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A MODEL OF DRIFT WAVES MODIFIED BY ELECTRON TEMPERATURE GRADIENTS AND $\vec{E}\times\vec{B}$ ROTATION IN CYLINDRICAL MAGNETIZED PLASMA.

By

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ABSTRACT

A general investigation of linear drift-waves phenomena in cylindrically bounded plasmas, immersed in a magnetic field without sheared and curvature, is described by the two-fluid equations, taking into account: (a) electron-temperature oscillation, (b)the radial variation of density and temperature, (c)the radial electric field $\vec{E}$*; and* $\vec{E}\times\vec{B}$ *rotation, as well as (d)the electron motion parallel to the magnetic field lines. For plasmas in which the electron temperature strongly exceeds the ion temperature the problem is reduced to an ordinary complex second-order differential equation describes the radial distribution of the oscillating electric potential. It is shown that the presence of the* $\vec{E}\times\vec{B}$ *rotation and the radial gradients in the undisturbed electron temperature leads to an important modification of the theory of drift waves in cylindrical plasma compared with previous models in which these phenomena were disregarded. The theory is applied to an experimental data of helium plasma using Runge-Kutta integration method. Our calculation shows that the temperature variation and the* $\vec{E}\times\vec{B}$ *rotation are important in the predictions of drift wave frequency and radial position of the maximum wave amplitude.*

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CHAPTER 1

INTRODUCTION

The plasma is sometimes referred to as the “fourth” state of matter. When a solid is heated sufficiently that the thermal motion of the atoms break the crystal lattice structure apart, usually a liquid is formed. When a liquid is heated enough that atoms vaporize off the surface faster than they recondense, a gas is formed. When a gas is heated enough that the atoms collide with each other and knock their electrons off in the process, a plasma is formed [11, 23].

Any ionized gas cannot be called a plasma; there is always some degree of ionization in any gas. A useful definition is:

A plasma is a quasi-neutral gas of charged and neutral particles which exhibits collective behavior. Quasi-neutral means that, the ion density and electron density are equal to each other. Collective behavior means that, the charged particles can generate local concentrations of positive or negative charge, which give rise to electric fields. Motion of charges also generates currents, and hence magnetic fields. These fields affect the motion of other particles away [2].

1.1 EQUILIBRIUM AND STABILITY

Since a moving charged particle in a magnetic field performs a spiral motion along a magnetic lines of force, this alter is used to confine the plasma. We need only make sure that the lines of force do not hit the vacuum wall and arrange the symmetry of the system in such a way that all the particle drifts $\vec{V}_E$ and $\vec{V}_{\nabla B}$, and so forth are parallel to the walls. In fact, the situation is totally different in confining a plasma, which has charged particles. As these charges move around, they can generate local concentration of positive or negative charge, which gives rise to electric fields $\vec{E}$, and then cause $\vec{E}\times\vec{B}$ drifts to the wall. Motion of charged particles also generate currents, and hence magnetic fields $\vec{B}$ which causes grad-B drifts outward [2].

We can divide the problem of confinement into two parts: The problem of equilibrium and the problem of stability. An equilibrium is a state in which all the forces are balanced, so that a time-independent solution to the equations describing the interaction between electromagnetic fields and plasmas is possible. The equilibrium is stable or unstable according to whether small perturbations are damped or amplified. A rubber ball at the bottom of a stick is in a stable equilibrium state (Fig. 1-1a). After any slight displacement of the ball, it will return to the bottom of the stick. On the other hand, a rubber ball on the top of a valley is in an unstable equilibrium. Any slight displacement of the ball will result in the destruction of the equilibrium state (Fig. 1-1b). In any equilibrium system the criterion for stability is as follows: If a small perturbation from the equilibrium state is followed by a return to equilibrium by a nongrowing oscillation about the equilibrium state, the system is stable; if a small perturbation from the equilibrium state results in a growth of the perturbation, the system is unstable [2, 25].

We shall consider some fluid plasma states that are not in perfect thermodynamic equilibrium, although they are in equilibrium in the sense that all forces are in balance and a time-independent solution is possible. The free energy which is available can cause waves to be self-excited; the equilibrium then is unstable one. An instability is always a motion which decrease the free energy and brings the plasma closer to true thermodynamic equilibrium [2].

FIG. 1-1 MECHANICAL ANALOGY OF THE TYPES OF EQUILIBRIUM (CHEN, 1984).

In plasmas instabilities may be classified according to the type of free energy available to drive them [2, 12]:

1. Streaming instabilities: In this case, either a beam of energetic particles travels through the plasma, or a current is driven through the plasma so that the different species have drift relative to one another. The drift energy is used to excite waves.

2. Rayleigh-Taylor instabilities: In this case, the plasma has a density gradient or sharp boundary, so that it is not uniform. An external nonelectromagnetic force is also applied to the plasma, and this force can drive the instability.
3. Universal instabilities: In laboratory conditions free energy can occur naturally in any finite plasma, e.g. gradients in temperature, density and pressure are inevitable. These tend to make the expansion of the plasma, and the expansion energy can drive an instability. This type of free energy is always present in any finite plasma, and the resulting waves are called “universal instabilities”. The ubiquitous appearance of drift waves and their deleterious effect on plasma confinement have made it a prime candidate for both theoretical and experimental study [8, 13, 17, 20, 27].

To check the equilibrium state of the plasma whether stable or not, we should find a solution to the dispersion relation which connect the frequency w and the wave-vector $\vec{k}$. If w is real, the plasma is stable, otherwise w is complex, and then there is an instability. For instance consider the spatial and temporal variation of a one-dimensional plane wave is in the form $\exp[i(k_x x - wt)]$. Let w be a complex quantity given by

$$w = a + ig \qquad (1\text{-}1)$$

Where a and g are $R_e(w)$ and $I_m(w)$, respectively. Under this condition the spatial and temporal variation of the wave becomes

$$\exp(ik_x x - ia\,t + g\,t) \qquad (1\text{-}2)$$

The propagation exponent contains a real quantity $g\,t$. If $g > 0$, the wave will increase in amplitude as time increases (instability state). If $g < 0$, the wave will decrease in amplitude as time increases (stability state). So g is called the growth rate,

$$\text{Growth rate} = g = I_m(w) \qquad (1\text{-}3)$$

The out line of this work is as follows: The second chapter describes the streaming instabilities including a detail study of their dispersion relation. An example about this relation is given for two streams with different velocities in the same direction. Another example treats two streams with the same velocity but in opposite directions. A third example studies another two streams, one is stationary and the other has some velocity.

Chapter three is concerned with the Rayleigh-taylor instabilities. This chapter illustrates the physical mechanism of the Rayleigh-taylor instabilities. The growth rate of the instabilities, the gravitational instability as well as, the flute instability due to field curvature are also studied.

In the fourth chapter, the theory of the universal instabilities is derived on the bases of the two-fluid hydrodynamical equations in order to obtain a model of a drift wave. A second order differential equation is also formulated from the theory. This equation will include the electron-temperature oscillation, the gradients in density and temperature as well as, the $\vec{E}\times\vec{B}$ rotation and the electron motion parallel to the magnetic field lines. It will be shown that the presence of the $\vec{E}\times\vec{B}$ rotation and the radial gradients in electron temperature leads to a modification of the theory previously formulated in different papers [8, 17, 21]. The theory will be applied to experimental data from Egger et al. (1986). Numerical results using Runge-Kutta integration method and shooting method will be given at the end of this chapter, in addition to the conclusions.

1.2 OBJECTIVE OF THE STUDY

The main objective of this study is to show that, the drift wave frequency and the radial distribution of the oscillating electric potential of a cylindrical magnetized plasma are influenced by the gradients in electron temperature, and the $\vec{E}\times\vec{B}$ rotation. To reach the objective:

- The theory is formulated by two-fluid hydrodynamical equations, taking into account the electron-temperature oscillation, the radial variation of density and temperature, the $\vec{E}\times\vec{B}$ rotation, and the electron motion parallel to the magnetic field lines.
- This theory is obtained in the form of a second order differential equation for the oscillating electric potential as an eigenfunction and the drift wave frequency as an eigenvalue.
- Comparison between the theoretical model and an experimental data of helium plasma from Egger et al. (1986) using Runge-Kutta integration method will be carried out throughout this work.

CHAPTER 2

STREAMING INSTABILITY

In this chapter we shall consider a plasma that is composed of two or more streams of particles with different velocities. For simplicity, we shall assume that each stream is of zero temperature. This leads to the important concept of "two-stream instability" in its simplest form.

2.1 PARTICLE STREAMS OF ZERO TEMPERATURE

Consider a collection of cold interpenetrating plasma streams with the goal of considering the instabilities that can arise in such systems. We consider different "species" of particles denoted by s (s= 1, 2, …), particles of species s have mass m_s and charge q_s. We use the procedure of linearization, by this we mean that the amplitude of oscillation is small and terms containing higher powers of amplitude factors can be neglected. We first separate the dependent variables into two parts: an "equilibrium" part indicated by a subscript o, and a "perturbation" part indicated by a subscript 1. We assume that, in the equilibrium state, the density and velocity of species s are n_{s^o} and $\vec{V}_{s^o}$ respectively. We consider the perturbations as,

$$n_{s^o} \rightarrow n_{s^o} + n_{s^1} \qquad (2\text{-}1)$$

$$\vec{V}_{s^o} \rightarrow \vec{V}_{s^o} + \vec{V}_{s^1}$$

We also assume a quasi-neutrality that is

$$\sum_s n_{s^o} q_{s^o} = 0 \,, \qquad (2\text{-}2)$$

so that we may assume that there is no electric field in the unperturbed state. Then the only electric field that exist is $\vec{E}_1$, that satisfies Poisson's equation:

$$e_o \nabla \cdot \vec{E}_1 = \sum_s q_s n_{s^1} \qquad (2\text{-}3)$$

We assume that there is no magnetic field in the unperturbed state, and we consider nonrelativistic velocities, so that the effect of the magnetic field produced by the particle streams may be neglected [23]. Then the equation of motion is [22]

$$\frac{\partial \vec{V}_s}{\partial t} + \left(\vec{V}_s \cdot \nabla\right)\vec{V}_s = \frac{q_s}{m_s} E_1 \qquad (2\text{-}4)$$

Hence, we obtain the following equation for the perturbation $\vec{V}_{s^1}$:

$$\frac{\partial \vec{V}_{s^1}}{\partial t} + \left(\vec{V}_{s^o} \cdot \nabla\right)\vec{V}_{s^1} = \frac{q_s}{m_s} \vec{E}_1 \qquad (2\text{-}5)$$

In order to complete our system of equations, we also need to include the continuity equation [5],

$$\frac{\partial n_s}{\partial t} + \vec{\nabla} \cdot \left(n_s \vec{V}_s\right) = 0 \qquad (2\text{-}6)$$

The linearized form of this equation is

$$\frac{\partial n_{s^1}}{\partial t} + \vec{V}_{s^o} \cdot \vec{\nabla} n_{s^1} + n_{s^o} \vec{\nabla} \cdot \vec{V}_{s^1} = 0 \qquad (2\text{-}7)$$

We consider a plane electrostatic wave of the form

$$y_{s^1} = y_s e^{i(kx - wt)} \hat{x} \qquad (2\text{-}8)$$

where y_{s^1} represents $n_{s^1}, \vec{V}_{s^1}, and\ E_1$.

The time derivative $\frac{\partial}{\partial t}$ can therefore be replaced by $-iw$, and the gradient $\vec{\nabla}$ by $i\vec{k}$.

