \text{(B.18)}$$

$$A_{(n+1)2} = \frac{\varepsilon_B}{\varepsilon_2 - \varepsilon_1}\left\{B_{(n+1)1} - B_{(n+1)2}\right\}e^{A1} - \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_2 - \varepsilon_1}A_{(n+1)1}e^{2A1} \quad \text{(B.19)}$$

The equation (B.17) can be written as:

$$\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}A_{(n+1)2} = \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\left\{B_{n1}e^{B1} - B_{n2}e^{B2}\right\} - A_{(n+1)1} \quad \text{(B.20)}$$

Eliminating $A_{(n+1)1}$ from the equations (B.20) and (B.16) the result is:

$$\left\{1 + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_2 + \varepsilon_1}\right\}A_{(n+1)2} = \left\{1 - \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{n1}e^{B1} + \left\{1 + \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{n2}e^{B2} \quad \text{(B.21)}$$

The equation (B.19) can be written as:

$$\left\{\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_2 + \varepsilon_1}\right\}A_{(n+1)2} = \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\left\{B_{(n+1)1} - B_{(n+1)2}\right\}\cdot e^{A1} - A_{(n+1)1}e^{2A1} \quad \text{(B.22)}$$

The equations (B.18) and (B.22) gives the following result :

$$\left\{1 + \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_2 + \varepsilon_1}\right\}A_{(n+1)2} = \left\{1 - \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{(n+1)1}e^{A1} + \left\{1 - \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{(n+1)2}e^{A1} \quad \text{(B.23)}$$

Eliminating $A_{(n+1)2}$ from the equations (B.21) and (B.23) we have :

$$\left\{1 - \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{n1}e^{B1} + \left\{1 + \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{n1}e^{B2} = \left\{1 - \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{(n+1)1}e^{A1} + \left\{1 + \frac{\varepsilon_B}{\varepsilon_2 + \varepsilon_1}\right\}B_{(n+1)2}e^{A1} \quad \text{(B.24)}$$

We use the equations (A.19) and (B.24) to find :

$$-xB_{n1}e^{B1} + zB_{n2} = -xB_{(n+1)1}e^{A1} + zB_{(n+1)2}e^{A1} \quad \text{(B.25)}$$

In the matrix form, the equations (B.25) and (B.15) can be written as:

$$\begin{bmatrix} -x\cdot e^{A1} & z\cdot e^{A1} \\ (\varepsilon_1-\varepsilon_2)z\cdot e^{A2} & -(\varepsilon_1-\varepsilon_2)\cdot e^{A2} \end{bmatrix}\cdot\begin{bmatrix} B_{(n+1)1} \\ B_{(n+1)2} \end{bmatrix}=\begin{bmatrix} -x\cdot e^{B1} & z\cdot e^{B2} \\ w(\varepsilon_1+\varepsilon_2)\cdot e^{B1} & -y(\varepsilon_1+\varepsilon_2)\cdot e^{B2} \end{bmatrix}\cdot\begin{bmatrix} B_{n1} \\ B_{n2} \end{bmatrix} \quad \text{(B.26)}$$

Now the matrix T_2 comes directly from the equation (B.26) as:

$$T_2 = R\begin{bmatrix} x^2(\varepsilon_1-\varepsilon_2)e^{A2+B1}-zw(\varepsilon_1+\varepsilon_2)e^{A1+B1} & zy(\varepsilon_1+\varepsilon_2)e^{B2+A1}-xz(\varepsilon_1-\varepsilon_2)e^{A2+B2)} \\ xz(\varepsilon_1-\varepsilon_2)e^{B1+A2}-xw(\varepsilon_1+\varepsilon_2)e^{A1+B1} & xy(\varepsilon_1+\varepsilon_2)e^{B2+A1}-z^2(\varepsilon_1-\varepsilon_2)e^{A2+B2} \end{bmatrix} \quad \text{(B.27)}$$

where

$$R=\frac{1}{(\varepsilon_1-\varepsilon_2)(x^2-z^2)}$$

Now we want to find T, the transfer matrix between the cells $(n,\ n+1)$ such that we have just two layers $\varepsilon_A, \varepsilon_B$ in each cell :

$$T = T_2 T_1 \quad \text{(B.28)}$$

The constants between the cells $(n,\ n+1)$ are :

$$\begin{bmatrix} C_{(n+1)1} \\ C_{(n+1)2} \end{bmatrix} = T\cdot\begin{bmatrix} C_{n1} \\ C_{n2} \end{bmatrix} \quad \text{(B.29)}$$

where the transfer matrix T gives the dispersion relation :

$$\cos\left[Q_n(d_A+d_B)\right]=\frac{1}{2}tr(T) \quad \text{(B.30)}$$

where $d_A + d_B =$ the depth of the cell .

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