Equations (2-3), (2-5) and (2-7) now becomes

$$ie_o\left(\vec{k} \cdot \vec{E}_1\right) = \sum_s q_s n_{s^1} \qquad (2\text{-}9)$$

$$-iw\vec{V}_{s^1} + i\left(\vec{k} \cdot \vec{V}_{s^o}\right)\vec{V}_{s^1} = \frac{q_s}{m_s}\vec{E}_1 \qquad (2\text{-}10)$$

and

$$-iw\, n_{s^1} + i\left(\vec{k} \cdot \vec{V}_{s^o}\right) n_{s^1} + i n_{s^o} \vec{k} \cdot \vec{V}_{s^1} = 0 \qquad (2\text{-}11)$$

Equation (2-10) leads to the following expression for the velocity perturbation in terms of the electric-field perturbation:

$$\vec{V}_{s^1} = \frac{\frac{q_s}{m_s}\vec{E}_1}{-i\left(w - \vec{k} \cdot \vec{V}_{s^o}\right)} \qquad (2\text{-}12)$$

Equation (2-11) leads to

$$n_{s^1} = \frac{n_{s^o} \vec{k} \cdot \vec{V}_{s^1}}{w - \vec{k} \cdot \vec{V}_{s^o}} \qquad (2\text{-}13)$$

using equation (2-12), we obtain

$$n_{s^1} = \frac{n_{s^o} \frac{q_s}{m_s} \vec{k} \cdot \vec{E}_1}{-i\left(w - \vec{k} \cdot \vec{V}_{s^o}\right)^2} \tag{2-14}$$

Substituting (2-14) into (2-9), we obtain

$$ie_o\left(\vec{k} \cdot \vec{E}_1\right) = i\left\{\sum_s \frac{q_s{}^2 n_{s^o}}{m_s} \frac{1}{\left(w - \vec{k} \cdot \vec{V}_{s^o}\right)^2}\right\} \vec{k} \cdot \vec{E}_1 \tag{2-15}$$

that is

$$\left\{\sum_s \frac{w^2{}_{ps}}{\left(w - \vec{k} \cdot \vec{V}_{s^o}\right)^2} - 1\right\} \vec{k} \cdot \vec{E}_1 = 0 \tag{2-16}$$

where $w_{ps} = \left(\frac{n_{s^0} q_s{}^2}{e_0 m_s}\right)^{\frac{1}{2}}$ is the plasma natural frequency. The dispersion relation is found upon dividing equation (2-16) by $\vec{k} \cdot \vec{E}_1$:

$$\sum_s \frac{w_{ps}{}^2}{\left(w - \vec{k} \cdot \vec{V}_{s^o}\right)^2} = 1 \tag{2-17}$$

If we consider that all streams have zero velocity, this dispersion relation becomes:

$$w^2 = \sum_s w_{ps}{}^2 \tag{2-18}$$

Now consider only a single stream, say an electron stream, then the relation (2-17) becomes

$$\left(w - \vec{k} \cdot \vec{V}_{e^o}\right)^2 = w_{pe}{}^2 \tag{2-19}$$

It is clear that the group velocity $\frac{\partial w}{\partial k}$ is equal to $\vec{V}_{e^o}$, the unperturbed velocity of the electron stream. Hence we find, once more, that simple plasma oscillation propagate with the electron plasma.

2.2 TWO-STREAM INSTABILITY

Now suppose that we need to consider only two streams of particles. These may be a stream of electrons and a stream of ions, of equal but opposite charge density, with different velocities. On the other hand, we might consider two streams of electrons, limiting our attention to sufficiently high frequencies that the motion of the background ions may be neglected. We also consider only waves with wave vectors parallel to the velocities of the streams V_1, V_2. Hence the problem has become one dimensional, and the dispersion relation (2-17) becomes [23]

$$\frac{w_{p1}{}^2}{(w - V_1 k)^2} + \frac{w_{p2}{}^2}{(w - V_2 k)^2} = 1 \tag{2-20}$$

Since this is a quatric equation in w, there is no simple general expression for w in terms of k. If we introduce the phase velocity V_f defined by

$$V_f = \frac{w}{k} \tag{2-21}$$

the dispersion relation becomes

$$\frac{w_{p1}{}^2}{k^2 (V_f - V_1)^2} + \frac{w_{p2}{}^2}{k^2 (V_f - V_2)^2} = 1 \tag{2-22}$$

This equation may be expressed alternatively as

$$G(v_f) \equiv \frac{w_{p1}{}^2}{(V_f - V_1)^2} + \frac{w_{p2}{}^2}{(V_f - V_2)^2} = k^2 \tag{2-23}$$

since this equation is a fourth order equation with real coefficients, there will be four solutions for V_f that can be arranged in two pairs, each pair being a complex-conjugate pair.

The general form of the function $G(V_f)$ is shown in Fig. 2-1. This function will have singularities at $V_f = V_1 \, and \, V_f = V_2$. We see that there is a critical value of k, that we label k_c. For $k^2 > k_c{}^2$, equation (2-23) has four real roots so that the system is in a stable state. On other hand, if $k^2 < k_c{}^2$, equation (2-23) has only two real roots. Hence the other two roots must be complex, corresponding to complex frequencies. Since these two frequencies are complex conjugates, one of them will correspond to an unstable wave. Hence any small perturbation of the system will grow to arbitrary large values. Thus, the system will be unstable, and it is called “two-stream instability”.

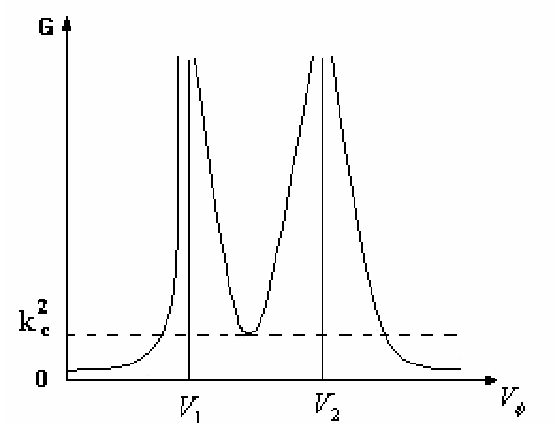

FIG. 2-1 SCHEMATIC REPRESENTATION OF THE FUNCTION $G(V_f)$ DEFINED BY EQUATION (2-23) (AFTER P. A. STURROCK, 1994).

We may find the value k_c by finding the value of V_f for which the derivative $\frac{dG}{dV_f}$ is zero. This is found to be

$$V_{f,c} = \frac{w_{p1}^{\frac{2}{3}} V_2 + w_{p2}^{\frac{2}{3}} V_1}{w_{p1}^{\frac{2}{3}} + w_{p2}^{\frac{2}{3}}} \tag{2-24}$$

from which we find that

$$k_c^{\ 2} = \frac{\left(w_{p1}^{\frac{2}{3}} + w_{p2}^{\frac{2}{3}} \right)^3}{(V_1 - V_2)^2} \tag{2-25}$$

From this relation we see that as $(V_1 - V_2)$ becomes arbitrary small, k_c becomes arbitrarily large. This indicates that the system becomes unstable for a larger and larger range of wave number. This does not make much physical sense, since $(V_1 - V_2)$ is the source of energy driving the instability. The difficulty comes from our use of the fluid equations. Any real plasma has a finite temperature, and thermal effects should be taken into account by a kinetic-theory treatment. A phenomenon known as Landau damping will then occur for $(V_1 - V_2) \le V_{th}$ where V_{th} is the thermal velocity, and no instability is predicted if $(V_1 - V_2)$ is too small [2].

2.3 TWO IDENTICAL BUT OPPOSING STREAMS

If the streams have the same density and equal but opposite velocities, we may adopt [5]

$$V_1 = -V_2 = V, \quad w_{p1} = w_{p2} = w_p \tag{2-26}$$

So that equation (2-17) becomes

$$\frac{w_p^{\ 2}}{(w - Vk)^2} + \frac{w_p^{\ 2}}{(w + Vk)^2} = 1 \tag{2-27}$$

This is now a quadratic equation in w^2:

$$2w_p^{\ 2}\left(w^2 + V^2k^2\right) = \left(w^2 - V^2k^2\right)^2 \tag{2-28}$$

rearranging this equation we obtain

$$w^4 - 2\left(w_p^{\ 2} + V^2k^2\right)w^2 - 2w_p^{\ 2}V^2k^2 + V^4k^4 = 0 \tag{2-29}$$

Hence we obtain the following expression for w^2:

$$w^2 = w_p^{\ 2} + V^2k^2 \pm w_p\left(w_p^{\ 2} + 4V^2k^2\right)^{\frac{1}{2}} \tag{2-30}$$

This relationship between w^2 and k^2 is shown graphically in Fig. 2-2. We see that, for $k^2 > k_c^{\ 2}$, all values of w are real. However, for $k^2 < k_c^{\ 2}$, two values of w will be complex, and one of these will represent a growing wave, that is, an instability. We find that

$$k_c = \frac{\sqrt{2}w_p}{V} \tag{2-31}$$

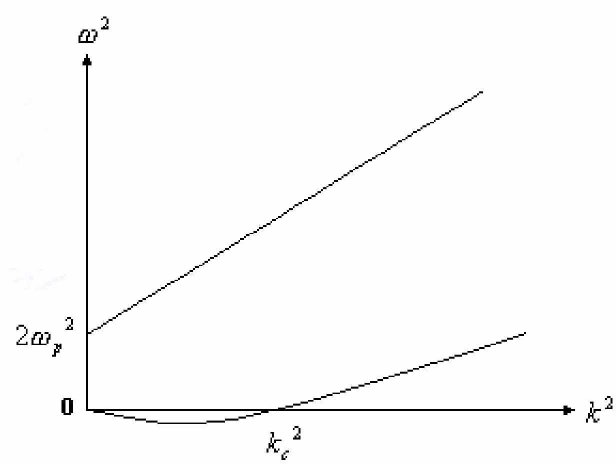

FIG. 2-2 REPRESENTATION OF RELATIONSHIP BETWEEN w^2 AND k^2 GIVEN BY EQUATION (2-30) (AFTER P. A. STURROCK, 1994).

Once again, we see that the range of unstable wave numbers becomes increasingly large as V becomes small.

We may find the wave number k_m that corresponds to the most unstable mode by finding the value of k^2 for which $\frac{dw^2}{dk^2}=0$. Hence, we obtain from equation (2-30)

$$k_m = \frac{\sqrt{3}}{2}\frac{w_p}{V} \tag{2-32}$$

Also it was reported by P. A. Sturrock, 1994 that the maximum imaginary value of the frequency $I_m(w)$ is given by

$$I_m(w) = \frac{1}{2}w_p, \tag{2-33}$$

which represents the maximum growth rate predicted by equation (2-30).

2.4 ELECTRON STREAM MOVING THROUGH STATIONARY IONS

Consider a uniform plasma in which the ions are stationary and the electrons have a velocity v relative to the ions. Then the dispersion relation (2-17) becomes [2, 5]

$$\frac{{w_{pi}}^2}{w^2} + \frac{{w_{pe}}^2}{(w-Vk)^2} = 1 \tag{2-34}$$

Substituting about the plasma natural frequencies w_{pi}, w_{pe}, this equation becomes:

$$ {w_{pe}}^2\left[\frac{m_e/m_i}{w^2} + \frac{1}{(w-Vk)^2}\right] = 1 \tag{2-35}$$

Let us see whether oscillation with real k are stable or unstable. Upon multiplying through by the common denominator, one would obtain a fourth-order equation for w. If all roots w_j are real, each root would indicate a possible oscillation

$$\vec{E}_1 = E\, e^{i(kx - w_j t)}\hat{x}$$

If some of the roots are complex, they will occur in complex conjugate pairs. Let these complex roots be written as

$$w_j = a_j + ig_j$$

The time dependence is now given by

$$\vec{E}_1 = E\,e^{i(kx - a_j t)} e^{g_j t}\,\hat{x}$$

Positive $I_m(w)$ indicates an exponentially growing wave; negative $I_m(w)$ indicates a damped wave. Since the roots w_j occur in conjugate pairs, one of these will always be unstable unless all the roots are real. The damped roots are not self-excited and not of interest.

The dispersion relation (2-35) can be analyzed without actually solving the fourth-order equation. Let us define

$$x \equiv \frac{w}{w_{pe}}, \quad y \equiv \frac{Vk}{w_{pe}} \tag{2-36}$$

Then equation (2-35) becomes

$$F(x, y) \equiv \frac{m_e/m_i}{x^2} + \frac{1}{(x-y)^2} = 1 \tag{2-37}$$

For any given value of y, we can plot $F(x, y)$ as a function of x. This function will have singularities at $x = 0$ and $x = y$ (Fig. 2-3). The intersections of this curve with the line $F(x, y) = 1$ give the values of x satisfying the dispersion relation. In the case depicted in Fig 2-3, there are four intersections, so that there are four real roots w_j. However, if we choose a smaller value of y, the graph would look as shown in Fig. 2-4. Now there are only two intersections and, therefore, only two real roots. The other roots must be complex, and one of them corresponds to an unstable wave. Thus for sufficiently small Vk, the plasma is unstable. For any given V, the plasma is always unstable to long-wavelength oscillations. The maximum growth rate is predicted by equation (2-35) for $\frac{m_e}{m_i} << 1$ as [2]

$$I_m\left(\frac{w}{w_p}\right) \approx \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \tag{2-38}$$

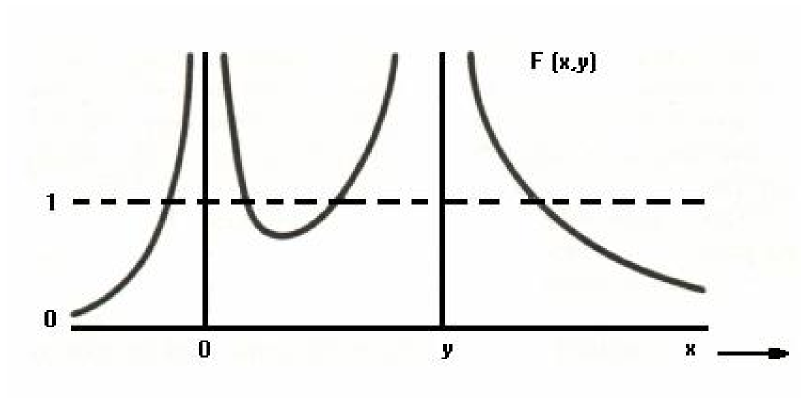

FIG. 2-3 THE FUNCTION $F(x, y)$ IN THE TWO-STREAM INSTABILITY, WHEN THE PLASMA IS STABLE (AFTER CHEN, 1984 AND DENDY, 1993).

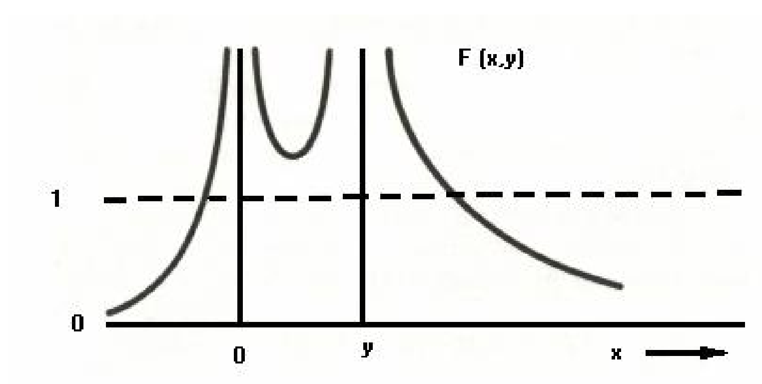

FIG. 2-4 THE FUNCTION $F(x, y)$ IN THE TWO-STREAM INSTABILITY, WHEN THE PLASMA IS UNSTABLE (AFTER CHEN, 1984 AND DENDY, 1993)

CHAPTER 3

RAYLEIGH-TAYLOR INSTABILITY

3.1 PHYSICAL MECHANISM OF THE RAYLEIGH-TAYLOR INSTABILITY

The instability of a heavy fluid supported by a light fluid against the gravitational field is a classical problem in hydrodynamics. An analogy is available in the example of an inverted glass of water (Fig. 3-1). Although the plane interface between the water and air is in a state of equilibrium in that the weight of the water is supported by the air pressure, it is an unstable equilibrium. Any ripple in the surface will tend to grow at the expanse of potential energy in the gravitational field. A similar problem arises in a plasma (acts as a heavy fluid) supported by a magnetic field (a light fluid) against gravity or some other force field. Such instabilities are generally categorized as the Rayleigh-Taylor instability. These involving gravity are in particular called the gravitational instability [2, 14].

FIG. 3-1 HYDRODYNAMIC RRAYLEIGH-TAYLOR INSTABILITY OF A HEAVY FLUID SUPPORTED BY A LIGHTER ONE (AFTER CHEN, 1984).

To treat the simplest case, we consider a plasma that is only non-uniform in the x direction and is immersed in a uniform magnetic field in the z direction. To be specific, we suppose that the density gradient $\vec{\nabla} n_0$ is in the negative x direction and the gravitational field $\vec{g}$ is opposite to it, i.e. in the positive x direction. This corresponds to the case of a dense plasma supported against gravity by a magnetic field, as shown in

Fig. 3-2. For simplicity we may let $k_B T_i = k_B T_e = 0$, where k_B is Boltzmann's constant, T_i and T_e are the ion and electron temperatures.

The physical mechanism at work in the Rayleigh-Taylor instability can be understood in terms of the gravitational drifts of the ions and electrons. An external force $\vec{F}$ (such as a gravitational force $\vec{F} = m\vec{g}$) perpendicular to a magnetic field $\vec{B}$ causes a charged particle (in particular, an ion with charge +e) to drift with a velocity [12]

$$\vec{V}_0 = \frac{m_i \vec{g} \times \vec{B}}{eB^2} = -\frac{m_i g}{eB}\hat{y} \tag{3-1}$$

The electrons have an opposite drift which can be neglected in the limit $\frac{m_e}{m_i} \to 0$.

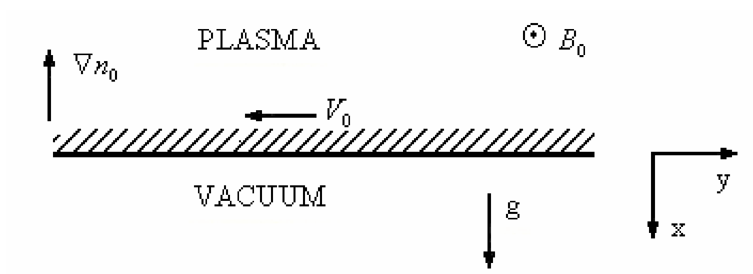

FIG 3-2 A PLASMA SURFACE SUBJECT TO A GRAVITATIONAL INSTABILITY (AFTER SETSUO ICHIMARU, 1986).

Suppose a small wave like ripple should develop on a plasma-vacuum interface as a result of random fluctuations, as shown in Fig. 3-3. The gravitational drift of ions on the plasma side of the interface will cause positive charge to build up on one side of the ripple, as illustrated in Fig. 3-4; the depletion of ions causes a negative charge to build up on other side of the ripple. Due to this separation of charges, a small electric field $\vec{E}_1$ develops, and this electric field changes sign going from crest to trough of the perturbation, again, as shown in Fig. 3-4. It is apparent that the resulting $\vec{E}_1 \times \vec{B}_0$ drift is always upward in those regions where the interface has already moved upward, and downward in those regions where the interface has already moved downward. Thus the initial ripple grows larger, as a result of $\vec{E} \times \vec{B}$ drifts that are phased so as to amplify the initial perturbation [12].

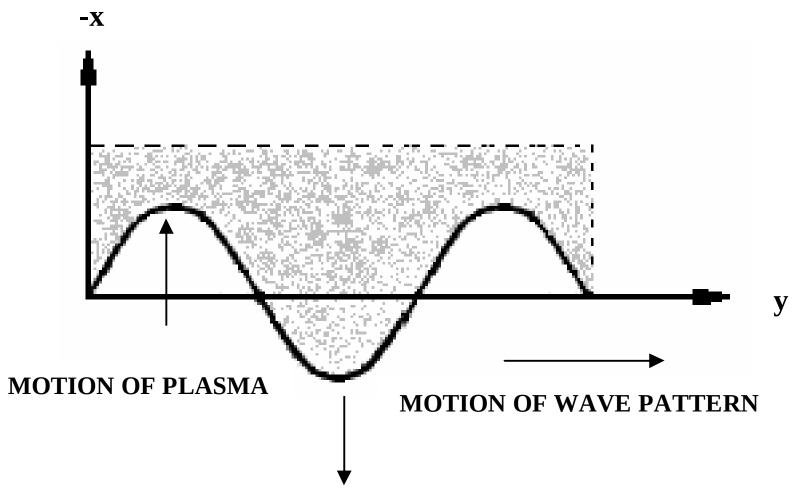

FIG. 3-3 A WAVE-LIKE PERTURBATION OF THE PLASMA-VACUUM INTERFACE SHOWN IN FIG.3-2 (AFTER R. S. GOLDSTON AND P. H. RUTHERFORD, 1995).

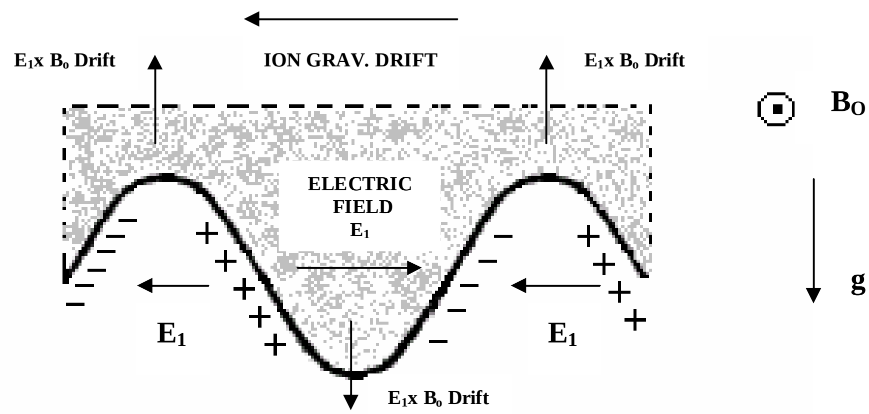

FIG. 3-4 THE MECHANISM OF THE RAYLEIGH-TAYLOR INSTABILITY. THE ION GRAVITATIONAL DRIFT LEADS TO CHARGE SEPARATION ON THE PLASMA-VACUUM INTERFACE, PRODUCING ELECTRIC FIELDS AND $\vec{E} \times \vec{B}$ DRIFTS THAT INCREASE THE AMPLITUDE OF THE PERTURBATION (AFTER R. S. GOLDSTON AND P. H. RUTHERFORD, 1995).

3.2 THE GROWTH RATE OF THE INSTABILITY

In the equilibrium state, the ions obey the equation [2]

$$m_i n_0 (\vec{V}_0 \cdot \vec{\nabla})\vec{V}_0 = e n_0 \vec{V}_0 \times \vec{B}_0 + m_i n_0 \vec{g} \tag{3-2}$$

To find the growth rate, we can perform the usual linearized wave analysis for waves propagating in the y direction (the wave number is $\vec{k} = k\,\hat{y}$). The perturbed ion equation of motion is, [2]

$$\begin{aligned} & m_i (n_0 + n_1)\left[\frac{\partial}{\partial t}(\vec{V}_0 + \vec{V}_1) + (\vec{V}_0 + \vec{V}_1)\cdot\vec{\nabla}(\vec{V}_0 + \vec{V}_1)\right] \\ & = e(n_0 + n_1)\left[\vec{E}_1 + (\vec{V}_0 + \vec{V}_1)\times\vec{B}_0\right] + m_i (n_0 + n_1)\vec{g} \end{aligned} \tag{3-3}$$

Multiply equation (3-2) by $1 + \left(\frac{n_1}{n_0}\right)$ to obtain

$$m_i (n_0 + n_1)(\vec{V}_0 \cdot \vec{\nabla})\vec{V}_0 = e(n_0 + n_1)\vec{V}_0 \times \vec{B}_0 + m_i (n_0 + n_1)\vec{g} \tag{3-4}$$

Subtracting this from equation (3-3) and neglecting second-order terms we obtain

$$m_i n_0 \left[\frac{\partial \vec{V}_1}{\partial t} + (\vec{V}_0 \cdot \vec{\nabla})\vec{V}_1\right] = e n_0 (\vec{E}_1 + \vec{V}_1 \times \vec{B}_0) \tag{3-5}$$

Note that $\vec{g}$ has cancelled out. Information regarding $\vec{g}$, however, is still contained in $\vec{V}_0$. For perturbation of the form $\exp[i(ky - w\,t)]$, we have

$$m_i (w - k V_0)\vec{V}_1 = ie(\vec{E}_1 + \vec{V}_1 \times \vec{B}_0) \tag{3-6}$$

This equation can be solved and the solution for $w_{ci}{}^2 >> (w - kv_0)^2$ is as follows

$$V_{ix} = \frac{E_y}{B_0}, \qquad V_{iy} = -i\frac{w - kV_0}{w_{ci}}\frac{E_y}{B_0} \tag{3-7}$$

where w_{ci} is the ion cyclotron frequency given by $\frac{eB_0}{m_i}$. The quantity V_{ey} for electrons vanishes in the limit $\frac{m_e}{m_i} \to 0$. For electrons we therefore have

$$V_{ex} = \frac{E_y}{B_0}, \qquad V_{ey} = 0 \tag{3-8}$$

The perturbed equation of continuity for ions is

$$\frac{\partial n_1}{\partial t} + \vec{\nabla} n_1 \cdot \vec{V}_0 + n_0 \vec{\nabla} \cdot \vec{V}_1 + \vec{\nabla} n_0 \cdot \vec{V}_1 = 0 \tag{3-9}$$

The zeroth-order term vanishes since $\vec{V}_0$ is perpendicular to $\vec{\nabla} n_0$, and the $n_1 \vec{\nabla} \cdot \vec{V}_0$ term vanishes if $\vec{V}_0$ is constant. Therefore, the first order equation is

$$-iw\,n_1 + ikV_0 n_1 + ikn_0 V_{iy} + n_0' V_{ix} = 0 \qquad (3\text{-}10)$$

where $n_0' = \frac{\partial n_0}{\partial x}$. The electrons follow a simpler equation, since $\vec{V}_{e^0} = 0$ and $V_{ey} = 0$ so, we obtain

$$-iw\,n_1 + V_{ex} n_0' = 0 \qquad (3\text{-}11)$$

Note that we have used the plasma approximation and have assumed $n_{i1} = n_{e1} = n_1$. This is possible because the unstable waves are of low frequency $w << w_{ci}$. Equations (3-7) and (3-10) yield

$$(w - kV_0)n_1 + i\frac{E_y}{B_0} n_0' + ikn_0 \frac{w - kV_0}{w_{ci}} \frac{E_y}{B_0} = 0 \qquad (3\text{-}12)$$

Equations (3-8) and (3-11) yield

$$w\,n_1 + i\frac{E_y}{B_0} n_0' = 0 \;, \qquad \frac{E_y}{B_0} = \frac{iw\,n_1}{n_0'} \qquad (3\text{-}13)$$

Substituting this into equation (3-12), we have

$$(w - kV_0)n_1 - \left(n_0' + kn_0 \frac{w - kV_0}{w_{ci}} \right) \frac{w\,n_1}{n_0'} = 0$$

$$w - kV_0 - \left(1 + \frac{kn_0}{w_{ci}} \frac{w - kV_0}{n_0'} \right) w = 0$$

$$w(w - kV_o) = -V_0 w_{ci} \frac{n_0'}{n_0} \qquad (3\text{-}14)$$

Substituting for V_0 from equation (3-1), we obtain a quadratic equation for w:

$$w^2 - kV_0 w - g\left(\frac{n_0'}{n_0} \right) = 0 \qquad (3\text{-}15)$$

which has the following solutions

$$w = \frac{1}{2} kV_0 \pm \left[\frac{1}{4} k^2 V_0{}^2 + g\left(\frac{n_0'}{n_0} \right) \right]^{\frac{1}{2}} \qquad (3\text{-}16)$$

There is an instability if w is complex; that is, if

$$-g\frac{n_0'}{n_0} > \frac{1}{4} k^2 V^2{}_0 \qquad (3\text{-}17)$$

From this, we see that instability requires g and $\frac{n_0'}{n_0}$ to have opposite sign. This is just the statement that the light fluid is supporting the heavy fluid; otherwise, w is real and the plasma is stable. For sufficiently small k (long wavelength), the growth rate is given by

$$g = I_m(w) \approx \left[-g\left(\frac{n_0'}{n_0}\right)\right]^{\frac{1}{2}} \tag{3-18}$$

The real part of w is $\frac{1}{2}kV_0$. Since V_0 is an ion velocity, this is a low-frequency oscillation, as previously assumed.

This instability, which has $\vec{k}\perp\vec{B}_0$, is sometimes called a "flute" instability [2].

3.3 FLUTE INSTABILITY DUE TO FIELD CURVATURE

The driving force of the gravitational instability discussed above was provided essentially by the electric field produced by the charge separation at the boundary. This charge separation, in turn, was brought about by the gravitational field, which is a charge-insensitive force. We can thus develop a similar line of argument to derive a stability criterion, whenever a charge-insensitive force field is involved in a direction perpendicular to the magnetic field. Hence for a plasma confined in a magnetic field, if there is a force acting on the charged particles, regardless of their signs, in the direction from the plasma to the vacuum region, then the system is found to be unstable with respect to the Rayleigh-Taylor mode. If the force is directed in the opposite directions, the system may be stable as for as the Rayleigh-Taylor mode is concerned [14].

A most important example of such a charge-insensitive field arises when the magnetic lines of force that confine the plasma have finite radii of curvature (Fig. 3-5) [2, 12, 14, 25]. If $V_{//}^{2}$ denotes the average square of the component of random velocity along $\vec{B}$, the average centrifugal force is

$$\vec{F}_{cf} = \frac{mV_{//}^{2}}{R_c}\hat{r} = mV_{//}^{2}\frac{\vec{R}_c}{R^2{}_c} \tag{3-19}$$

The curvature also involves the field gradient in the directions perpendicular to the magnetic lines of force. From a vacuum field relationship $\vec{\nabla}\times\vec{B}=0$, which we can reasonably assume for a plasma-confining system, in cylindrical coordinates of Fig.3-5,

$\vec{\nabla} \times \vec{B}$ has only a z-component, since $\vec{B}$ has only a q component and $\vec{\nabla} B$ has only an r component. We then have

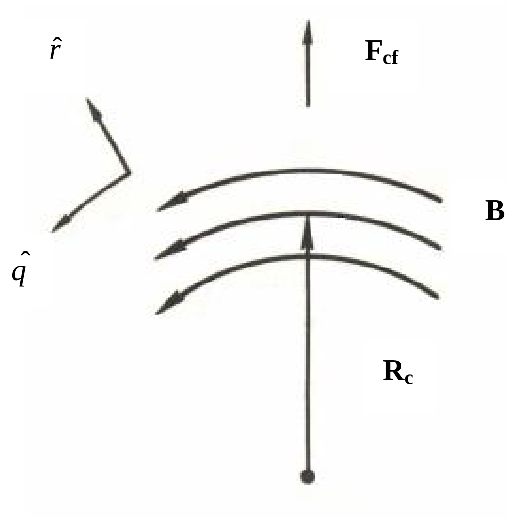

FIG. 3-5 A CURVED MAGNETIC FIELD (AFTER CHEN, 1984).

$$\left(\vec{\nabla} \times \vec{B}\right)_z = \frac{1}{r}\frac{\partial}{\partial r}(rB_q) = 0, \qquad B_q \propto \frac{1}{r} \tag{3-20}$$

Thus

$$|B| \propto \frac{1}{R_c}, \qquad \frac{\vec{\nabla} B}{B} = -\frac{\vec{R}_c}{R^2{}_c} \tag{3-21}$$

It is known that when a spatial gradient of the field strength exists, a charged particle experiences an effective force given by [14]

$$\vec{F}_G = -\frac{1}{2} m V_\perp{}^2 \; \frac{\vec{\nabla} B}{B} \tag{3-22}$$

Substitution of equation (3-21) in equation (3-22) yields an effective force in the perpendicular directions,

$$\vec{F}_G = \frac{1}{2} m V_\perp{}^2 \; \frac{\vec{R}_c}{R^2{}_c} \tag{3-23}$$

Collecting the two forces, equations (3-19) and (3-23), we may apply the results of the foregoing gravitational-instability analysis to the present case through a replacement

$$\vec{g} \rightarrow \frac{m V_{//}{}^2 + \frac{1}{2} m V_\perp{}^2}{m} \frac{\vec{R}_c}{R_c{}^2} = -\left(V_{//}{}^2 + \frac{V_\perp{}^2}{2}\right)\frac{\vec{\nabla}|B|}{B} \tag{3-24}$$

From the criterion of the Rayleigh-Taylor instability, we then find that a system in which the magnetic field confines the plasma in a convex shape (as in Fig. 3-6a) is unstable because $\vec{R}_c$ (and hence the non-electromagnetic force) in this case is directed from the plasma to vacuum; on the contrary a system like Fig. 3-6b is stable because $\vec{R}_c$ is directed from the vacuum to the plasma.

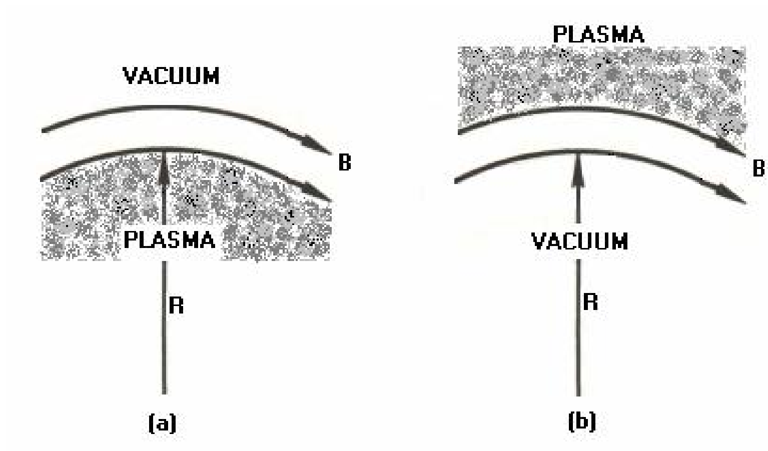

FIG. 3-6 STABILITY OF MAGNETIC CONFINEMENT (AFTER SETSUO ICHIMARU, 1986).

3.4 PHYSICAL MECHANISM OF THE CURVATURE INSTABILITY

The physical reason for the instability and stability of the plasma in Fig. 3-6 is illustrated by Martin A. Uman, 1964, as follows:

We shall assume that the plasma has a uniform kinetic pressure P_0. The magnetic pressure at the plasma surface must be equal to the pressure of the plasma,

$$P_0 = \frac{|B_0|^2}{2m_0} \tag{3-25}$$

where B_0 is the magnetic field at the plasma surface, and m_0 is the permeability of free space. The magnetic field strength diminishes with increasing distance from the center of curvature of the field lines. In order to test the stability for any system, we must perturb the plasma surface and then determine whether perturbations will grow larger or diminish in size. We shall perturb the plasma surface in Fig. 3-6a with a wave whose crests and troughs run parallel to the magnetic field lines as shown in Fig. 3-7. The kinetic pressure of the plasma will be unchanged if the plasma is perturbed in such a manner that its volume does not change.

CHAPTER 4

UNIVERSAL INSTABILITY

4.1 INTRODUCTION

This type of instability occurs as a result of gradients in density and temperature in a plasma. In laboratory conditions, any finite plasma confined by a static magnetic field possess these gradients, hence the resulting waves are called "universal instabilities". These waves are low frequency oscillations $w << w_{ce}$ (where w_{ce} is the electron cyclotron frequency) propagating azimuthaly mainly perpendicular to both the magnetic field and the gradients with the well-known electron diamagnetic drift velocity V_{de}, so it is termed "drift waves" or "drift instability". Usually they are found to localize where the radial plasma density gradient is largest: a region which typically falls about midway between the center of the column and the edge [2, 16].

The drift instability has been observed in many devices both linear and toroidal, and in both the collisionless and collisional dominated regimes [4, 6, 8, 9, 10, 13, 16, 20, 26]. The instability amplitude can attain very high levels and in many cases drift waves lead to anomalous transport of plasma across magnetic field lines [3, 7, 8, 9]. The overall appearance of the drift mode and its harmfull effect on plasma confinement have made it a prime candidate for both theoretical and experimental study.

In the last few years, some theoretical work has been done in cylindrical plasma geometry. In these papers, which always use the two-fluid equations of motion [6, 8, 10, 16], some simplifying theoretical assumptions are made that are inconsistent with the real situation as,

(i) The electron- density distribution is usually taken into account, but the electron-temperature distribution is always assumed to be constant.In fact, however, in a bounded plasma there must be some cooling near the boundary, and hence gradient in temperature [21].

(ii) Electron-temperature oscillations are not usually considered. In fact, electron-energy conservation demands that they always exist [21].

(iii) The radial electric field are often not considered. This does not make much physical sense, since in cylindrical plasma, the particles diffuse in a direction opposite the gradient in density. The step length is the magnitude of the Larmor radius r_L which is the radius of gyration. As a result, the ions move faster than electrons because of their higher Larmor radius, and hence a radial electric field is build up in the direction of density gradient as shown in Fig. 4-1 [2].

When drift waves occur in a plasma column which has a radial electric field, the drift-wave frequency is affected by plasma column rotation by an "$\vec{E}\times\vec{B}$ drift". The frequency of plasma column rotation w_{rot} caused by the $\vec{E}\times\vec{B}$ drift is given by [2, 16]

$$w_{rot} = \frac{E}{rB} \tag{4-1}$$

In his work, L. Zhang, (1991), had considered this field, but he eliminated the electron temperature oscillation in the theory.

(iv) Electron motion parallel to the magnetic field lines is considered by Marden-Marshall et al. (1986). However the electron temperature gradient is not included into the theory, which contradicts the real situation.

In the present chapter, both electron-density and temperature gradients, electron-temperature oscillations, electron motion parallel to the magnetic field lines and the radial electric field are taken into account. We first formulate our theory independently of any given laboratory plasma, using the full non-viscous electron-energy equation and considering the radial dependent of the collision frequencies, making assumptions that are usually well satisfied in the plasma used to study drift waves, so that the theory is valid for a variety of plasmas. We then apply it to specific experimental data (from Egger et al. 1986) and demonstrate its usefulness.

The outline of this chapter is as follows. In sec. 4.2, we derive the electron and ion diamagnetic drift velocity. In sec. 4.3, we explain the physical features of drift waves. In sec. 4.4, we formulate our theory based on the inclusion of electron-temperature oscillations gradients (and radial electric field as well as, electron motion parallel to B) and give all the formulae relevant for the calculation of the theory. Sec. 4.4.1 describes the basic two-fluid hydrodynamical equations. These equations are used to derive the final differential equation for both the oscillating potential and the eigenvalue which is the complex drift-wave frequency. From the eigenfunction, representing the radial distribution of the oscillating potential, the radial distributions of all other oscillating quantities can be calculated with the formula given. Sec. 4.5 is a description of the numerical method used to solve the eigenvalue differential equation problem. Sec. 4.6 presents the numerical results of the application of this equation to experimental data from Egger et al., (1986).

4.2 DIAMAGNETIC DRIFT

To estimate the electron and ion diamagnetic drift velocity, we consider a column of a plasma confined by a magnetic field in the z-direction $\vec{B} = B\hat{z}$. In this geometry the density n, the pressure P, and the temperature T have gradients perpendicular to the magnetic field are unidirectional (Fig. 4-1). The equation of motion for each species [22]

$$m_a n_a \left(\frac{\partial \vec{V}_a}{\partial t} + \left(\vec{V}_a \cdot \vec{\nabla}\right)\vec{V}_a \right) = n_a e_a \left(\vec{E} + \vec{V}_a \times \vec{B}\right) - \vec{\nabla} P_a - m_a n_a n_a \vec{V}_a \qquad (4\text{-}2)$$

$$(1) \qquad (2) \qquad\qquad (3) \qquad\qquad (4)$$

where a is either e or i denoting electrons or ions, $m_a, e_a, and \vec{V}_a$ are the electron (or ion) mass, charge and velocity. n_a is the charged particle-neutral atom collision frequency.

Consider the ratio of each term of (1) & (4) to term (3):

$$\frac{(1)}{(3)} = \left| \frac{m_a n_a i w V_{\perp a}}{e_a n_a V_{\perp a} B} \right| = \frac{w}{w_{ca}}$$

$$\frac{(4)}{(3)} = \left| \frac{m_a n_a n_a V_{\perp a}}{e_a n_a V_{\perp a} B} \right| = \frac{n_a}{w_{ca}}$$

where $w_{ca} = \frac{e_a B}{m_a}$ is the electron (or ion) cyclotron frequency. Here we have taken $\frac{\partial}{\partial t} = iw$ and are concerned only with $\vec{V}_\perp$. For drifts slow compared with the time scale of w_{ca}, we may neglect term (1) [2]. In general $n_a << w_{ca}$ as a result of strong $\vec{B}$ [21], this allows us to neglect term (4). We shall also neglect the $(\vec{V}_a \cdot \vec{\nabla})\vec{V}_a$ term and show later that this is fit to the theory. Taking the cross product of equation (4-2) with $\vec{B}$ (neglecting the left-hand side),we get

$$0 = e_a n_a \left[\vec{E}\times\vec{B} + (\vec{V}_{\perp a}\times\vec{B})\times\vec{B}\right] - \vec{\nabla}P_a \times \vec{B}$$
$$0 = e_a n_a \left[\vec{E}\times\vec{B} + (\vec{V}_{\perp a}\cdot\vec{B})\vec{B} - \vec{V}_{\perp a}B^2\right] - \vec{\nabla}P_a \times \vec{B}$$

Therefore

$$\vec{V}_{\perp a} = \frac{\vec{E}\times\vec{B}}{B^2} - \frac{\vec{\nabla}P_a \times \vec{B}}{e_a n_a B^2} \equiv \vec{V}_E + \vec{V}_{da} \qquad (4\text{-}3)$$

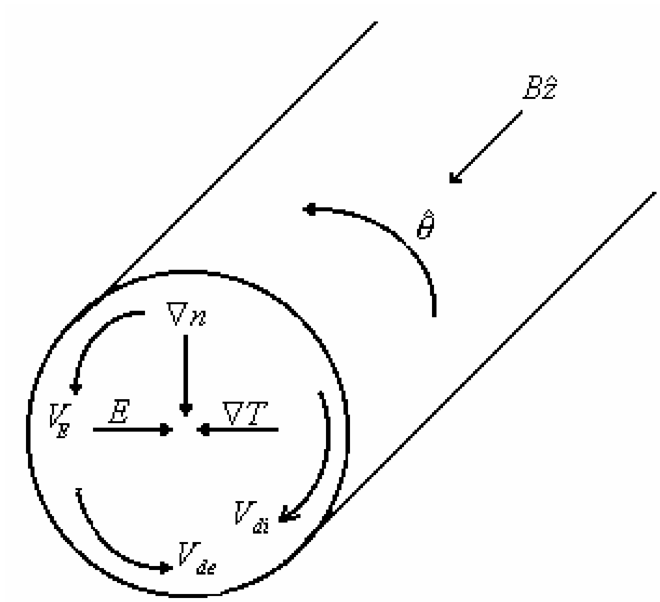

FIG. 4-1 CYLINDRICAL PLASMA COLUMN. GRADIENT IN DENSITY AND TEMPERATURE GIVE RISE TO THE ELECTRON (ION) DIAMAGNETIC VELOCITY (AFTER CHEN, 1984).

where

$$\vec{V}_E = \frac{\vec{E}\times\vec{B}}{B^2} \qquad (4\text{-}4)$$

$$\vec{V}_{da} = -\frac{\vec{\nabla}P_a \times \vec{B}}{e_a n_a B^2} \qquad (4\text{-}5)$$

direction of the gradients, our neglect of $(\vec{V}_a \cdot \vec{\nabla})\vec{V}_a$ is justified. If we make the substitution $P_a = n_a k_B T_a$; where k_B is the Boltzmann's constant, equation (4-5) becomes

$$\vec{V}_{da} = -\frac{1}{e_a n_a B^2}\left(k_B T_a \vec{\nabla} n_a + k_B n_a \vec{\nabla} T_a\right) \times \vec{B}$$

In the geometry shown in Fig. 4-1 $\vec{\nabla} n_a = n'_a \hat{r}$ and $\vec{\nabla} T_a = T'_a \hat{r}$, so we have the following formula

$$\vec{V}_{da} = -\frac{k_B T_a}{e_a B}\left(\frac{n'_a}{n_a} + \frac{T'_a}{T_a}\right)\hat{r} \times \hat{z} \qquad (4\text{-}6)$$

For ions
$$\vec{V}_{di} = \frac{k_B T_i}{eB}\left(\frac{n'_i}{n_i} + \frac{T'_i}{T_i}\right)\hat{q} \qquad (4\text{-}7)$$

For electrons
$$\vec{V}_{de} = -\frac{k_B T_e}{eB}\left(\frac{n'_e}{n_e} + \frac{T'_e}{T_e}\right)\hat{q} \qquad (4\text{-}8)$$

Where $n'_a = \frac{\partial n_a}{\partial r} < 0$ and $T'_a = \frac{\partial T_a}{\partial r} < 0$

In the cylindrical geometry shown in Fig. 4-1 both the electron ($e_a = -e$) and the ions ($e_a = e$) diamagnetic drifts as derived here give rise to currents in the plasma that serve to reduce the magnetic field inside the plasma. Hence the name "diamagnetic" drift [12], the diamagnetic current is given by

$$\vec{j}_d = ne\left(\vec{V}_{di} - \vec{V}_{de}\right) \qquad (4\text{-}9)$$

The diamagnetic drift in a uniform magnetic field is the result of adding the Larmor orbits of the charged particles in the presence of a density or temperature gradients or both together. The effect of the density gradient is specially easy to see by examining Fig. 4-2a, which shows the Larmor orbit of positively charged particles (ions) about a magnetic field directed into the paper. It is clear that, there is a greater current moving to the left than to the right, despite the fact that the guiding centers are stationary [23]. Similarly when a temperature gradient is involved, the difference between the average velocities in the high and low temperature domains accounts for the appearance of current flow as shown in Fig. 4-2b [14]. The situation is totally different in a homogeneous plasma (no gradient in density nor temperature), the reason is shown clearly in Fig. 4-2c. At each point we can imagine the upper part of one Larmor orbit to be in contact with the lower part of another, the current flows in opposite directions in the upper and lower parts of the circle. The total diamagnetic current in a homogeneous plasma is therefore equal to zero [23].

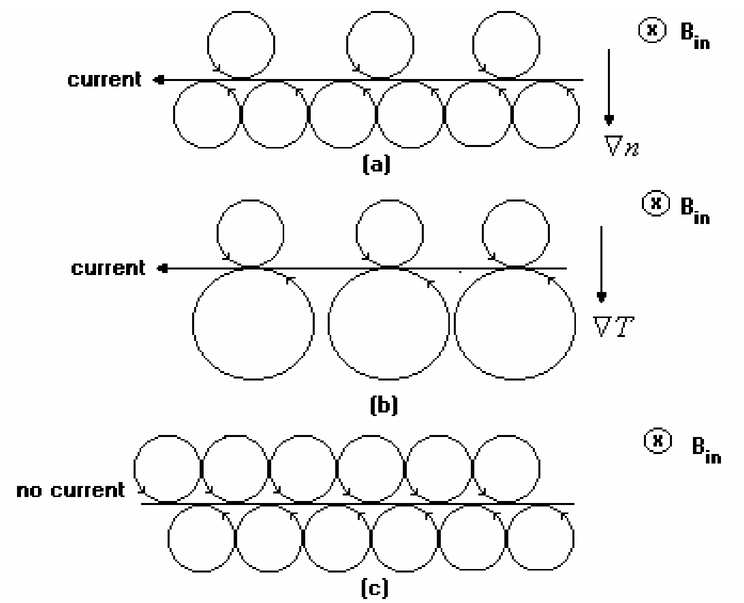

FIG. 4-2 ORIGIN OF THE DIAMAGNETIC CURRENT, (A) GRADIENT IN DENSITY, (B) GRADIENT IN TEMPERATURE, (C) HOMOGENEOUS PLASMA (AFTER P. A. STURROCK, 1994).

4.3 PHYSICAL FEATURES OF DRIFT WAVES (PLASMA MODEL)

The cylindrical geometry shown in Fig. 4-1 acts as a simple example for universal instability . In contrast to gravitational modes, drift waves have a smaller but finite component of $\vec{k}$ along $\vec{B}$ (in addition to $\vec{k}\perp\vec{B}$). Suppose that the plasma is initially in equilibrium state, then any constant density surface will be sharp. Now suppose that a perturbing ripple is developed due to random thermal fluctuations, then the density surface will be with a slight helical twist, as indicated by the solid line in Fig. 4-3 [24].

If we enlarge the cross section enclosed by the box in Fig. 4-3 and straighten it out into Cartesian geometry, it would appear as in Fig. 4-4. The only driving force for the instability is the pressure gradient $\vec{\nabla}P_0$ (we assume K_BT =constant, for simplicity). In this case the diamagnetic drifts are [2]

$$\vec{V}_{i0} = \vec{V}_{di} = \frac{k_B T_i}{eB_0}\frac{n_0'}{n_0}\hat{y} \tag{4-10}$$

$$\vec{V}_{e0} = \vec{V}_{de} = -\frac{k_B T_e}{eB_0}\frac{n_0'}{n_0}\hat{y} \qquad (4\text{-}11)$$

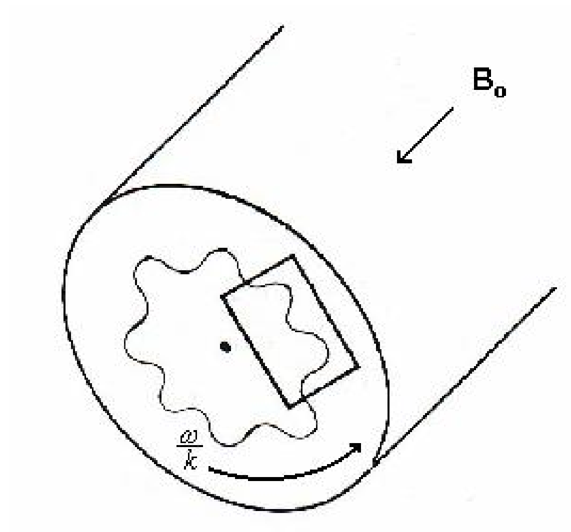

FIG. 4-3 GEOMETRY OF A DRIFT INSTABILITY IN A CYLINDER. THE REGION IN THE RECTANGLE IS SHOWN IN DETAIL IN FIG. 4-4 (AFTER H. SUK, 1998) .

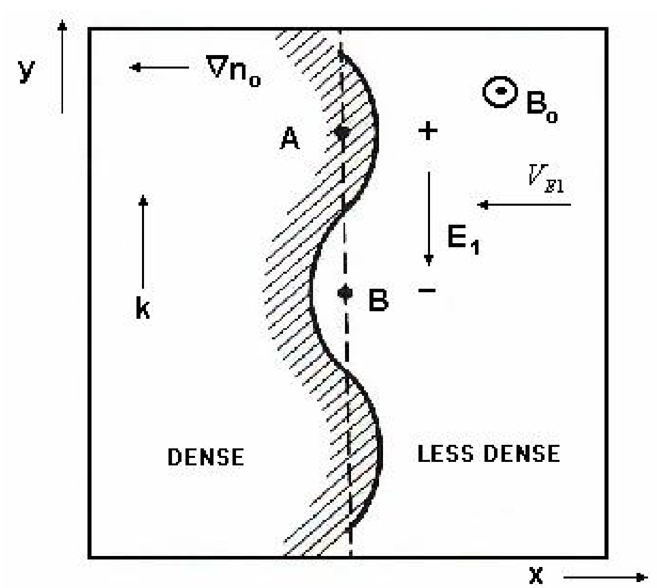

FIG. 4-4 PHYSICAL MECHANISM OF A DRIFT WAVE (AFTER CHEN, 1984).

We shall show that the phase velocity $\frac{w}{k_y}$ of the drift instability is approximately equal to the electron diamagnetic drift velocity V_{de}.

Since drift waves have finite k_z, electron can flow along $\vec{B}_0$ to establish a thermodynamic equilibrium among themselves. They will then obey the Boltzmann relation:

$$n = n_0 \exp\left(\frac{ef_1}{k_B T_e}\right)$$

where f_1 is the oscillating potential, and T_e is the electron temperature. In the perturbation state $n = n_0 + n_1$. For regions where $ef_1 << k_B T_e$, the exponential can be expanded in the form:

$$n_0 + n_1 = n_0\left(1 + \frac{ef_1}{k_B T_e}\right)$$

From which we obtain

$$\frac{n_1}{n_0} = \frac{ef_1}{k_B T_e} \qquad (4\text{-}12)$$

At point A in Fig. 4-4 the density is larger than in equilibrium, n_1 is positive, and therefore f_1 is positive. Similarly, at point B, n_1 and f_1 are negative. The difference in potential means that an electric field $\vec{E}_1$ exists between A and B. This field causes a perturbed drift velocity $\vec{V}_{E1} = \frac{\vec{E}_1 \times \vec{B}_0}{{B_0}^2}$ in the x direction. As the wave passes by, traveling in the y direction, an observer at point A will see n_1 and f_1 oscillating in time. The drift $\vec{V}_{E1}$ will also oscillate in time (because $\vec{E}_1$ changes sign by oscillation). Since there is a gradient $\vec{\nabla} n_0$ in the negative x-direction, the drift $\vec{V}_{E1}$ will bring plasma of different density to a fixed observer A. Therefore, a drift wave has a motion such that the fluid moves back and forth in the x-direction although the wave travel in the y direction.

To be more quantitative, the drift speed V_{1x} which is the magnitude of $\vec{V}_{E1}$ is given by

$$V_{1x} = \frac{-E_1}{B_0} = \frac{-ik_y f_1}{B_0} \qquad (4\text{-}13)$$

we shall assume that V_{1x} does not vary with x and that k_z is much less than k_y; that is, the fluid oscillates incompressibly in the x direction. Consider now the number of guiding centers brought into cubic meter at a fixed point A; it is obviously

$$\frac{\partial n_1}{\partial t} = -V_{1x}\frac{\partial n_0}{\partial x} \tag{4-14}$$

This is just the equation of continuity for guiding centers, in which the fluid drift $\vec{V}_d$ is not included. The term $n_0\vec{\nabla}\cdot\vec{V}_{E1}$ vanishes according to the previous assumption. Substitute about v_{1x} from (4-13) into (4-14)

$$-iw\,n_1 = \frac{ik_y f_1}{B_0}n_0' \tag{4-15}$$

Substitute about n_1 from (4-12) into (4-15) yield

$$-iw\frac{ef_1}{k_B T_e}n_0 = \frac{ik_y f_1}{B_0}n_0'$$

Thus the phase velocity is given by

$$\frac{w}{k_y} = -\frac{k_B T_e}{eB_0}\frac{n_0'}{n_0} = V_{de} \tag{4-16}$$

These waves, therefore, travel with the electron diamagnetic drift velocity and are called drift waves. This is the velocity in the y with respect to the Cartesian coordinates, or azimuthal direction with respect to the cylindrical coordinates.

4.4 FORMULATION OF THE THEORY

4.4.1 BASIC EQUATIONS

We shall consider a weakly ionized cylindrical low-b plasma (coordinates r, q, z) in which collisions between charged particles and neutrals are important but coulomb collisions can be neglected [11], this plasma is immersed in a strong, constant and homogeneous magnetic field pointing in the axial direction $\hat{z}$:

$$\vec{B}_0 = B_0\hat{z} \tag{4-17}$$

The two-fluid equations (a =e, i) are as follows

(i) The equation of motions [22]:

$$n_a m_a\left[\frac{\partial \vec{V}_a}{\partial t} + \left(\vec{V}_a\cdot\vec{\nabla}\right)\vec{V}_a\right] = -\vec{\nabla}P_a + e_a n_a\left(\vec{E} + \vec{V}_a\times\vec{B}_0\right) - m_a n_a n_a \vec{V}_a \tag{4-18}$$

where collisions are represented by the drag term $m_a n_a n_a\vec{V}_a$.

(ii) The equation of continuity [5]:

$$\frac{\partial n_a}{\partial t} + \vec{\nabla} \cdot \left(n_a \vec{V}_a \right) = 0 \qquad \text{(4-19)}$$

(iii) The equations of state, for which we take the ideal gas law [19]

$$P_a = n_a K_B T_a \qquad \text{(4-20)}$$

(iv) The non-viscous electron-energy equation in its reduced form, i.e. the kinetic energy and the Ohmic dissipation terms have been eliminated with the aid of the electron equations of motion and continuity [1]:

$$\frac{3}{2} n_e \left(\frac{\partial}{\partial t} + \vec{V}_e \cdot \vec{\nabla} \right) K_B T_e = -\vec{\nabla} \cdot \vec{q}_e - n_e K_B T_e \vec{\nabla} \cdot \vec{V}_e \qquad \text{(4-21)}$$

$\vec{q}_e$ is the electron thermal flux vector.

The system of equations will be closed with the quasi-electrostatic approximation as $\vec{E} = -\vec{\nabla} f$ and the assumption of quasi-neutrality as $n_e = n_i$.

4.4.2 SECOND ORDER DIFFERENTIAL EQUATION FOR DRIFT WAVES

Drift waves are low-frequency electron and ion-density oscillations propagating azimuthally with the well known diamagnetic drift velocity. They are characterized by their azimuthal mode number m (number of cycles occuring azimuthally). Usually they are observed to propagate also axially. Drift-wave instabilities are driven by the radial pressure gradient in the plasma [2].

Equations (4-18)-(4-21) can easily be solved by the procedure of linearization. By this we assume that the amplitude of oscillation is small. Thus, terms with higher power of amplitude factors can be neglected [2]. The most important features in identifying the drift instability are the oscillation (fluctuation) level in plasma density, electron temperature and electrostatic potential. We separate the dependent variables, namely the particle density n_a, the fluid velocities $\vec{V}_a$, the electric potential f, and the electron temperature T_e, into two parts: an "equilibrium" part, indicated by a subscript 0, and a comparatively small oscillating perturbation part indicated by a subscript 1. Considering the propagation properties of drift waves mentioned above, we can represent $n_a, \vec{V}_a, f$ and T_e in the form [21]

$$\Psi_a = \Psi_{a^0} + \Psi_{a^1}(r) e^{i(mq + k_z z - wt)} \qquad \text{(4-22)}$$

where w is the complex drift wave frequency given by

$$w = w_R + i w_I \tag{4-23}$$

Where w_R is the real part of w, and imaginary part w_I represents the growth rate of the corresponding drift wave mode m.

Applying the general set of equations (4-18)-(4-21) to drift waves, and adopting the following assumptions as described in different literatures [16, 20, 27], we have

(i)The ions are relatively cold: $T_{i^0} << T_{e^0}$. This allows us to neglect the pressure gradient $-\vec{\nabla} P_i$, and the convective term $(\vec{V}_i \cdot \vec{\nabla})\vec{V}_i$ in the ion equation of motion and the terms containing the drift velocity $\vec{V}_{i^0}$ in the ion equation of continuity. However, we take into account the dependence of the ion-neutral collision frequency n_i on T_{i^0}.

Aebischer and Sayasov, (1988) indicated that T_{i^0} is usually 5-6 times smaller than T_{e^0} in plasmas used to study drift waves so that we considered the ions to be cold $(T_{i^0} \approx 0)$.

(ii) The applied magnetic field is strong enough such that the total electron collision frequency is much lower than the electron cyclotron frequency: $n_e << w_{ce}$. This allows us to neglect the collision term in the electron motion perpendicular to $\vec{B}_0$. This assumption further allows us to assume that the electron diamagnetic velocity $\vec{V}_{e^0}$ has a component in the q-direction only. It follows from the equilibrium electron equation of motion (4-8) that,

$$\vec{V}_{e^0} = \vec{V}_{de} = -\frac{k_B T_{e^0}}{eB_0}\left(\frac{1}{T_{e^0}}\frac{dT_{e^0}}{dr} + \frac{1}{n_{e^0}}\frac{dn_{e^0}}{dr}\right)\hat{q} \tag{4-24}$$

The temperature-gradient term enters naturally when T_{e^0} is allowed to vary radially in a magnetized plasma. For simplicity, let us introduce the following shorthand notation:

$$\vec{K} = \frac{1}{n_{e^0}}\frac{dn_{e^0}}{dr}\hat{r}, \qquad \vec{K}' = \frac{1}{T_{e^0}}\frac{dT_{e^0}}{dr}\hat{r}$$

So equation (4-24) becomes:

$$\vec{V}_{e^0} = -\frac{k_B T_{e^0}}{eB_0}(K' + K)\hat{q} \tag{4-25}$$

The thermal flux vector $\vec{q}_e$ can also be assumed to be parallel to the magnetic field, and is given by [1]

$$\vec{q}_e = -l_z \frac{\partial T_e}{\partial z}\hat{z} \tag{4-26}$$

where l_z is the thermal conductivity along the magnetic field B_0.

(iii) We assume a sheared radial electric potential $f_0(r)$ i.e. an electric field $\vec{E}$ in the direction of $\vec{\nabla} n$, this field produces an $\vec{E}\times\vec{B}$ drift velocity $\vec{V}_E$,

$$\vec{V}_E = \frac{\vec{E}\times\vec{B}_0}{B_0^{\,2}} = \frac{E}{B_0}\hat{q} \tag{4-27}$$

(iv)The plasma may carry a current along $\vec{B}_0$, which can be represented by a drift of the electrons at speed u_0 in the z-direction.

(v)The phase velocity of the drift wave parallel to $\vec{B}_0$ is much higher than the ion thermal velocity $\frac{w}{k_z} >> \left(\frac{k_B T_{i^0}}{m_i}\right)^{\frac{1}{2}}$. This allows us to neglect ion motion parallel to $\vec{B}_0$.

(vi)The plasma is quasi-neutral at all times: $n_e = n_i$. The condition required for this approximation to be valid is that the Debye length $l_D = \left(\frac{e_0 k_B T_e}{n_0 e^2}\right)^{\frac{1}{2}}$ of the plasma is small compared with the physical size of the plasma L ($l_D << L$) [23]. This assumption implies that $n_{e^0} = n_{i^0} = n_0$ and $n_{e^1} = n_{i^1} = n_1$ at all times.

(vii)The drift wave frequencies are low: $w << w_{ci} << w_{ce}$ [8].

With the seven assumptions stated above, and neglecting electron inertia, we obtain the following set of linearized equations from the general hydrodynamical equations (4-18)-(4-21)

$$-iw\, n_{i^0} m_i \vec{V}_{i^{1\perp}} = -en_{i^0}\vec{\nabla}_\perp f_1 - en_{i^1}\vec{\nabla}_\perp f_0 + en_{i^0}B_0\vec{V}_{i^{1\perp}}\times\hat{z} + en_{i^1}V_E B_0\hat{r} - m_i n_{i^0} n_i \vec{V}_{i^{1\perp}} \tag{4-28}$$

$$-iw\, n_{i^1} + \frac{1}{r}\frac{\partial n_{i^1}}{\partial q}V_E + \vec{\nabla} n_{i^0}\cdot\vec{V}_{i^{1\perp}} + n_{i^0}\vec{\nabla}_\perp\cdot\vec{V}_{i^{1\perp}} = 0 \tag{4-29}$$

$$\begin{aligned}0 = -k_B n_{e^1}\vec{\nabla}_\perp T_{e^0} - k_B T_{e^0}\vec{\nabla}_\perp n_{e^1} - k_B\vec{\nabla}_\perp\left(T_{e^1}n_{e^0}\right) + en_{e^1}\vec{\nabla}_\perp f_0 + en_{e^0}\vec{\nabla}_\perp f_1 \\ - eB_0 n_{e^0}\vec{V}_{e^{1\perp}}\times\hat{z} - eB_0 n_{e^1}\left(V_{de} + V_E\right)\hat{r}\end{aligned} \tag{4-30}$$

$$0 = ik_z\left(-k_B T_{e1} n_{e0} - k_B n_{e1} T_{e0} + n_{e0} e f_1\right) - m_e n_e n_{e0} V_{e1z} \qquad (4\text{-}31)$$

$$-i(w - w_1)n_{e1} + \frac{V_{de} + V_E}{r}\frac{\partial n_{e1}}{\partial q} + V_{e1r}\frac{dn_{e0}}{dr} + n_{e0}\vec{\nabla}\cdot\vec{V}_{e1} = 0 \qquad (4\text{-}32)$$

$$\frac{3}{2}n_{e0}\left[k_B T_{e0} V_{e1r} K' + i\left(V_{de}\frac{m}{r} - (w - w_1 - w_E)\right)k_B T_{e1}\right] = -l_z k_z^2 k_B T_{e1}$$
$$-ik_B T_{e0}(w - w_1 - w_E)n_{e1} + k_B T_{e0}\left(iV_{de}\frac{m}{r}n_{e1} + n_{e0}V_{e1r}K\right) \qquad (4\text{-}33)$$

where

$$w_1 = k_z u_0 \qquad (4\text{-}34)$$

and

$$w_E = k_q V_E = \frac{m}{r}\frac{E}{B_0} = \frac{m}{r}\frac{1}{B_0}\frac{df_0}{dr} \qquad (4\text{-}35)$$

represent the drift wave frequency due to the electron motion parallel B_0 and $\vec{E}\times\vec{B}$ rotation respectively. Equation (4-28) is the ion equation of motion, (4-29) is the ion equation of continuity, (4-30) is the electron equation of motion perpendicular to the magnetic field lines, (4-31) is the electron parallel equation of motion, (4-32) is the electron equation of continuity, and (4-33) is the full non-viscous electron-energy equation.

It is possible to reduce the above system of equations to one single ordinary differential equation for the oscillating potential f_1 by the procedure described below. The intermediate formulas that will be obtained are useful to describe the drift wave phenomena and to reveal important relations between the physical quantities.

From the linearized ion equation of motion (4-28), we can express the oscillating ion velocity $\vec{V}_{i1\perp}$ in terms of the oscillating potential f_1:

$$\vec{V}_{i1\perp} = \frac{1}{B_0}\left[i\frac{w + in_i}{w_{ci}}\left(\vec{\nabla}_\perp f_1 + \frac{n_{i1}}{n_{i0}}\vec{\nabla}_\perp f_0 - B_0\frac{n_{i1}}{n_{i0}}V_E\hat{r}\right) + \hat{z}\times\vec{\nabla}_\perp f_1 + \frac{n_{i1}}{n_{i0}}\hat{z}\times\vec{\nabla}f_0 - B_0\frac{n_{i1}}{n_{i0}}V_E\hat{q}\right] \qquad (4\text{-}36)$$

Substituting this expression into the ion equation of continuity (4-29), yields the ion-density oscillation n_{i1} in terms of f_1:

$$\frac{n_{i1}}{n_{i0}} = \frac{e}{k_B T_{e0}}\left[\frac{w + in_i}{(w - w_E)}r^2\left(\frac{\partial^2 f_1}{\partial r^2} + \frac{1}{r}\frac{\partial f_1}{\partial r} - \frac{m^2}{r^2}f_1 + K\frac{\partial f_1}{\partial r}\right) + \frac{w_{de}}{(w - w_E)}f_1\right] \qquad (4\text{-}37)$$

where r is the ion Larmor radius, but with the electron temperature T_{e^0} in the numerator instead of the ion temperature T_{i^0} [21]:

$$r^2 = \frac{k_B T_{e^0}}{m_i w^2{}_{ci}} \tag{4-38}$$

We further define

$$w_{de} = -\frac{k_B T_{e^0}}{eB_0}\frac{m}{r}K, \qquad w'_{de} = -\frac{k_B T_{e^0}}{eB_0}\frac{m}{r}K' \tag{4-39}$$

The procedure for the electron equations is more involved. First, the electron perpendicular equation of motion (4-30) can be solved for the components $V_{e^{1r}}$ and $V_{e^{1q}}$. The electron parallel equation of motion (4-31) can also be solved for the component $V_{e^{1z}}$ of the oscillating electron velocity V_{e^1}. This yields

$$V_{e^{1r}} = -i\frac{k_B T_{e^0}}{eB_0}\frac{m}{r}\left(\frac{ef_1}{k_B T_{e^0}} - \frac{n_{e^1}}{n_{e^0}} - \frac{T_{e^1}}{T_{e^0}}\right) \tag{4-40}$$

$$V_{e^{1q}} = \frac{1}{eB_0 n_{e^0}}\left[k_B\left(-T_{e^0}\frac{\partial n_{e^1}}{\partial r} - n_{e^1}\frac{dT_{e^0}}{dr} - T_{e^1}\frac{dn_{e^0}}{dr} - n_{e^0}\frac{\partial T_{e^1}}{\partial r}\right) + en_{e^1}\frac{df_0}{dr} + en_{e^0}\frac{\partial f_1}{\partial r} - eB_0 n_{e^1}\left(V_{de} + V_E\right)\right] \tag{4-41}$$

$$V_{e^{1z}} = -i\frac{k_z}{m_e n_e}\left(k_B T_{e^0}\frac{n_{e^1}}{n_{e^0}} + k_B T_{e^1} - ef_1\right) \tag{4-42}$$

The above equations can then be substituted into the electron equation of continuity (4-32). After a mathematical treatments, we have obtained a simple form for the electron equation of continuity in terms of f_1, n_{e^1}, and the electron-temperature oscillation T_{e^1}:

$$-i\left(w - w_1 - w_E + in_{//}\right)n_{e^1} + \frac{n_{//}n_{e^0}}{k_B T_{e^0}}\left(k_B T_{e^1} - ef_1\right) - i\frac{m}{r}\frac{n_{e^0}}{B_0}Kf_1 = 0 \tag{4-43}$$

where $n_{//}$ is a shorthand notation for the expression

$$n_{//} = \frac{k_z{}^2 k_B T_{e^0}}{m_e n_e} \tag{4-44}$$

We then continue by substituting the expression (4-40) for $V_{e^{1r}}$ into the electro energy equation (4-33) and solving for the electron-temperature oscillation T_{e^1}. We get

$$T_{e^1}=\frac{iT_{e^0}\left[\left(w_{de}-\frac{3}{2}w'_{de}\right)\frac{ef_1}{k_BT_{e^0}}+\left(\frac{5}{2}w'_{de}-w_s\right)\frac{n_{e^1}}{n_{e^0}}\right]}{w_z+\frac{i}{2}\left(5w_{de}-3w_s\right)} \tag{4-45}$$

where

$$w_s=w-w_1-w_E \tag{4-46}$$

$$w_z=\frac{1}{n_0}l_zk_z^{\;2} \tag{4-47}$$

If we then substitute equation (4-45) for T_{e^1} into the equation of continuity (4-43), the electron density oscillation in terms of f_1 can be produced, as we did for the ions in (4-37):

$$\frac{n_{e^1}}{n_{e^0}}=\frac{\left(n_{//}-iw_{de}\right)\left[w_z+i\frac{3}{2}\left(w_{de}^{\;*}-w_s\right)\right]+w_{de}\left(w_{de}-\frac{3}{2}w'_{de}\right)}{\left(n_{//}-iw_s\right)\left[w_z+i\frac{3}{2}\left(w_{de}^{\;*}-w_s\right)\right]+w_s\left(w_{de}-\frac{3}{2}w'_{de}\right)+in_{//}\left(w_{de}^{\;*}-w_s\right)}\frac{ef_1}{k_BT_{e^0}} \tag{4-48}$$

where

$$w_{de}^{\;*}=w_{de}+w'_{de} \tag{4-49}$$

Clearly, this perturbed electron density is similar to the form of the perturbed ion density that is given in equation (4-37).

From the quasi-neutrality condition, it then follows that (4-37) and (4-48) can be put equal to each other. This leads to the desired differential equation for the radial distribution of the oscillating potential f_1:

$$\frac{d^2f_1(r)}{dr^2}+\left[\frac{1}{r}+K(r)\right]\frac{df_1(r)}{dr}+\left[Q(r,w)-\frac{m^2}{r^2}\right]f_1(r)=0 \tag{4-50}$$

where

$$Q(r,w)=\frac{w-w_E}{r^2\left(w+in_i\right)}\times\left\{\frac{w_{de}}{w-w_E}-\frac{\left(n_{//}-iw_{de}\right)\left[w_z+i\frac{3}{2}\left(w_{de}^{\;*}-w_s\right)\right]+w_{de}\left(w_{de}-\frac{3}{2}w'_{de}\right)}{\left(n_{//}-iw_s\right)\left[w_z+i\frac{3}{2}\left(w_{de}^{\;*}-w_s\right)\right]+w_s\left(w_{de}-\frac{3}{2}w'_{de}\right)+in_{//}\left(w_{de}^{\;*}-w_s\right)}\right\} \tag{4-51}$$

with boundary conditions at the plasma beam center r = 0, and the plasma beam radius $r = r_0$ [16, 21]

$$f_1(0) = 0, \qquad f_1(r_0) = 0 \tag{4-52}$$

The difference between equation (4-50) and the equation obtained by Aebischer and Sayasov, (1988) is the $Q(r,w)$ value. We have noticed that when the plasma column rotation is not considered, and the electron motion parallel to $\vec{B}_0$ is neglected, then $Q(r,w)$ in equation (4-51) becomes:

$$Q(r,w) = \frac{w}{r^2(w + in_i)} \times \left\{ \frac{w_{de}}{w} - \frac{(n_{//} - iw_{de})\left[w_z + i\frac{3}{2}(w_{de}^* - w)\right] + w_{de}\left(w_{de} - \frac{3}{2}w'_{de}\right)}{(n_{//} - iw)\left[w_z + i\frac{3}{2}(w_{de}^* - w)\right] + w\left(w_{de} - \frac{3}{2}w'_{de}\right) + in_{//}(w_{de}^* - w)} \right\} \tag{4-53}$$

which is the expression obtained by Aebischer and Sayasov, (1988). On other hand, if the radial electron temperature is also considered constant, i.e.

$$\frac{dT_{e^0}}{dr} = 0, \text{ and } w_{de} = w_{de}^* \tag{4-54}$$

then, equation (4-51) becomes similar to the expression derived by Ellis' et al. (1980).

Equations (4-50)-(4-52) represent a complex-eigenvalue problem for the complex drift-wave frequency w and the complex eigenfunction f_1(r), the radial distribution of the oscillating electric potential. It can be solved numerically, especially for arbitrary given undistributed density and temperature profiles $n_0(r)$ and $T_{e^0}(r)$. Once $f_1(r)$ is known, the remaining oscillating quantities can be computed with the aid of (4-36)-(4-49). In addition, the maximum wave amplitude position of the drift wave can be determined. This would show where the drift wave localize in the plasma beam region.

4.5 NUMERICAL METHOD

Equation (4-50) is a second order differential equation which predicts an eigenvalue problem for the complex drift-wave frequency w and the eigenfunction $f_1(r)$, which represents the radial distribution of the oscillating electric potential. One suitable general strategy for numerical solution of an eigenvalue problem is an iterative one (this strategy is sometimes called the "shooting method"). We guess a trial eigenvalue and generate a solution by integrating the differential equation as an initial value problem. If the resulting solution does not satisfy the boundary conditions, we change the trial eigenvalue and integrate again, repeating the process until a trial eigenvalue is found for which the boundary conditions are satisfied, such that if one integrates the differential equation (4-50), starting at one boundary and considering the boundary condition there, the resulting solution $f_1(r)$ automatically satisfies the condition at the other boundary. The boundary value problem is thus transformed into an initial-value problem. This can be solved with the aid of the Runge-Kutta integration method if one transforms the original complex second-order equation (4-50) into a set of coupled first-order equations [15].

Once the eigenvalue w is computed, the eigenfunction $f_1(r)$ can be found by Runge-Kutta integration as described above. The boundary condition on the boundary into which one integrates will then be automatically satisfied, since the proper value w is now used. With the aid of the equations given in section 4.4 the radial distributions of all other oscillating quantities can then be calculated from $f_1(r)$.

4.6 NUMERICAL RESULTS

We apply our theory to the weakly ionized helium plasma described by Egger et al. (1986). The main plasma parameters are B_0 = .077T, r_o = 2.8cm, $k_B\overline{T}_{e^o}$ = 3.5ev, $n_i = 2.1\times10^5$, $n_{//} = 7.7\times10^5$, $w_z = 1.7n_{//}$. The drift wave frequency for m = 6 mode is $w_R = 3.5\times10^5 s^{-1}$ and the growth rate $w_I = 1\times10^4 s^{-1}$. The measured radial number density profile can be approximated by the relation $n(r) = n_0 \Big/ \left(1 + r^2 / a^2\right)$, where a is constant depends on the magnetic field and pressure. The fitting curve for the measured electron temperature is also approximated as $T_e(r) = T_{e^o} \Big/ \left(1 + r^2 / c^2\right)$, where c is similar to, but smaller than a (a = 7mm, c = 4mm). In such circumstances, equation (4-50) becomes:

$$\frac{d^2 f_1(u)}{du^2} + \left[\frac{1}{u} - \frac{2u}{1+u^2}\right]\frac{df_1(u)}{du} + \left[Q(u,w) - \frac{m^2}{u^2}\right]f_1(u) = 0 \qquad (4\text{-}55)$$

where

$$u = \frac{r}{a} \tag{4-56}$$

and $Q(u,w)$ is the same as in equation (4-51), but r is replaced by u and $\frac{1}{r^2}$ by A, where A is:

$$A = \frac{a^2}{r^2} \tag{4-57}$$

and

$$r^2 = 4\times 10^{-8}\left(\frac{k_B T_{e^0}}{eB_0{}^2}\right) \tag{4-58}$$

To see how the variation of the electron temperature affects the drift-wave characteristics, we solved the radial wave equation (4-55) by ignoring the $\vec{E}\times\vec{B}$ rotation terms (i.e. the equation that obtained by Aebischer and Sayasov, 1988). To demonstrate that the radial temperature profile $T_e(r)$ influences the radial drift-wave amplitude distribution, the radial wave equation was also solved for a hypothetical plasma column which has the same density profile as the typical case, but a constant electron temperature in radius (i.e. the equation that obtained by Ellis et al. 1980).
In this process for the hypothetical plasma with T_e is constant (equation of Ellis) the eigen-frequency for m = 6 mode is $3.3\times 10^5 s^{-1}$ is purely real. For the typical plasma in which the radial variation of T_e is taken into account we find that the eigen-frequency for m = 6 mode is: $w_R = 4.3\times 10^5 s^{-1}$, $w_I = 0.86\times 10^4 s^{-1}$ i.e. $w_I > 0$ This proves the importance of the effect of the electron-temperature variation and the usefulness of the theory of Aebscher and Sayasov, (1988). The two eigen-functions for the typical and the hypothetical plasmas are shown in Fig. 4-5a, b. Clearly, the position of the maximum drift-wave amplitude for the typical plasma moves further towards the plasma edge than in the hypothetical plasma. The maximum wave amplitude is found at 2.39 for the hypothetical case and 2.63 for the real laboratory plasma.

In the present work the electric field is included by considering a sheared electric potential given by $f_0(r) = b_1 + b_2 r^2$, where b_1 and b_2 are constants determined by measuring the equilibrium potential at the boundary $r = 0, r = r_0$ [17]. This potential produces a sheared electric field $\vec{E}(r)$ and hence a non-sheared rotation frequency w_E. If $\vec{E} = 0$ then, $w_E = 0$ and the eigenfunction $f_1(r)$ and the eigenvalue w become as those obtained by Aebischer and Sayasov, (1988).

The numerical results of the present work are found for the same mode number m = 6 as follows: $w_R = 4.9 \times 10^5 s^{-1}$, $w_I = 1.1 \times 10^4 s^{-1}$ which is slightly different from the measured value given above. This indicates that the inclusion of the $\vec{E} \times \vec{B}$ rotation into the theory is important and has an effect on the drift wave frequency. In Fig. 4-5c we have plotted the eigen-function of our theory, the distribution of the radial wave shape from the solution with $\vec{E} \times \vec{B}$ rotation becomes more pronounced and the maximum amplitude is shifted further more to the edge of the plasma beam. The maximum wave amplitude is found at 2.9. This shift is attributed to the inclusion of both temperature gradient and $\vec{E} \times \vec{B}$ rotation in the theory. Fig. 4-6 shows the eigenfunction $f_1(u)$ for the three cases described above.

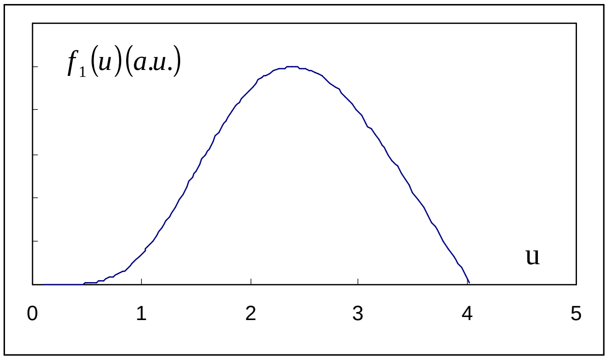

FIG. 4-5a THE EIGENFUNCTION $f_1(u)$ IN ARBITRARY UNITS (a.u.) FOR M = 6 MODE WITH NO $\vec{E}\times\vec{B}$ ROTATION; FOR HYPOTHETICAL PLASMA (WITH A CONSTANT RADIAL ELECTRON TEMPERATURE).

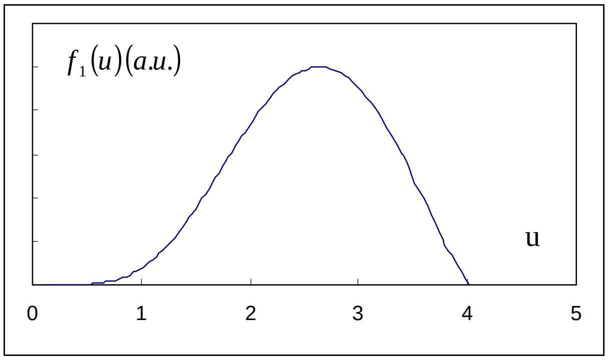

FIG. 4-5b THE EIGENFUNCTION $f_1(u)$ IN ARBITRARY UNITS (a.u.) FOR M = 6 MODE WITH NO $\vec{E}\times\vec{B}$ ROTATION; FOR REAL LABORATORY PLASMA (WITH THE ELECTRON TEMPERATURE VARIATION).

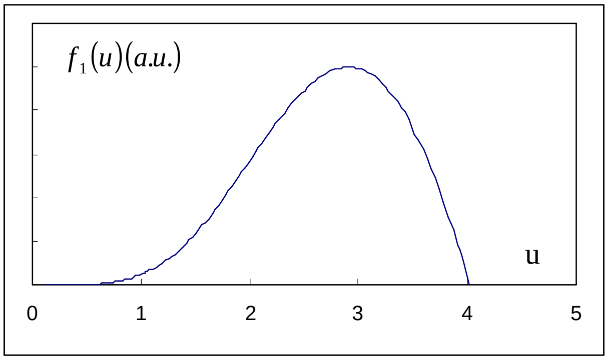

FIG. 4-5c THE EIGENFUNCTION $f_1(u)$ IN ARBITRARY UNITS (a.u.) FOR M = 6 MODE, WITH THE $\vec{E}\times\vec{B}$ ROTATION IN ADDITION TO THE RADIAL VARIATION OF THE ELECTRON TEMPERATURE.

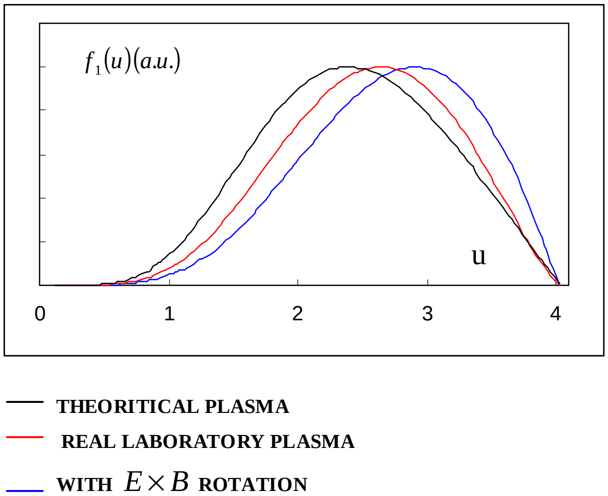

FIG. 4-6 THE EIGENFUNCTION $f_1(u)$ IN ARBITRARY UNITS (a.u.) FOR M = 6 MODE, FOR THE THREE CASES DESCRIBED ABOVE.

CONCLUSIONS

A theory has been formulated to describe drift-wave phenomena in cylindrical plasmas, based on the linearized two-fluid hydrodynamical equations, including the full non-viscous electron-energy equation in order to be able to take into account electron-temperature oscillations. This has led to an important modification of the previously published theory of drift waves in cylindrical geometry. In the present work, the distributions of the undisturbed electron density and temperature, the $\vec{E}\times\vec{B}$ rotation as well as the dependence of the charged-particle-neutral collision frequencies have also been considered. The theory has been presented in terms of an ordinary second-order differential equation for the radial distribution of the oscillating electric potential. This equation, together with the appropriate boundary conditions, define a complex-eigenvalue problem for the drift wave frequency and the growth rate of the considered drift-wave mode. Analytical expressions have been given that are fully consistent among themselves and relate all other oscillating physical quantities of the plasma to the oscillating electric potential. Thus the theoretical formulation presented in this work allows realistic calculations of any oscillating quantities related to drift waves in cylindrical plasmas to be performed simply and accurately. In addition, the radial oscillation profile of the wave amplitude, and the maximum wave position can be determined.

The major value of the present work lies in the detection of the drift wave and to identify the effect of both electron temperature gradient and $\vec{E}\times\vec{B}$ rotation on the drift wave frequency and the radial drift-wave structure. Although we were not able to find out a good fit of experimental data, we have compared our results with the experimental data of Egger et al. (1986). Our calculations have been carried out by two procedures. In the first procedure, we only consider the variation effect of the radial temperature. We find that the drift-wave frequency is sensitive to different electron temperature profiles. A comparison of the m = 6 eigenmode in the real laboratory plasma with that in the hypothetical plasma indicates that the temperature variation influences the radial wave structure. The second consideration was concerned with the radial electric field that usually accompanies the radial density and temperature gradients in a magnetized plasma. A radial electric field in combination with an axial magnetic field, produces an $\vec{E}\times\vec{B}$ rotation of plasma column, which typically in the direction of propagation of drift wave. This rotation is usually assumed to be uniform and generates a Doppler shift to the drift wave frequency. In addition we have noticed that the wave mode maximum position moves a little further from the beam center towards the beam edge. This is attributed to the inclusion of electron temperature gradient and $\vec{E}\times\vec{B}$ rotation into the theory. In the present work, the role of $\vec{E}\times\vec{B}$ rotation in the instability of the wave is investigated by including the radial potential in the two fluid model that used. The numerical results obtained for the drift-wave frequency, the growth rate and the radial distributions of the oscillating electric potential agree quite with the experimental results of Egger et al. (1986). The calculation also shows that the radial drift-wave structure is influenced by this rotation. This implies that the $\vec{E}\times\vec{B}$ rotation, like pressure gradient, can play a role in exciting the drift waves.

Finally as we mentioned above, our work was concentrated to show the influence of the two parameters: the electron temperature gradient and $\vec{E}\times\vec{B}$ rotation on drift waves, and the modification of the theory is required. We believe that a more measurement and theoretical study of drift wave behavior is needed and hopefully, this investigation can be easily discussed in the near future.

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